Impacts of Correlation between Baselines on the Adjustment of GPS Control Network

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Keywords: Baseline correlation, GPS control network, high precision, adjustment.

SUMMARY

In recent years, more and more engineering projects with a precision of millimeter level are needed. In order to achieve such high precision, Special observation scheme is usually carried out to setup the high precision network and the observation data is processed with rigorous adjustment method. But when taking adjustment, the correlation between GPS baselines is often ignored. In this paper, the GPS data process and adjustment algorithms for GPS control network are presented. Then the impacts of correlation between baselines on adjustment are analyzed. Finally two real data sets of high precision GPS engineering control networks are used to process and analyze. It has been demonstrated in these samples that ±7mm position error will be caused if the correlation among the baselines is ignored.

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1. INTRODUCTION

Global Positioning System (GPS) is widely used in national geodetic network, urban control network and other engineering control network. Special observation scheme is usually carried out to setup for high precision network and the observation data is processed with rigorous adjustment method. With regard to the urban control network and other engineering network which aim at achieving centimeter-level precision, data process is usually performed with commerce GPS receiver processing software. In this way, the correlation between GPS baseline vectors' components is considered while the correlation between GPS baselines is ignored, thus some observation information may be lost.

In recent years, more and more engineering projects with a precision of millimeter level are needed, such as urban GPS monitoring network, special bridge network, high-speed railway network. In order to achieve such high precision, some special observation schemes are used in data collection, such as the observation station with compulsive center, choke antenna, long lasting observation time between 6h and 12h or more

In this paper, the GPS data process and adjustment algorithms for GPS control network are presented. The adjustment models in the coordinate reference systems of WGS-84, national or local systems are given firstly. Then the impacts of correlation between baselines on adjustment are analyzed. Finally two real data sets from the small scale high precision GPS control networks are used to process and analyze. It has been demonstrated in these samples that ±7mm position error will be caused by ignoring the correlation among the baselines and the precise satellite orbit and long lasting observation time are usually needed in order to achieve millimeter or higher precision.

2. BASIC MODELS FOR GPS NETWORK ADJUSTMENT

GPS network adjustment with GPS vector observations can be performed in local reference system with constraint or only in WGS 84 system without constraint. Combined adjustment and non-constraint estimate are presented here.

Combined with local reference system, national reference frame or independent local reference frame, the observation model for each vector can be written as (Liu et al, 1996)

$$\begin{bmatrix} V_{Xij} \\ V_{Yij} \\ V_{Zij} \end{bmatrix} = \begin{bmatrix} \delta\hat{X}_j \\ \delta\hat{Y}_j \\ \delta\hat{Z}_j \end{bmatrix} - \begin{bmatrix} \delta\hat{X}_i \\ \delta\hat{Y}_i \\ \delta\hat{Z}_i \end{bmatrix} + \begin{bmatrix} \Delta X_{ij}^{\ 0} & 0 & \Delta Z_{ij}^{\ 0} & -\Delta X_{ij}^{\ 0} \\ \Delta Y_{ij}^{\ 0} & -\Delta Z_{ij}^{\ 0} & 0 & \Delta Y_{ij}^{\ 0} \\ \Delta Z_{ij}^{\ 0} & \Delta X_{ij}^{\ 0} & -\Delta Y_{ij}^{\ 0} & 0 \end{bmatrix} \begin{bmatrix} \delta\hat{\mu} \\ \hat{\varepsilon}_x \\ \hat{\varepsilon}_y \\ \hat{\varepsilon}_z \end{bmatrix} - \begin{bmatrix} \Delta X_{ij} - \Delta X_{ij}^{\ 0} \\ \Delta Y_{ij} - \Delta Y_{ij}^{\ 0} \\ \Delta Z_{ij} - \Delta Z_{ij}^{\ 0} \end{bmatrix} \tag{1}$$

where subscript i and j are indices for a baseline end points; $(\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij})^T$ is GPS vector in the WGS-84 reference frame; $(\Delta X_{ij}^0, \Delta Y_{ij}^0, \Delta Z_{ij}^0)^T$ is the approximate vector; $\delta\hat{\lambda} = [\delta\hat{\mu} \quad \hat{\varepsilon}_x \quad \hat{\varepsilon}_y \quad \hat{\varepsilon}_z]^T$ is fiducial scale parameter and fiducial azimuth parameters; $\delta\bar{X} = (\delta X, \delta Y, \delta Z)^T$ is coordinate parameter. The observation equation can be rewritten in matrix form

$$V = A\delta\hat{X} + B\delta\hat{\lambda} - L \tag{2}$$

In addition, the datum condition equation of GPS control network adjustment is expressed as the following datum equation

$$G_x^T \delta\hat{X} + G_\lambda^T \delta\hat{\lambda} = 0 \tag{3}$$

If the variance matrix of every GPS vector observation is D_{ii}, covariance matrix between GPS vectors is D_{ij} and unit weight variance is σ_0^2; so that the variance matrix D of whole GPS control network and weight matrix P are

$$D = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{22} & \cdots & D_{12} \\ \cdots & \cdots & \cdots & \cdots \\ D_{n1} & D_{n2} & \cdots & D_{nn} \end{bmatrix}, \qquad P = \sigma_0^2 D^{-1} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{12} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix}$$

If no correlations between different GPS vectors are considered, the variance matrix of whole GPS control network and weight matrix can also be written as

$$\tilde{D} = diag(D_{ii})$$

$$\tilde{P} = diag(\tilde{P}_{ii}) = diag(\sigma_0^2 D_{ii}^{-1})$$

If adjustment is performed only in WGS-84 reference frame, the datum condition equation is expressed as

$$\delta\hat{\lambda} = 0 \tag{4}$$

Then Eq. (2) can be rewritten

$$V = A\delta\hat{X} - L \tag{5}$$

The least square solution can be obtained

$$\begin{bmatrix} \delta\hat{X}_w \\ \delta\hat{\lambda}_w \end{bmatrix} = \begin{bmatrix} (A^T PA)^{-1} A^T PL \\ 0 \end{bmatrix} = \begin{bmatrix} N^{-1} A^T PL \\ 0 \end{bmatrix} \tag{6}$$

The co-factor and variance of $\delta\hat{X}_w$

$$Q_{\hat{X}w} = (A^T PA)^{-1} = N^{-1}$$

$$D_{\hat{X}w} = \hat{\sigma}_0^2 (A^T PA)^{-1} = \hat{\sigma}_0^2 N^{-1} \tag{7}$$

The unit weight variance

$$\hat{\sigma}_0^2 = \frac{1}{r} V^T PV \tag{8}$$

Combined with local reference system, the least square estimation with constraint can be obtained with Eq. (2) and Eq. (3)

$$\begin{bmatrix} \delta\hat{X}_c \\ \delta\hat{\lambda}_c \end{bmatrix} = \begin{bmatrix} A^T PA + G_1 G_x^T & A^T PB + G_1 G_\lambda^T \\ B^T PA + G_2 G_x^T & B^T PB + G_2 G_\lambda^T \end{bmatrix}^{-1} \begin{bmatrix} A^T PL \\ B^T PL \end{bmatrix} \tag{9}$$

Supposing that $G^T = \begin{bmatrix} G_1^T & G_2^T \end{bmatrix}$, then

$$\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = AG_1 + BG_2 = 0 \tag{10}$$

So that Eq. (9) can be expressed with the following equation

$$\begin{aligned} \begin{bmatrix} \delta\hat{X}_c \\ \delta\hat{\lambda}_c \end{bmatrix} &= \left\{ I - \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \left(\begin{bmatrix} G_x^T & G_\lambda^T \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \right)^{-1} \begin{bmatrix} G_x^T & G_\lambda^T \end{bmatrix} \right\} \begin{bmatrix} \delta\hat{X}_w \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} I - G_1 (G_1 G_x^T + G_2 G_\lambda^T)^{-1} G_x^T \\ -G_2 (G_1 G_x^T + G_2 G_\lambda^T)^{-1} G_x^T \end{bmatrix} \delta\hat{X}_w = \begin{bmatrix} H_x \\ H_\lambda \end{bmatrix} \delta\hat{X}_w \end{aligned} \tag{11}$$

The result of co-factors of $\delta\hat{X}_c$ and $\delta\hat{\lambda}_c$ are

$$\begin{aligned} Q_{\hat{X}c} &= H_x (A^T PA)^{-1} H_x^T \\ Q_{\hat{\lambda}c} &= H_\lambda (A^T PA)^{-1} H_\lambda^T \end{aligned} \tag{12}$$

3. THE INFLUENCE OF CORRELATION BETWEEN GPS VECTORS TO THE ADJUSTMENT RESULT

The influence of the correlation between GPS vectors to the adjustment results will be showed by comparing the result obtained with and without considering the correlation factors. The result difference for the non-constraint in WGS-84 $\delta\hat{X}_w$ will be given first.
If no correlations between vectors in GPS network adjustment are considered, then P will be replaced by $\tilde{P}$

$$\tilde{P} = P - \Delta P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} - \begin{bmatrix} 0 & P_{12} & \cdots & P_{1n} \\ P_{21} & 0 & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & 0 \end{bmatrix} \tag{13}$$

Then Eq. (6) and Eq. (7) can be written as

$$\begin{aligned} \delta\tilde{X}_w &= (A^T \tilde{P} A)^{-1} A^T \tilde{P} L = (N - \Delta N)^{-1} A^T \tilde{P} L \\ Q_{\tilde{X}w} &= (A^T \tilde{P} A)^{-1} = (N - \Delta N)^{-1} \end{aligned} \qquad (14\square$$

Where $\Delta N = A^T PA$

According to the progression decompose equation (Liu et al.1993)

$$(\lambda I - A)^{-1} = \lambda^{-1}(I + \lambda^{-1}A + \lambda^{-2}A^2 + \cdots)$$
$$(N - \Delta N)^{-1} = N^{-1} + N^{-1}\Delta N N^{-1} + (N^{-1}\Delta N)^2 N^{-1} + \ldots. \tag{15}$$

If higher degree parts are ignored, then Eq.(14) can be rewritten

$$\delta\tilde{X}_w = N^{-1}A^T PL + N^{-1}A^T \Delta PL + N^{-1}\Delta N N^{-1}A^T PL + N^{-1}\Delta N N^{-1}A^T \Delta PL$$
$$Q_{\tilde{X}w} = N^{-1} + N^{-1}\Delta N N^{-1} \tag{16}$$

Compared with Eq. (6) and Eq. (7), the estimate difference in WGS84 due to correlation can be found

$$\Delta\tilde{X}_w = \delta\tilde{X}_w - \delta\hat{X}_w = N^{-1}A^T \Delta PL + N^{-1}\Delta N N^{-1}A^T PL + N^{-1}\Delta N N^{-1}A^T \Delta PL$$
$$\Delta Q_{\tilde{X}w} = Q_{\tilde{X}w} - Q_{\hat{X}w} = N^{-1}\Delta N N^{-1} \tag{17}$$

In the same way, the influence to V and $\hat{\sigma}_0^2$ will also be gotten. Replacing P with $\tilde{P}$, the influence of the correlation to V can be gotten

$$\tilde{V} = A\delta\tilde{X}_w - L = (A\delta\hat{X}_w - L) + A\Delta\tilde{X}_w$$
$$\Delta\tilde{V} = \tilde{V} - V = A\Delta\tilde{X}_w \tag{18}$$

and $\Delta\tilde{\sigma}_0^2$ is the influence of the correlation of GPS baselines to $\hat{\sigma}_0^2$.

$$\tilde{V}^T\tilde{P}\tilde{V} = \tilde{V}^T\tilde{P}L = V^T\tilde{P}L + \Delta\tilde{V}^T\tilde{P}L$$
$$= V^T PV - V^T\Delta PL + \Delta\tilde{V}^T\tilde{P}L$$
$$\Delta\tilde{\sigma}_0^2 = \tilde{\sigma}_0^2 - \hat{\sigma}_0^2 = \frac{1}{r}(-V^T\Delta PL + \Delta\tilde{V}^T\tilde{P}L) \tag{19}$$

Similarly, as to combined adjustment in local reference system with constraints, the result difference can also be obtained .

Eq. (11) can be rewritten

$$\begin{bmatrix} \delta\tilde{X}_c \\ \delta\tilde{\lambda}_c \end{bmatrix} = \begin{bmatrix} H_x \\ H_\lambda \end{bmatrix}\delta\tilde{X}_w = \begin{bmatrix} H_x \\ H_\lambda \end{bmatrix}(\delta\hat{X}_w + \Delta\tilde{X}_w) \tag{20}$$

Replacing P with $\tilde{P}$, the influence value to $\delta\hat{X}_c$ and $\delta\hat{\lambda}_c$ without considering the correlation can be written as

$$\begin{bmatrix} \Delta\tilde{X}_c \\ \Delta\tilde{\lambda}_c \end{bmatrix} = \begin{bmatrix} \delta\tilde{X}_c \\ \delta\tilde{\lambda}_c \end{bmatrix} - \begin{bmatrix} \delta\hat{X}_c \\ \delta\hat{\lambda}_c \end{bmatrix} = \begin{bmatrix} H_x \\ H_\lambda \end{bmatrix}\Delta\tilde{X}_w \tag{21}$$

And the influence to co-factor is

$$\Delta Q_{\tilde{X}c} = Q_{\tilde{X}c} - Q_{\hat{X}c} = H_x(N^{-1}\Delta N N^{-1})H_x^T$$
$$\Delta Q_{\tilde{\lambda}c} = Q_{\tilde{X}\lambda} - Q_{\hat{X}\lambda} = H_\lambda(N^{-1}\Delta N N^{-1})H_\lambda^T \tag{22}$$

4. DATA PROCESSING AND RESULTS

To evaluate the influence of the correlation between GPS baselines in high accuracy engineering GPS network, the result differences caused by correlation are calculated by the algorithms presented in this paper with two GPS control network data.

The first data sample is a GPS control network for an extra bridge where there are 15 control points and 11 long observation baselines. The observation time is 6 hours for the longer baselines and 2 hours for the shorter ones (Figure 1). The GPS baselines are processed by a high accuracy software and the adjustment is performed with GPS adjustment software TGPPSW2003 developed by Tongji University. The adjustment is carried out in WGS-84 reference frame and local BJ- 54 reference frame respectively with and without considering correlation, P or $\widetilde{P}$. There is no significant differences for the adjusted results in BJ- 54 reference frame and in WGS-84 reference frame.

The results differences for horizontal coordinate components with and without correlation in WGS-84 reference frame are showed in figure 3. It can be seen that the differences are within ±7mm and most of them are within ±2mm. For millimeter or higher accuracy requirement of engineering control network, the difference has to be considered carefully.

The influence of the correlation of GPS baselines to the transformation parameter and unit weight error are listed table 1. Ignoring the correlation of GPS baselines makes the numerical value of unit weight error smaller. The sign of correlation coefficient has also been tested to see the different influence on the result and it has been found that when the sign of all correlation coefficients are positive or negative, its influence to transformation parameter and unit weight error become greatly obvious.

The second experiment is a GPS control network for a high-speed railway. There are 6 control points and some of the baselines are longer than 100km (Figure 1). The observation time is 12 hours.

The data processing strategy adopted is the same as those in the first experiment. The results difference with and without considering correlation in WGS-84 reference frame are showed in table 2. It can be seen that the difference due to correlation are within ±5mm both for 3D and 2D coordinate components. Most of them are within ±3mm. It has been further proved that the impacts on estimates by baselines correlation are in the range of several millimeters.

5. CONCLUSION

Ignoring the correlation of GPS baselines in adjustment of GPS control network will cause about ±7mm coordinate component biases and it also has a certain influence on the transformation parameters and unit weight variance.

It is tolerable to ignore the correlation of GPS baselines for centimeter precision control network. As to the higher accuracy engineering control network, the influence of the correlation of GPS baselines must be considered in adjustment for GPS control network in addition to taking a series of measures in field work and careful baseline vector processing.
To ensure the reliable GPS vector with suitable GPS data processing software, longer observation is needed in fieldwork.

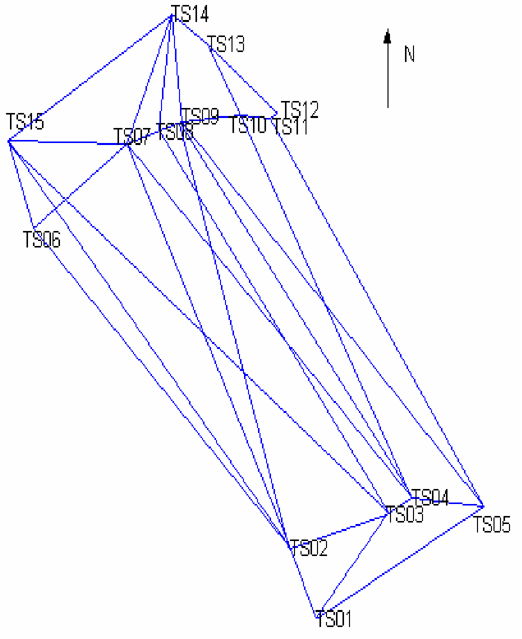

Fig.1: A GPS control network for bridge

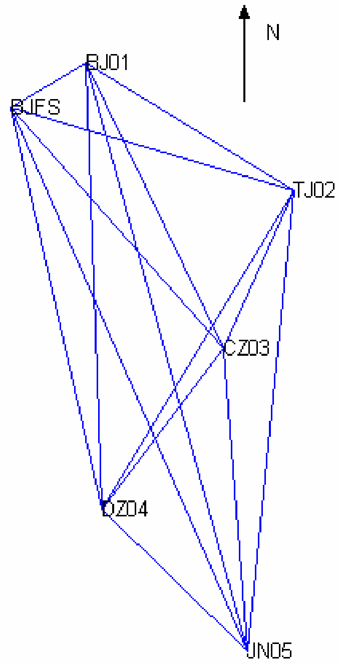

Fig.2: A GPS control network for high-speed railway

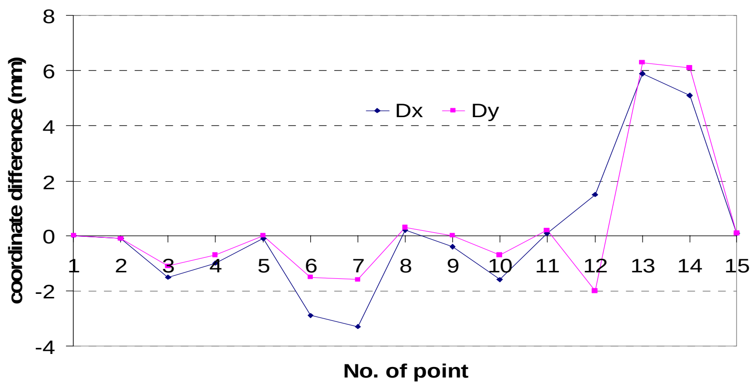

Fig.3: The coordinate difference caused by correlation

Tab.1: The influence of the correlation on the transformation parameter and unit weight error

	Unit weight (m)	Scale parameter (ppm)		Rotation parameter (sec)			
	σ	μ	δμ	εξ	εη	εA	δεA
With correlation	0.00807	0.923	0.002	0.000	0.000	1.556	0.002
No correlation	0.00723	0.985	0.009	0.000	0.000	1.546	0.009

Tab.2: The coordinate difference caused by correlation in the high-speed railway GPS control network (mm)

Point No.		1	2	3	4	5	6
3D coordinate	□X	0	-2.5	-0.8	-2.3	-4.4	-0.9
	□Y	0	-1.9	-1.4	-1.1	-0.7	-0.7
	□Z	0	-1.2	-1.1	-0.7	1.0	-1.0
2D coordinate	□x	0	-0.5	-0.3	-0.6	0.0	-0.7
	□y	0	3.0	1.3	2.6	4.2	4.2

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BIOGRAPHICAL NOTES

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Professor Dajie Liu, was the former head of Department of Surveying and Geo-Informatics, Tongji University. He got his Bsc. from Wuhan Technical University of Surveying and Mapping, and had worked there as a professor till 1992. He was specialized in data processing theory and application in traditional surveying adjustment, GPS data processing and quality control for GIS data.

Dr. Congwei Hu, is an associate professor in Department of Surveying, Tongji University. Curently he works as a research associate in the Dept. Of LSGI, the Hong Kong Polytiechnic University. He received his B.Eng. and M.Eng. from Tongji University in 1989 and 1992, and PhD. from Institute of Geodesy and Geophysics, Chinese Academy of Science in 2000. He is specialized in GPS data processing.

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