A Precise Geoidal Map of the Southern Part of Egypt by Collocation Toshka Geoid

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Key words: geoid, collocation, tailored models, data validation

SUMMARY

A gravimetric geoid was predicted for the newly developed Toshka sector in South Egypt. The Least-Squares collocation solution utilized scattered heterogeneous data types as input, using the remove-Restore technique. The input data included gravity anomalies, gravity disturbances and vertical deflection components, while the available GPS/Lev. geoidal height data were devoted for the evaluation of the gravimetric geoid accuracy. The dominant data type was the gravity disturbances, and hence, this data was used to predict the empirical covariance function. The RTM topographic effect was accounted for, using an appropriate DTM for the area under investigation. The long wavelength contribution was properly accounted for, using the locally fitted geopotential model EGM96EGCT. The results show that the mean collocation standard error is about 5 cm, while the comparison at the independent GPS/Lev. check points gives an external accuracy of about 16 cm. Based on the obtained results, the obtained Toshka geoid is recommended to be used for any future geodetic computations in that area. It is also recommended to be used for densifying leveling networks of lower order. Consequently, appropriate future planning for rigorous GPS measurements in these areas is highly recommended.

الملخص:

قي هذا البحث، تم حساب نموذج لسطح الجيوئيد في منطقة توشكي بجنوب مصر، باستخدام البيانات غير المتجانسة لمجال الجاذبية الأرضية، والمتاحة بالنسبة لتلك المنطقة. وقد تم استخدام طريقة (LSC) لنمذجة الجيوئيد بأسلوب الحذف- الحساب-الإضافة. البيانات المدخلة تضمنت بيانات شذوذ الجاذبية، واضطراب الجاذبية، ومركبتي حيود الرأسي عن العمودي في اتجاهي الزوال والرأسي الأول، بينما تم تخصيص قيم الجيوئد المحسوبة عند نقط أرصاد (GPS) معلومة المناسيب لحساب دقة الجيوئيد. هذا، وقد تم استخدام بيانات اضطراب الجاذبية في حساب دالة التباين الوضعية. وقد تم أخذ تأثير طوبوغرافية الأرض في الاعتبار (باستخدام طريقة RTM)، بينم ا استخدم النموذج التوافقي المحسن محلياً (EGM96EGCT)، في تمثيل المعالم الأساسية ذات التحليل المحدود لمجال الجاذبية بكفاءة.

وقد أظهرت النتائج أن متوسط الخطأ المعياري للجيوئيد و الناتج من طريقة (LSC) مقداره 5 سم و قد أظهرت المقارنة عند نقط أرصاد (GPS) خطأ معيارياً مقداره 16 سم للفروق بين قيم الجوئيد المرصودة والمحسوبة. وبناءاً على النتائج التي أظهرها البحث، يوصى باستخدام نموذج الجيوئيد المحسوب في التطبيقات الجيوديسية المستقبلية بتلك المنطقة. كما يوصى باستخدام نموذج الجيوئيد الناتج في تكثيف شبكات الميزانية ذات الدقة المنخفضة. وبذا، يجب التخطيط لعمل أرصاد(GPS) دقيقة لتلك المنطقة.

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1 INTRODUCTION

Recently, the virgin desert area of the southern part of Egypt have seen the beginning of a revolutionary era of agricultural and industrial development, since the Egyptian government have decided to begin one of its national mega projects for development in that area, named Toshka project. This led to the need for accurate maps of different kinds for this long time neglected area. The accuracy required for such maps necessitates, in turn, the knowledge of the accurate and precise geoid at the same area. This is why we have decided to make this study, especially after the relevant data of that area have been available to us. It should be noted that the region under study may be considered as composed of two areas, which were separately investigated by (Abd-Elmotaal, 1998 and Tscherning et. al., 2001). However, in the current study, a locally fitted geopotential model is used as a reference field, besides, a DEM is utilized for smoothing the available data.

The common defects of all classical techniques for geoid determination are the inadequate and inhomogeneous distribution of the used geodetic data. The Least-Squares Collocation (LSC) method, however, is the only which is capable of combining systematically all kinds of data (Moritz, 1973; 1980 and Tscherning, 1982; 1987), in one unique solution.

The solution by the LSC requires a necessary precondition, that the field data and hence the covariance function, representing the same field, to be of purely random nature (Moritz, 1973; 1978), which implies that the used data, during the prediction stage, should be of maximum smoothness. This smoothness can be achieved by removing all the computably possible systematic parts of the used data that depend on global or local information. In the current study, this have been done practically by extracting the global trend effect of a tailored global geopotential harmonic model, to the considered area, from all the local gravitational field elements, then subtracting the effect of the local topography for the same reason, by using a suitable digital elevation model (DEM), (Tscherning, 1982a & b).

After the removal of both effects, the remainders (signals) of the field data are referred to as residual field elements, from which the gravity disturbance signals were depicted and used for computing the isotropic empirical covariance function that represents the local gravitational field in the investigated area. This function was then utilized to determine the necessary (best fitted) model (or analytical) covariance function, required to perform the computations through the LSC method. The signals (residual data) remained at all the data points together with the obtained analytical covariance function were then used in the prediction process, utilizing the law of variance-covariance propagation, via the LSC method. Afterwards, the effects of both removed parts were added to the predicted signals, at the computation points each 5'x5', to get the final geoid values. The operation of subtracting and adding the effect of the systematic parts of the data, before and after the prediction process, is usually known in literatures as the Remove-Restore technique The final results were then used to produce two contour maps, representing the geoid and its error standard deviations, for the whole studded area.

2 DATA TYPES, REDUCTION, FILTRATION AND SMOOTHING

2.1 Data types, and their reduction to a unified datum

To achieve accuracy in geoid computation at the decimeter level, any datum inconsistencies inherent in the used data should be eliminated in advance. In the current study, the available data, include free air gravity anomalies, gravity disturbance, geoidal heights, computed at GPS stations combined with elevations (from spirit leveling) and the deflection of the vertical components in both the meridian and prime-vertical directions at some stations of the first order triangulation network. Since these data are referred to different datum's, they were reduced to one and the same datum, by using several sets of transformation parameters each of these sets was utilized to reduce the position of a specific collection of data related to certain datum to the unified geocentric datum (WGS84). The transformation of the horizontal coordinates of any data point given on certain datum into the geocentric datum WGS84 was simply performed, using the orthometric heights as an approximation to the relevant ellipsoidal heights, then applying the specific set of transformation parameters in each case (Bolbol, et al., 1997, El-Tokhey, 2000). This approximation showed to have a neglected effect on the transformed horizontal positions, as was previously concluded by (Abd-Elmotaal and El-Tokhey, 1997).

2.2 Data filtration through a validation process

The reduced rough data were then filtered, by rejecting the data part that was found subjected to gross errors. This step is very important to insure that the input data has the optimum quality, which is essential for reliable predicted features. For this respect, the whole heterogeneous data were subjected to a validation process that aimed to filter out any data element, which lack a minimum level of guarantee and reliability. This was done by using the reduced set of heterogeneous data in a simultaneous collocation prediction to estimate (predict) respective signals at the same data points. This has the effect of data filtration from noise and in this case, there would be nearly no effect from field roughness or data gaps (Tscherning, 1982 and Sevilla et al., 1990). Then, one would reject a specific observation having gross errors, if the following condition was satisfied:

$$| S_{observed} - S_{predicted} | > 3.0 \, (\sigma^2_{observed} + \sigma^2_{predicted})^{1/2} \qquad (1)$$

In which the right-hand side is the Gaussian sum of the a-priori input noise and posteriori error estimate. The same condition was also useful in filtering out very closely spaced stations, which would deteriorate the LSC solution. The remaining finally filtered data, except that of the geoidal height observations, contributed then to the collocation process.

The filtered geoidal height data was merely used as a tool of comparison for checking the accuracy of the final results.

The number of the available data points of each type in the investigated area and the number of the filtered ones are listed in Table (1), where their distribution is shown in Figure (1).

Table (1): The raw and filtered data numbers and types

Item	Total data No.	Filtered data No.
Geoidal heights (GPS-Lev.)	35	29
Gravity anomalies	108	83
Gravity disturbances	313	231
η (prime-vertical deflections)	2	2
ξ (meridian deflections)	30	26

2.3 Data smoothing (residual field elements)

2.3.1 The remove-restore technique

Any observable element (X) of the gravitational field can be consider as composed of three parts as follows:

$$X = x_1 + x_2 + x_3 , \tag{2}$$

where x_1, x_2 and x_3, are the contributions of the long, medium and short wavelength signals of the total spectrum of the gravity field element, respectively. The long wavelength signals x_1 can be represented by a finite series of spherical harmonics using a global geopotential model (GGM) coefficients up to degree and order n_{max}. The short wavelength signals x_3 can be recovered from terrain heights in terms of a digital elevation (terrain) model, DEM (DTM). Practically, its value can be obtained most conveniently via the residual terrain model (RTM), i.e. the deviation of topography from a mean height surface (reference surface), including 15′ x 15′ or 30′ x 30′ mean heights. The RTM reduction has the advantages that the height data is required only for a limited area, and that the indirect effect on the geoid is rather small.

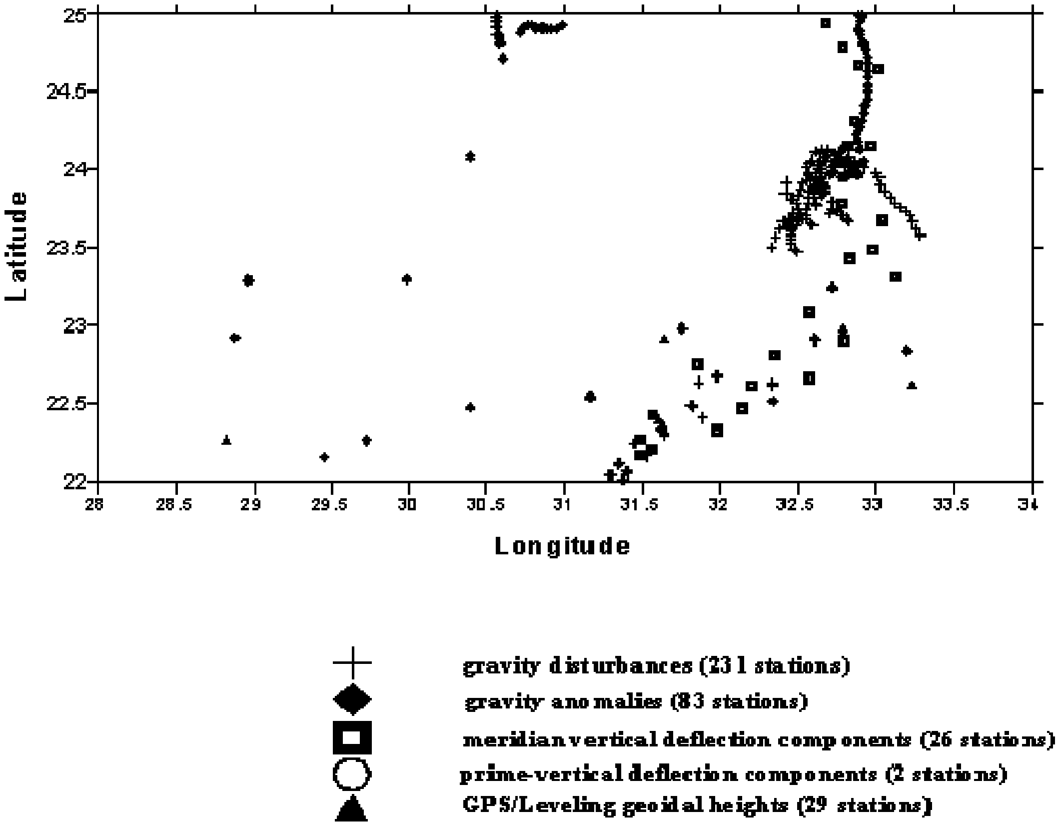

Figure (1): Heterogeneous data distribution in the considered area

Finally, the medium wavelength signals x_2 at the observation points can be obtained after arranging Eq. (2) in terms of the observed and the computed contributions of each of the long and short wavelengths as follows:

$$x_2 = X - x_1 - x_3, \tag{3}$$

x_2 is called the residual anomalous element, and belongs to the finally smoothed gravity field. at the computation points. Its value can be determined by the least-squares collocation (LSC) method, where the effects of the removed parts are restored afterwards to obtain the complete spectrum value of the required element. Thus, the remove-restore technique has a preprocessing and post processing stages. The preprocessing stage involves the computation and removal of the GGM and terrain contributions from all the elements of the field data, and the post processing step involves the restoration of the GGM contribution x_1 and the terrain contribution x_3 , via the indirect effect term, to the computed (predicted) element x_2 at the computation points, thus, obtaining the final result (full spectrum of the required element).

2.3.2 The choice of the best geopotential model for the investigated area

The subtraction of a suitable GGM from the local data results in a primary residual smoothed field that is essential for accurate and precise gravity field signal prediction by collocation. Thus the problem of choosing a geopotential model that matches the gravity field well, in local or regional areas, is of ultimate importance, because the removal of the long wavelength (low frequency) part, and restoration would be reliable and meaningful, only in case the used model contains local gravity information from the region under investigation. In Egypt, however, since the geodetic and gravimetric data are considered classified, none of these data have been used in the process of computing any of those models. Therefore, all those models fail in recovering optimally the long wavelength part of the total spectrum of the anomalous gravity field (El-Tokhey, 1995; Amin, 2002 and Hassouna, 2003), thus degrading the target precision of the local geoid solutions (Smith, 1998).

Several studies have shown that the geopotential models tailored to regional or local gravity data are best suited for high precision geoid computation (e.g. Wenzel, 1998).

In the current investigation, the above conclusion have been followed, where, we have used a tailored model, denoted as EGM96EGCT, generated by the authors, using the LSC method, and estimated up to degree and order 599, based on the coefficients of the global model EGM96 and all the available terrestrial geodetic data in Egypt. The reliability of this tailored model to recover the long wavelength features of the Egyptian gravity field in an efficient manner has been verified (Amin et al., 2003b). The respective low degree contribution of any of such global models, in spherical approximation, is related to the various gravimetric quantities (Lachapelle and Tscherning, 1978) as follows:

$$N_{GM} = (GM/r\gamma) \sum_{n=2}^{Nmax} (a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\psi) \tag{4}$$

$$\Delta g_{GM} = (GM/r^2) \sum_{n=2}^{Nmax} (n-1)\,(a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\psi) \tag{5}$$

$$\delta g_{GM} = (GM/r^2) \sum_{n=2}^{Nmax} (n+1)\,(a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\psi) \tag{6}$$

$$\xi_{GM} = -\,(GM/r^2\gamma) \sum_{n=2}^{Nmax} (a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\; \bar{P}_{nm}(\sin\psi)/d\psi \tag{7}$$

$$\eta_{GM} = -\,(GM/r^2\gamma\cos\psi) \sum_{n=2}^{Nmax} (a/r)^n \sum_{m=0}^{n} m(\bar{C}^*_{nm} (-\sin m\lambda) + \bar{S}_{nm} \cos m\lambda)\bar{P}_{nm}(\sin\psi) \tag{8}$$

with the zero and first degree terms taken equal to zero, and

N, Δg, δg, ξ and η are the relevant geoidal height, gravity anomaly, gravity disturbance, and the deflection of the vertical components in the meridian and prime vertical directions, respectively,

ψ the geocentric latitude,
λ the geodetic longitude,
r the geocentric radius to the geoid,
$\gamma(\psi,r)$ the normal gravity implied by the reference ellipsoid,
GM the Earth mass gravitational constant,
a the equatorial radius,

$\bar{C}^{*}_{nm}$ the fully normalized spherical harmonic C-coefficients of degree n and order m, reduced for the even zonal harmonics of the reference ellipsoid,

$\bar{S}_{nm}$ the fully normalized spherical harmonic S-coefficients of degree n and order m,

$\bar{P}_{nm}(\sin\psi)$ the fully normalized associated Legendre function of degree n and order m,

Note that, the used reference ellipsoid (WGS84, in our case) is assumed to have a mass equal to that of the Earth; and its center coincide with the geo-center, which means that the zero and first-degree terms are inadmissible (Heiskanen and Moritz, 1967, sect. 2.6). That is why, the spectral series in Eq. (4) through (8), begin from n=2.

2.3.3 The removal of the high frequency features of the gravity field in the investigated area

In a previous work by the authors (Amin et. al, 2003a), a detailed 5'x5' digital elevation model DEM for Egypt was computed by collocation, based on the available local height data and the global high-resolution topographic harmonic model GTM3a. This DEM model was utilized in removing the high frequency features of the gravity field in Egypt which also play an important role in the smoothing strategy of the data elements, where its contribution, related for example, to the gravity anomalies and the geoidal heights quantities were computed from the following expressions:

$$\Delta g_{DEM} = 2\pi G\rho(h-h_{ref}) - T_c$$

$$= 2\pi G\rho(h-h_{ref}) - (G\rho R^2 /2) (\Delta\varphi.\Delta\lambda)\Sigma((h'-h)^2 / l^3)) \quad (9)$$

$$N_{DEM} = - (2\pi G\rho/\gamma)(h-h_{ref})^2 - (G\rho R^2/6 \gamma))(\Delta\varphi.\Delta\lambda) \Sigma((h'^3-h^3) / l^3) \quad (10)$$

where Tc is the classical terrain correction with respect to the Bouguer plate, and the first term is the Bouguer plate effect on the anomaly or geoid data, and

h is the orthometric height of the computation point,
h' is the orthometric height of the running point,
G is the gravitational constant,
ρ is the mean crustal density,
h_{ref} is the relevant elevation of the average surface
l is the spatial distance between the computation point and the running point.

Table (2) through (5) show the statistics of the input residual gravity anomaly data, gravity disturbances, meridian and prime-vertical deflection components, respectively, based on both the tailored model EGM96EGCT and the DEM. These Tables show that the removal of the global and the local information had great smoothing affect on most of the raw gravimetric data, in terms of the mean, the standard deviation and root mean square value. It is also clear that the smoothing effect of the DEM is rather small than that of the used geopotential harmonic model.

Table (2): Statistics of original and residual gravity anomaly data (unit: mgals)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Free air gravity anomaly	4.965	17.857	18.431	-25.688	69.005
RTM reduced gravity anomaly	11.244	19.136	22.095	-20.911	75.002
Final (RTM + EGM96EGCT) residual gravity anomaly	4.092	16.591	16.991	-24.632	51.277

Table (3): Statistics of original and residual gravity disturbance data (unit: mgals)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Free air gravity disturbance	-2.159	8.182	8.445	-23.389	18.032
RTM reduced gravity disturbance	0.597	8.434	8.437	-19.081	22.584
Final (RTM + EGM96EGCT) residual gravity disturbance	-0.815	6.775	6.810	-19.021	17.858

Table (4): Statistics of original and residual meridian deflection component data (ξ) (unit: arc-seconds)

Item	Mean	Std. Dev.	RMS	Min.	Max.
ξ	-0.789	2.351	2.437	-7.021	3.380
RTM reduced ξ	-0.924	2.206	2.353	-6.637	2.588
Final (RTM + EGM96EGCT) residual ξ	-0.243	1.785	1.767	-3.660	3.346

Table (5): Statistics of original and residual prime-vertical deflection component data (η) (unit: arc-seconds)

Item	Mean	Std. Dev.	RMS	Min.	Max.
H	-5.527	1.527	5.632	-6.607	4.448
RTM reduced η	-5.465	1.575	5.577	-6.578	-4.351
Final (RTM + EGM96EGCT) residual η	-1.585	1.656	1.971	-2.756	-0.414

3 COMPUTATION TECHNIQUE, PROCEDURES AND RESULTS

The great advantage of LSC technique is its ability to utilize different data types, and their errors, as input. For example, the vector x_2 can contain as input (residual observations) any combination of N, Δ g, δ g, and of the vertical deflection components ξ , η . In the same time, it allows the data to be treated in any spatial distribution. It is not necessary for the data to be girded or for distribution to be continuous (gaps are tolerated), in the same time the estimated signals could be obtained at any optional point, distributed regularly or otherwise. Finally this technique produces also error estimates for the predicted quantities, which may be any single element or a combination of elements of the same set of observations.

Knowing the above-mentioned merits of the LSC algorithm, it was chosen in the current investigation, to be the routine technique used for modeling the local geoid.

3.1 Covariance function

To obtain the best LSC approximation to the true potential field in a certain area; an empirical covariance function should be determined and used in the computation. However, the practical solution by the LSC presupposes that the gravity field, and hence the relevant covariance functions, should be homogeneous and isotropic, i.e. location and azimuth independent, which implies that the used data should be as smooth as possible, so that they behave purely random. That is why; the above-mentioned removal steps have been conducted.

3.1.1 Computation of the local covariance function empirically

After the removal steps, it remains signals of the residual field data. One element of those signals, usually, the residual gravity anomaly, is depicted and used to obtain an isotropic empirical covariance function that represents the statistical characteristics of the local field.

As this function merely depends on the separation between the data points, it describes the spatial variability of the local residual field under consideration. The main features of this function are the variance (covariance at zero distance), the radius of curvature of the covariance function curve at the same point and the correlation length, which corresponds to a positive covariance value that is equal to half the variance value. Since the dominant data type used in this particular work was the gravity disturbance, the computed residual parts δg_r

of this element were therefore depicted and utilized to determine this function. Practically, this have been done by evaluating the product sum average of pairs of gravity disturbance signals, relevant to point pairs having spacing $(\psi-\Delta\psi)/2 \leq \psi \leq (\psi+\Delta\psi)/2$. Both $\Delta\psi$ and the ψ increments were chosen to be 2 minutes of arc and 100 covariance values (at 100 ψ values) were evaluated. The calculation of this function was done using the FORTRAN program EMPCOV written by C.C.Tscherning.

3.1.2 Computation of the model covariance function that fits the local area

In order to perform the different steps of calculation by collocation, we must have a covariance function model that represents the local area well. This is usually achieved by fitting the estimated values of the empirical covariance function to a model function in a none-linear iterative least-squares adjustment with three parameters. In this respect, we have used the well-known formula of the anomalous potential covariance function model that reads (Tscherning, 1993),

$$C(P,Q) = C(r, r', \psi)$$

$$= \sum_{n=2}^{N\max} c.\sigma^2_{neT.model} \left(\frac{R_b^2}{rr'}\right)^{n+1} P_n(\cos\psi) \quad +$$

$$+ \sum_{n=N\max+1}^{\infty} \frac{A}{(n-1).(n-2).(n+4)} \left(\frac{R_b^2}{rr'}\right)^{n+1} P_n(\cos\psi) \qquad (11)$$

where

ψ	the spherical distance between the two points P and Q,
r	the geocentric radial distance of point P $\approx R+H_P$,
r'	the geocentric radial distance of point Q $\approx R+H_Q$,
R	the mean radius of the Earth, taken $\approx$ 6371 km,
R_b	the radius of the Bjerhammar's sphere,
$\sigma^2_{neT.model}$	the n^{th} potential error degree variance based on the tailored model (EGM96EGCT) coefficients' standard error,
c	a positive unitless scale factor,
A	a positive constant ($mgal^2$),
N_{max}	599 (max degree of the tailored model EGM96EGCT),
H	orthometric height of the respective point,
$P_n(\cos\psi)$	are the Legendre polynomials.

The three parameters c, A and (R_b-R) are given firstly approximate values. The adjustment is then performed in an iterative manner until the convergence is arrived, resulting in the final three parameters: the unitless scale factor c, the constant A in $mgal^2$, and (R_B - R) in meters.

In the current study, the fitting have been done through the FORTRAN program COVFIT, written by P. Knudsen, which gives:

$C = 0.001037$
$A = 15.627\ \text{mgal}^2$
$R_B - R = -249.682\ \text{m}$

Figure (2) illustrates the input residual gravity disturbance isotropic empirical covariance function, and its associated fitted analytical function. The obtained final values were then used as input for the collocation process. In the LSC solution stage, the law of covariance propagation was executed to account for all possible varieties of auto-and cross-covariances related to the observed functions during the solution (Tscherning, 1974).

3.1 Least-squares collocation procedures and computations

The adoption of the LSC technique for geoid determination requires the solution of a set of linear equations with dimension equal to the number of observations (Moritz, 1978). The vector of the residual geoid undulation N_r is computed from the vector of observations X, from which the GGM and DTM contributions have been computed and removed i.e., from the vector x_2, of the residual data (along with their noise), by the expression

$$N_r = C_{Nx} (C_{xx} + C_{nn})^{-1} x_2, \tag{12}$$

where C_{xx} and C_{Nx} are the auto- and cross- covariance matrices between the observations and the predicted geoid, whose values are based on the computed parameters of the selected analytical model approximating the empirical covariance function. C_{nn} is the error (noise) covariance matrix of the observations. The observational errors propagate into the results, yielding the following error covariance matrix of the estimated geoid undulations;

$$C_{ee} = C_{NN} - C_{Nx} (C_{xx} + C_{nn})^{-1} C_{xN}, \tag{13}$$

where C_{NN} is the auto-covariance matrix of N.

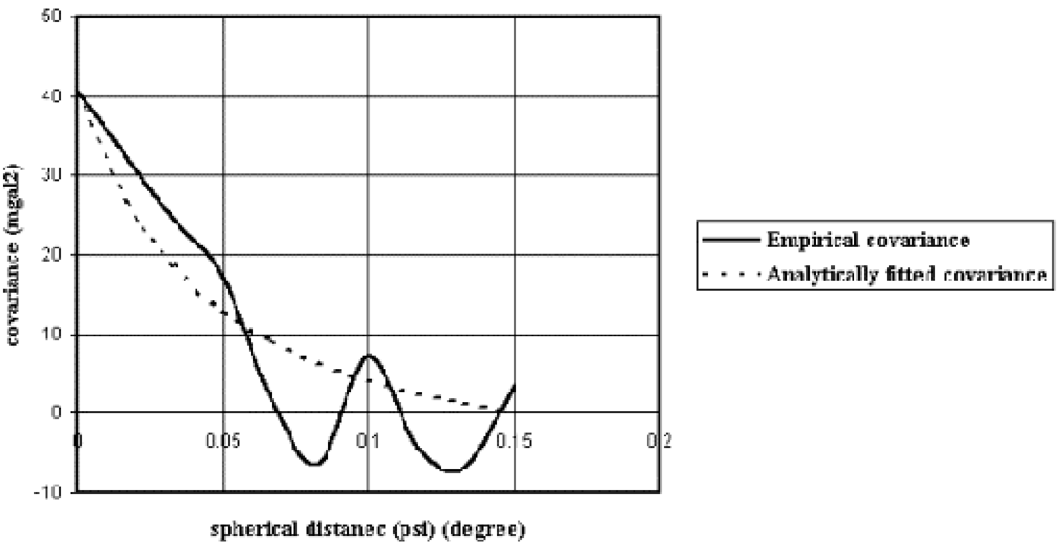

Figure (2): Residual gravity disturbance empirical and analytically fitted covariance functions

4 FINAL RESULTS, CONCLUSIONS AND RECOMMENDATIONS

The contributions of both the tailored harmonic model Eq.(4), and DEM in terms of the indirect effect of topography Eq.(10), were then added back (restored) to the residual geoid values predicted at the 5'x5' grid nodes, in order to obtain the respective 5'x5' full spectrum geoid values as well as their error estimates. Table (6) shows the statistics of the predicted geoid.

In order to estimate the accuracy of the geoid solution, discrete geoid values were predicted at the available check points, where geoidal heights are known in terms of GPS and leveling observations. Table (7) shows the resulting accuracy, as a function of the statistics of the differences between the observed and predicted geoid values at those check points, which have not been used as data points in the solution. Finally, Figures (3) and (4) show the contour maps of the geoid and its error standard deviations, respectively.

Table (6): Statistics of the residual and full predicted 5'x5' Toshka geoid
(Unit: meters)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Residual N	0.038	0.059	0.071	-0.074	0.390
Final N	11.036	1.819	11.185	7.489	16.641
Std. error	0.046	0.003	0.046	0.019	0.047

Table (7): Statistics of the differences among the observed and predicted geoidal heights at GPS/Leveling check stations
(Unit: meter)

Item	Mean	Std. Dev.	RMS	Min.	Max.
N(obs.) – N(pred.)	0.017	0.161	0.159	-0.355	0.380

Based on the results of the determined geoid, as outlined by Tables (6) and (7) and shown by Figures (3) and (4), it is clear that the accuracy obtained by the LSC technique in the considered area, in terms of the standard deviation of the differences that amounts to 0.16 m, is considered rather satisfactory.

Therefore, the obtained Toshka geoid is highly recommended to be used for any future rigorous geodetic computation in the newly developed sector of the southern part of Egypt. It is also recommended to be utilized for densifying leveling networks of lower order through rigorous GPS observations, especially in remote areas of the western or eastern desert of that part of Egypt. Consequently, appropriate future planning for rigorous GPS measurements in these areas is highly recommended.

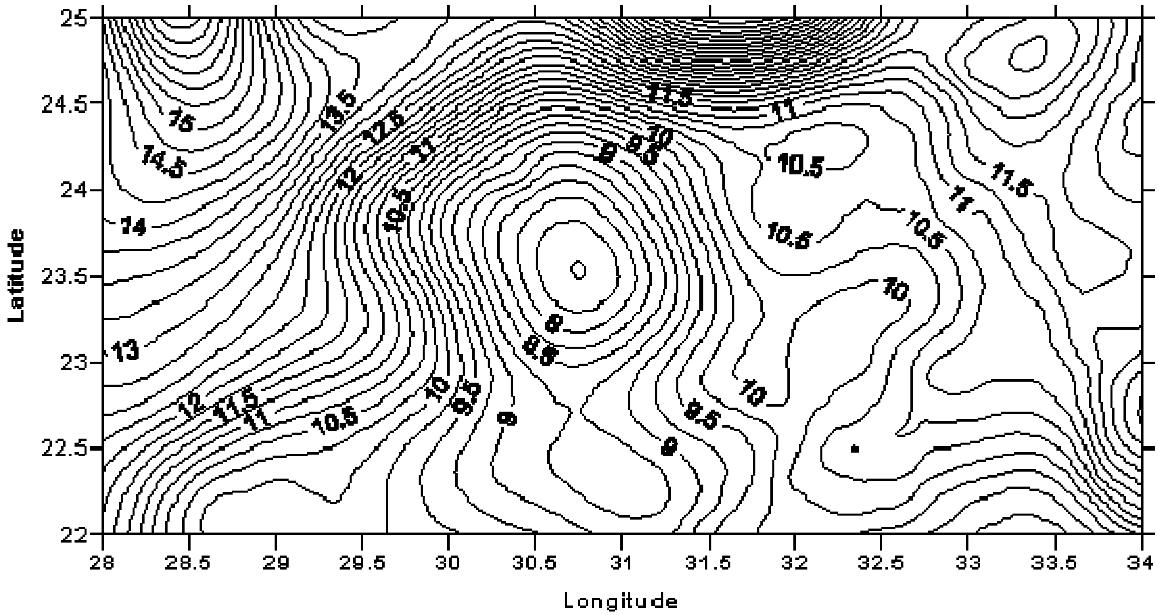

Figure (3): Contour map for the 5'x5' Toshka geoid (Interval: 0.25 m)

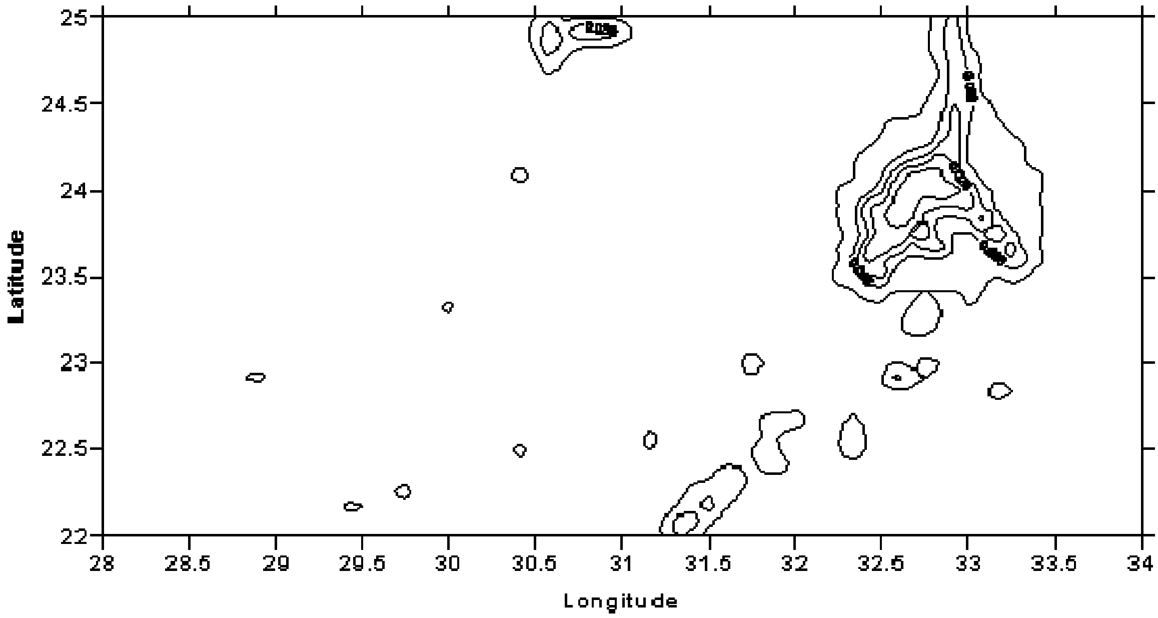

Figure (4): Contour map for the standard errors of the 5'x5' Toshka geoid
(Interval: 5 mm)

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