Direct Projection–
An Efficient Approach for Datum Transformation of Plane Co-ordinates

Lars E. ENGBERG and Mikael LILJE, Sweden

Key words: datum transformation, map-grid co-ordinates.

SUMMARY

Lantmäteriet has decided that SWEREF 99 shall be the Swedish official reference frame and replace RT 90 for surveying and mapping purposes. Transformation between global and local reference frames often leads to lengthy computations involving several consecutive transformation steps. The concept of the *direct projection* is to project the geodetic (global) system directly to the local system. The result is very good; the residuals are normally around 5 centimetres when local systems are transformed and a few more centimetres for the national system RT 90. Comparisons between direct projection and 7-parameter transformation show that it could be used especially when only 2D-position (map-data) is to be transformed.

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1 BACKGROUND

Implementation of a globally adapted reference frame as a national standard will affect all those who rely on co-ordinate data. Transformation between global and local reference frames often leads to lengthy computations involving several consecutive transformation steps. A typical sequence of transformations might look like

$$(\varphi,\lambda)_{global} \leftrightarrow (X,Y,Z)_{global} \leftrightarrow (X,Y,Z)_{national} \leftrightarrow (\varphi,\lambda)_{national} \leftrightarrow (x,y)_{national} \leftrightarrow (x,y)_{local} \qquad (1)$$

In 1997 B-G Reit proposed a method (Reit, 1997) that in most cases makes it possible to shorten the above sequence. This approach is based on the assumption that "*given a geodetic datum A and a plane rectangular system of another datum B, it is possible to find a set of projection parameters (using the same projection as used for the given plane coordinates of datum B) to define a plane system of datum A, which approximates the plane system of datum B"*. In strict mathematical sense the two systems will not be coincident, but the differences may be acceptable for some applications.
If the differences between the plane systems are acceptable, the transformation sequence (1) can be reduced to the simple expression

$$(\varphi,\lambda)_{global} \leftrightarrow (x,y)_{local} \qquad (2)$$

At the Geodetic Department of Lantmäteriet (the National Land Survey of Sweden), B-G Reit has developed an algorithm and also implemented it in a software that can be used for computation of the parameters of the projection.

2 THE CHANGE OF REFERENCE SYSTEM

2.1 Introduction of the ETRS 89 realisation SWEREF 99

Lantmäteriet has decided that SWEREF 99 shall be the Swedish official reference frame and replace RT 90 for surveying and mapping purposes.
A formal decision regarding map projections for national mapping purposes as well as for local surveying was taken in 2003 (Lantmäteriet 2003), all the projections are of Transverse Mercator type.

The work with the introduction of SWEREF 99 among other authorities in Sweden, such as local authorities, is in progress. Approximately 70 of the 290 Swedish municipalities have started the process to replace their old reference frames with SWEREF 99 and 11 have so far finalised the replacement.

To rectify distorted geometries of local reference frames, correction models (Kempe et al, 2006) can be used by the municipalities together with the transformation parameters from the RIX 95-project[1]. The models are based on residuals existing after transformation and the rectification is done by a so-called rubber sheeting algorithm. The result is a homogenous network in SWEREF 99 and geographical data with less deformation.

2.2 Old Systems Nationally and Locally

The Swedish old system was introduced for triangulation works in the southern part of the country in the first decade of the 20th century. Latitude and longitude on Bessel's ellipsoid are projected with the Gauss-Krüger (Transverse Mercator) projection. A system of six zones with 2.5 gon (2° 15') between central meridians was also introduced for cadastral works in rural areas. The towns mostly had their own local systems. As a result of the third national triangulation there are also 12 regional systems mostly used in cadastral works.
Therefore, the situation is rather complicated with several hundreds of local systems in the municipalities and dozens and dozens of 'national' systems.

3 TRANSFORMATION STRATEGIES

Traditionally, the transformation between two reference systems (different geodetic datums) is performed with three-dimensional similarity transformation, so-called 7-parameter transformation. One complication in that respect is that even though only plane co-ordinates will be transformed information about the altitude has to be provided. In the transformation sequence (1), the step between (φ,λ) and (X,Y,Z) requires the height above the ellipsoid. Thus, the transformed plane co-ordinates are dependent on height information. Of course, there is a possibility to always put in zero as height value and still use this 7-parameter transformation.
However, many local systems do not have a rigorous geodetic definition, which means that there is no way to calculate latitude and longitude from plane co-ordinates. In these cases a straightforward application of the 7-parameter transformation is not possible.

3.1 The *Direct Projection* Approach

The concept of the *direct projection* is to project the geodetic (global) system directly to the local system.
The approach is as follows (from Reit, 2003):
Given n points with co-ordinates known in both the geodetic system and the local system, $(\varphi,\lambda)_G$ and $(x,y)_L$, respectively.
Estimate values of the Transverse Mercator projection parameters λ_0, k_0, F_N and F_E by a least squares fit which minimise the sum of the squares of the quantities v_{xi} and v_{yi} of the equations

$$y_i = TM_y(\varphi_i, \lambda_i, \lambda_0, k_0, F_N, F_E, a, f) + v_{yi} \qquad (3a)$$

[1] RIX 95 is a national project, which aims at creating high quality connections between local, national and global reference frames.

$$x_i = TM_x(\varphi_i, \lambda_i, \lambda_0, k_0, F_N, F_E, a, f) + v_{xi} \quad (3b)$$

where TM_x and TM_y are the Transverse Mercator functions that maps the surface of the ellipsoid to a plane.
The equations are linearised by Taylor expansion around approximate values of the parameters λ_0, k_0, F_N and F_E. The solution is iterated until the corrections to the parameters are negligible.
Normally and provided the system covers a reasonably wide area, a suitable projection of the global (latitudes, longitudes) will produce co-ordinates in good agreement with the local system.

3.1.1 Networks with bad orientation

If the local co-ordinate systems are rotated compared to the global system, the projection must be done in combination with a transformation. There are basically two different methods that have been used due to the fact that different software's can not use the same type of transformations see figure 1.
These two methods give almost the same accuracy but not the same co-ordinates. Lantmäteriet calculates both combinations and the final decision on which combination that should be the official one is done by the local authorities in the municipality.

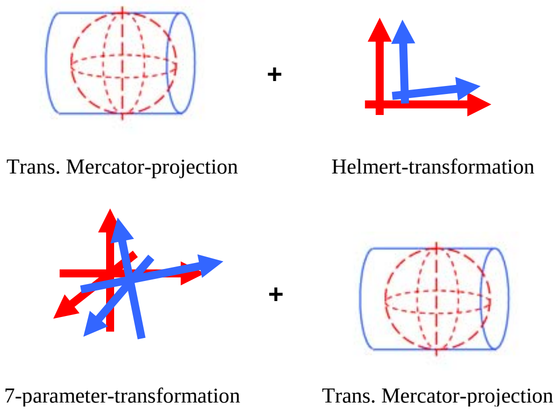

Figure 1: Different transformation combinations to handle rotated co-ordinate systems

4 APPLICATIONS OF THE *DIRECT PROJECTION* METHOD

Since the *direct projection* method was introduced it has been applied within Sweden. In the beginning it was seen as a completion to the 7-parameter transformation but in course of time it became more or less the main method in all cases.

4.1 Transformation of Local Systems

Changing from a local system into a SWEREF 99 system needs a transformation that includes a datum shift. Since on the one hand most of the control points in the local triangulation networks do not have heights and on the other many of the local systems do not have a rigorous geodetic definition it was obvious to use the direct projection approach.
The result is very good, the RMS value for the residuals is normally between 0.01 and 0.08 m, the maximum residual is normally less than twice these values. Taking in account that the internal geometric quality in these local networks is of the same order we can not expect better results.

4.2 Transformation of the National System RT 90

No transformation between RT 90 and SWEREF 99 is perfect since the system differences can not be modelled in analytical form.
Traditionally, the transformation between two reference systems (different geodetic datum) is performed with 7-parameter transformation. Using that method two problems arose, the need of height information and the absence of 7-parameter transformation module in some software.

Tests of the direct projection approach proved that it was as good as the 7-parameter transformation. The two methods will give almost the same accuracy in the transformation; a mean error not quite 0.07 m and a maximal error slightly more than 0.2 m.
The difference between plane co-ordinates obtained by direct projection and co-ordinates transformed with 7-parameter transformation is generally less than 0.1 m but in certain areas come to 0.3 m.
Figure 4a and *4b* will show the residuals in the fundamental points in the network of our permanent GPS stations (SWEPOS™).

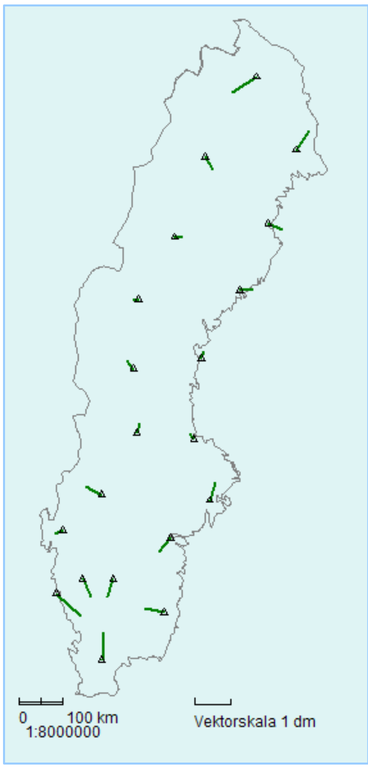

Figure 2a: 7-parameter transformation

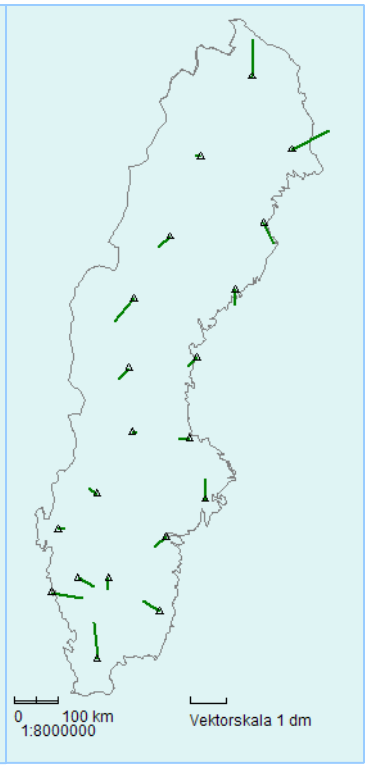

Figure 2b: Direct projection

5 DIRECT PROJECTION VS 7-PARAMETER TRANSFORMATION

The experiences from our work with transformation of local as well as national systems with the direct projection put up some questions. Is it possible to use direct projection elsewhere or are the conditions in Sweden special?

A procedure was created to calculate differences between direct projection and 7-parameter transformation. Using parameters from the CRS_{EU} web-site http://crs.bkg.bund.de/crs-eu/ co-ordinates in different national systems were calculated from latitudes and longitudes in the European reference frame ETRS 89. After that we calculated parameters for a *direct projection* with our least squares fit procedure. In table 1 result from some European countries all of which have a TM projection are listed.

Table 1: Relative comparison between 7-parameter transformation and direct projection

Country	Number of points	RMS (m)	Max residual (m)	Min latitude	Min longitude	Max latitude	Max longitude
Austria, west	6	0.011	0.013	47°00′	10°00′	47°30′	11°50′
Austria, middle	11	0.040	0.064	46°30′	11°50′	48°30′	14°50′
Austria, east	8	0.031	0.047	46°30′	14°50′	48°30′	17°00′
Croatia, west	8	0.052	0.094	43°30′	14°00′	46°00′	16°30′
Croatia, east	8	0.068	0.096	43°00′	16°30′	46°00′	19°00′
Denmark	12	0.015	0.022	55°00′	8°30′	57°30′	12°30′
Finland	47	0.235	0.587	60°00′	20°30′	70°00′	31°30′
Great Britain	41	0.048	0.142	50°00′	-8°00′	60°00′	2°00′
Ireland	23	0.013	0.029	51°30′	-10°30′	55°20′	-6°00′
Italy, west	23	0.018	0.043	42°00′	7°00′	47°00′	12°00′
Italy, east	29	0.029	0.066	38°00′	12°00′	47°00′	18°00′
Luxembourg	11	0.001	0.001	49°30′	5°45′	50°10′	6°30′
Slovenia	9	0.016	0.027	45°30′	14°00′	46°30′	16°00′
Sweden	69	0.090	0.271	55°00′	10°00′	69°00′	24°00′

It seems from these examples that direct projection could be used also in other areas than Sweden.

These comparisons just reflect the difference between 7-parameter transformation and direct projection but are the only way to do it without real data (co-ordinates in both systems). For Sweden and Finland also we have compared results from 7-parameter transformation and direct projection. In table 2 results from these tests and tests in Australia by Featherstone (Featherstone & Reit, 1998) are listed.

Table 2: Statistics of the differences between transformed and projected co-ordinates, the number of points in Sweden was 185, in Finland 90 and in Australia 82.

	Sweden		Finland		Australia	
	7-par transf.	Dir. proj.	7-par transf.	Dir. proj.	7-par transf.	Dir. proj.
RMS (m)	0.066	0.073	0.903	1.073	-	-
Max (m)	0.189	0.221	2.149	2.379	2.036	2.036
Min (m)	0.001	0.004	0.047	0.106	0.031	0.052
Mean (m)	0.055	0.063	0.806	0.970	0.630	0.663
Std (m)	0.038	0.040	0.408	0.462	0.340	0.444

From table 2 it is obvious that the difference between the two methods relative the old coordinates is not so big. It means that the accuracy is almost the same but the figures are different.

6 CONCLUDING REMARKS

The direct projection offers a simple and efficient method to transform co-ordinates between the existing and new reference frame (datum) and map projection. The accuracy is commensurable with the 7-parameter transformation and much more easy to use.

When using some GIS software that have implemented the 7-parameter transformation for transformation of the 2D-position (horizontal) also the heights will be altered and special arrangements have to be inaugurated. Using direct projection no such problem will occur.

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BIOGRAPHICAL NOTES

Lars E Engberg

Mr Engberg obtained his masters degree from the Royal Institute of Technology in Stockholm 1973. He has been working as a lecturer in geodesy at the School of Surveying for many years. Between 1989 and 1996 he was at the City Surveying Department in Stockholm and responsible for the establishment of an improved reference network in Greater Stockholm. Since 1996 he is working at the Geodetic Research Department at Lantmäteriet. At present, he is involved in a national project aiming to implement the new reference frame SWEREF 99 as a national standard. He is also engaged as an international adviser.

Mr Engberg is a member of the Swedish Association of Chartered Surveyors, the Nordic Geodetic Commission as well as the Swedish Cartographic Society.

Mikael Lilje

Mr Lilje graduated in 1993 from the Royal Institute of Technology as a Land Surveyor with emphasis on Geodesy and Photogrammetry. He is working at Lantmäteriet since 1994 with various topics, mainly at the Geodetic Research Department. Currently he is the head of a group working with reference frame and co-ordinate system questions. Mr Lilje is chair of the Swedish Map and Measuring Technique Society and chair of the FIG Commission 5 Working Group on “Reference Frame in Practice”. Mr Lilje was also secretary for FIG Commission 5 during the period 1998 – 2002.

CONTACTS

Lars E. Engberg
Geodetic Research Department
Lantmäteriverket
801 82 Gävle
SWEDEN
Tel. +46 26 63 30 37
Fax +46 26 61 06 76
Email: Lars.E.Engberg@lm.se
Web site: http//www.lantmateriet.se