The Meaning of Redundancy – 3D Topology and Geometric Parameterization

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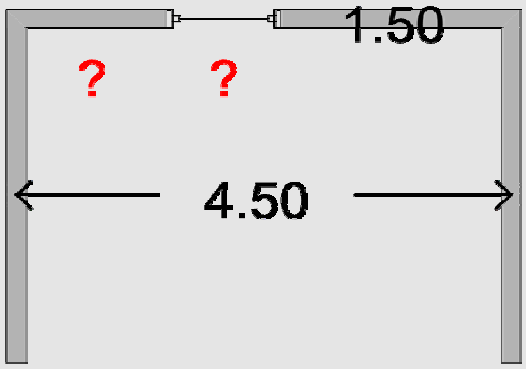
Motivation

Technische Universität Berlin
Department of Engineering Surveying and Adjustment Techniques

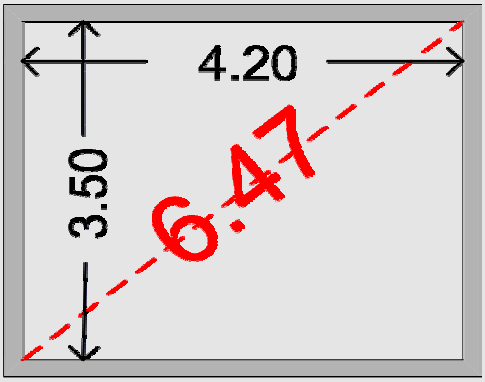
What if…



…measurements have been forgotten …

Motivation

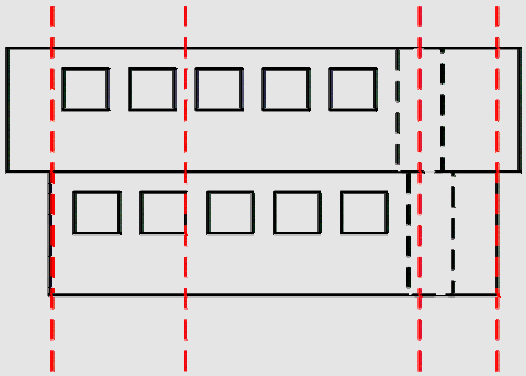
What if…



…measurements disagree

Motivation

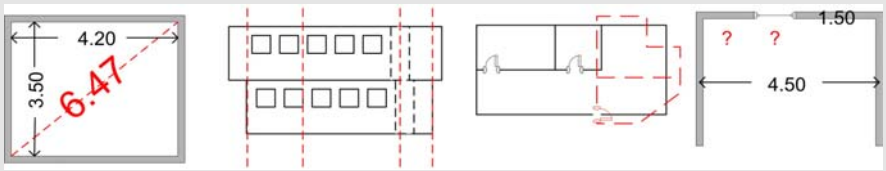
What if…



…resulting walls do not aligne

Motivation

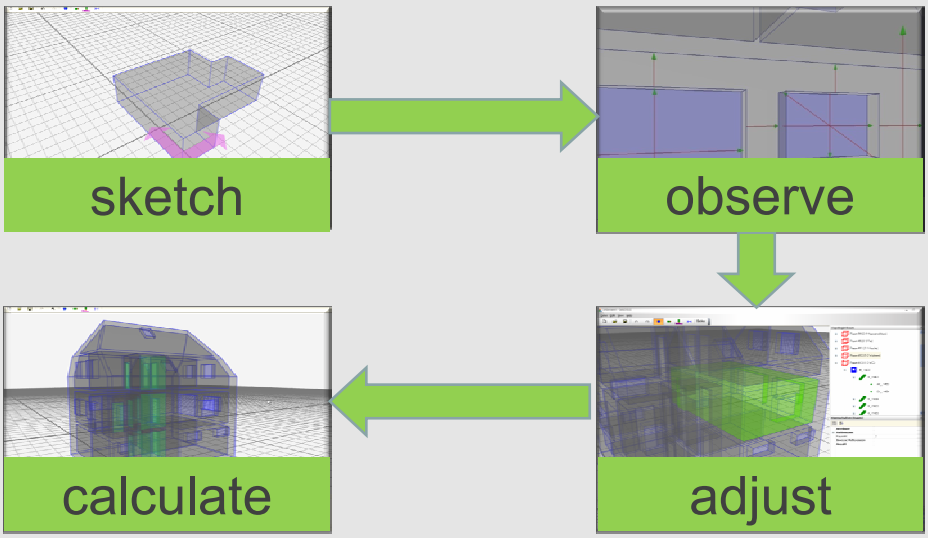
What if typical mistakes occur during 3D as-build-documentation ?



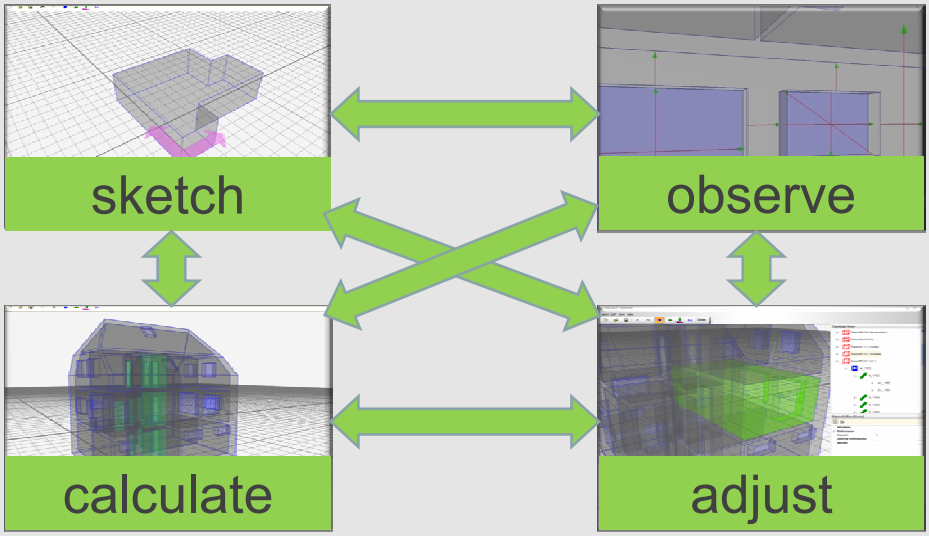
Staff has to return to the building for re-measuring.

Mistakes remain undetected, thus the resulting building geometry is wrong.

Motivation



Motivation

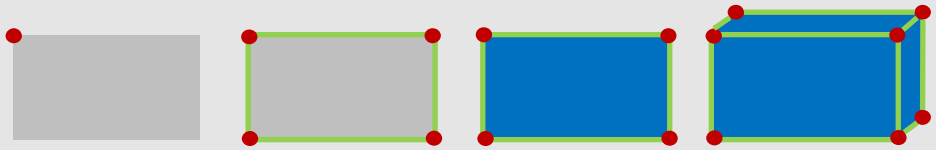


Agenda

- Topology
  - topological primitives
  - why topology ?
  - topological normalization
- Geometry
  - Redundancy of point-based geometry
  - Advantages of surface-based geometry
- Adjustment
  - functional model
  - statistical testing

topological primitives

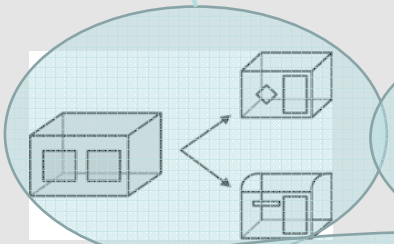
- A **node** is a 0-dimensional topological object. A **node** has no boundary.
- An **edge** is a 1-dimensional topological object. An **edge** is bounded by two nodes
- A **face** is a 2-dimesional topological object. A **face** is bounded by a finite set of edges. The boundary is closed and has no self intersection.
- A **topological solid** is a 3-dimensional topological object. A **solid** is bounded by faces. The surface (boundary) is closed, orientable, connected and free of intersection.



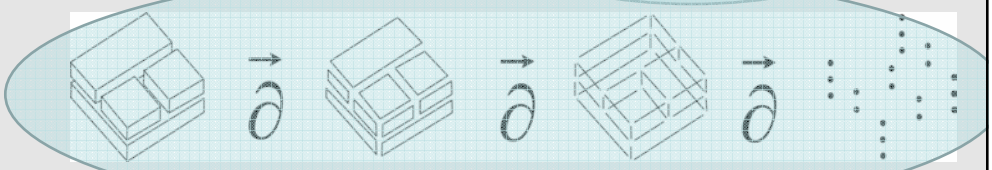
node edge face solid

why topology ?

Right Question: Why explicitly specified topology in three dimensional boundary represeniation models ?



Database tables/attributes…
XML tags…
Java objects…
that represent topological primitives and topological relations



why topology ?

Right Question: Why explicitly specified topology in three dimensional boundary representation models ?

ISO 19103: The most important productive use of topology is to accelerate computational geometry

Surveying Engineer: The most important productive use of topology is to ensure consistency during inserting, updating and deleting operations

Database designer: Consistency is ensured by minimizing redundancy (normalization)

deterministic redundancy vs. stochastic redundancy

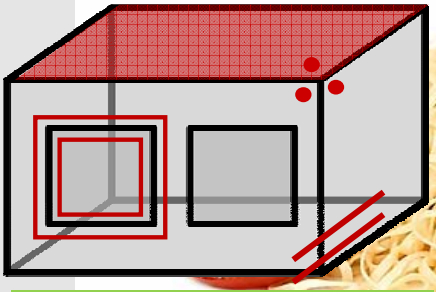
why topology ?

Right Question: Why explicitly specified topology in three dimensional boundary representation models ?

Today: Reduce deterministic redundancy !

….Upcoming slides: stepwise reduction of deterministic redundancy

Topological Normalization



Spaghetti – Polygon Soup

Noderedundancy
Multiple storage of points

Edgeredundacy
Multiple storage of node connectivity

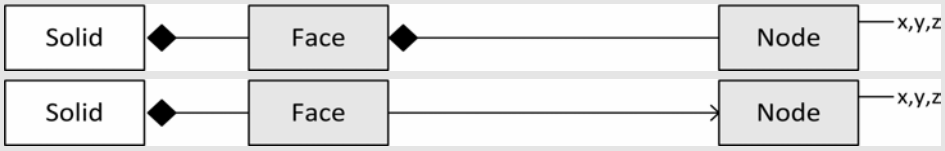
Loopredundancy
Multiple storage of edge loops

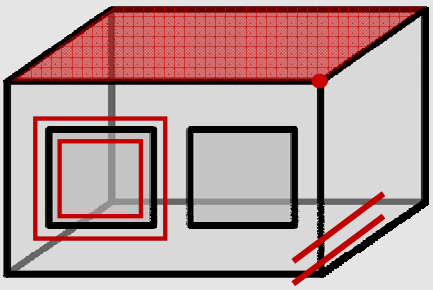
Faceredundancy
Multiple storage of faces

Now:
Stepwise „Normalization“

Bildquelle: www.marions-kochbuch.de/rezept/0592.htm

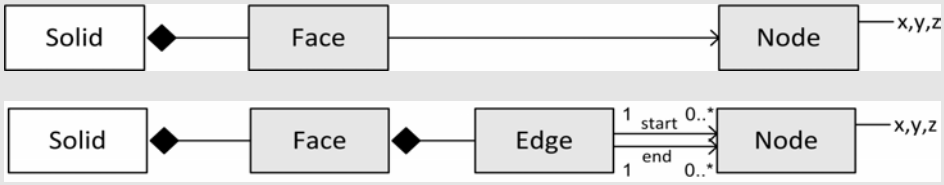
topological normalization

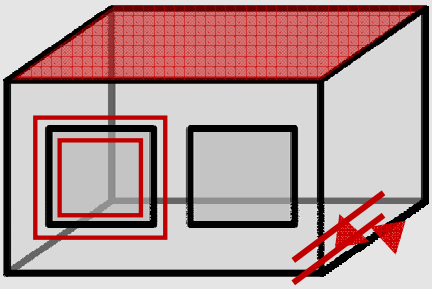




Noderedundancy is avoided
by swapping node composition to node
aggregation

topological normalization

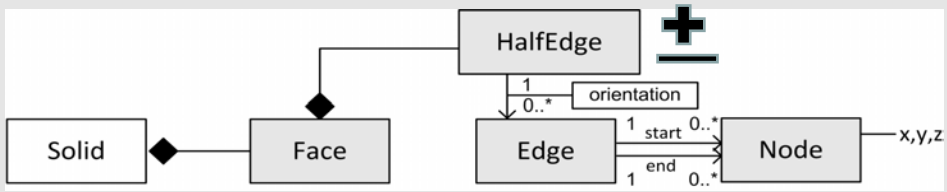


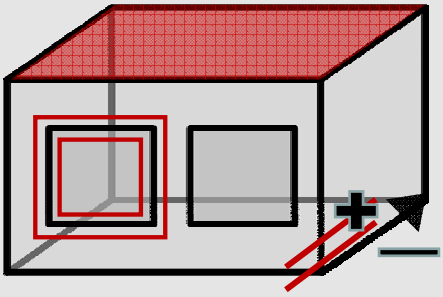


Orientation of edges
by defining start-node and end node

Problem:
Each faces stores its own edges

topological normalization

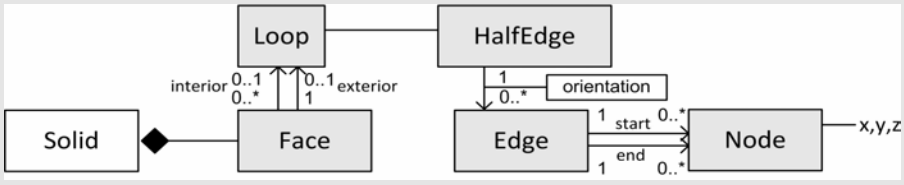


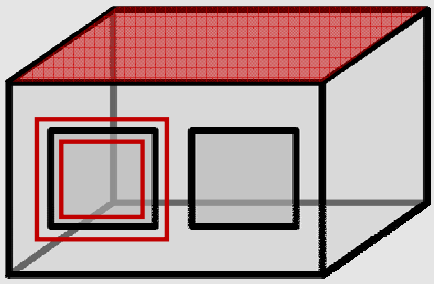


Orientation is stored seperately in the half-edge entity

… thus a face is oriented since ordering of edges (and therefore nodes) is given

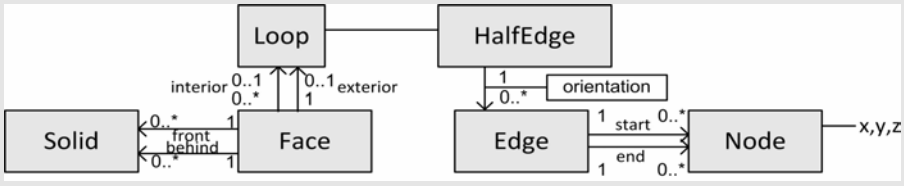
topological normalization

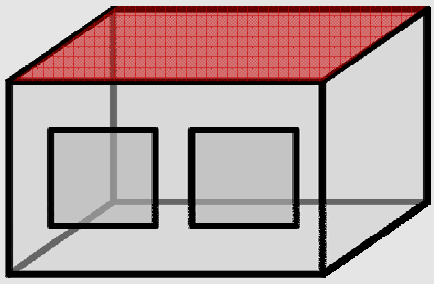




Loops allow for unique storage of half-edge collections, Thus the interior ring and the exterior is topologically the same entity

topological normalization

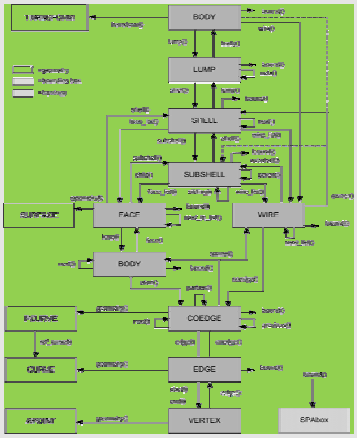




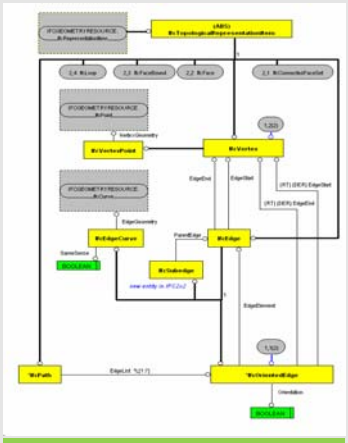
By letting the face refer to the solid (not the other way around) a face (collection of loops) is only stored once.

No redundancy any more ???

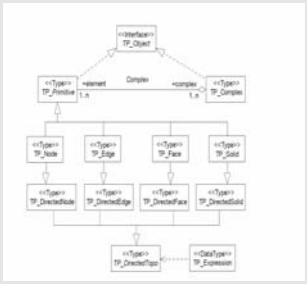
topological normalization



ACIS



IFC



GML/ ISO 19107

…and many more !!!!!!!

The choise of topological model strongly depends on the application

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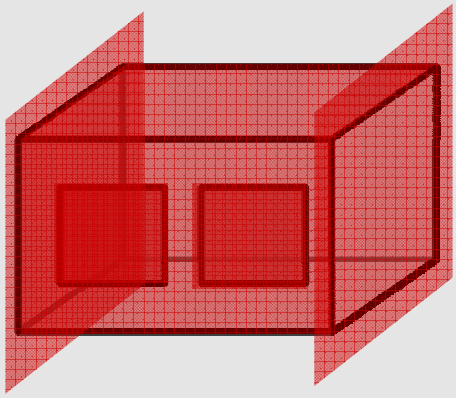
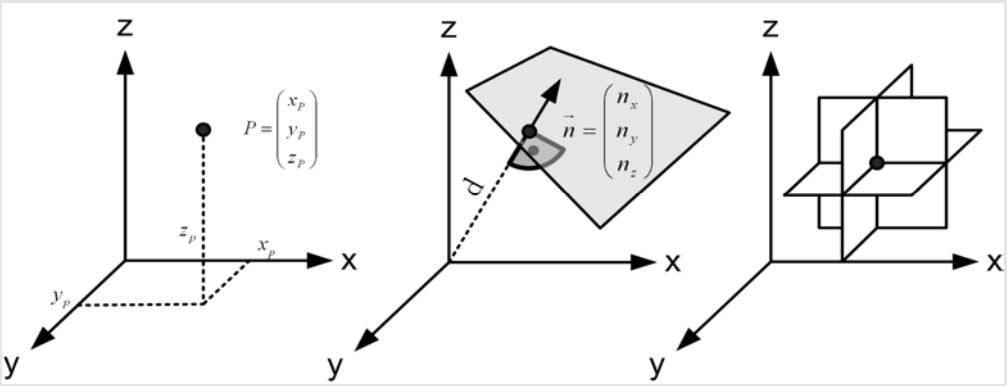
Geometric normalization

- Geometric Redundancy
- Planarity
- Co-planarity
- Paralellism



Geometric normalization



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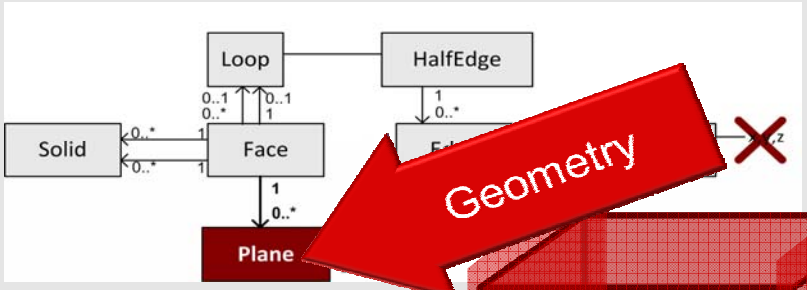
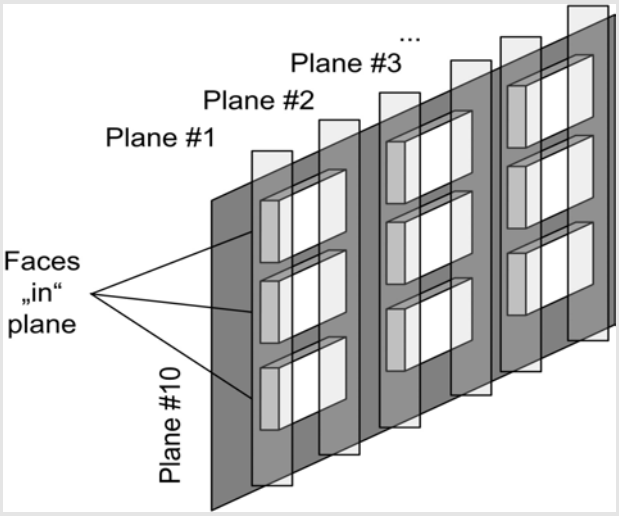


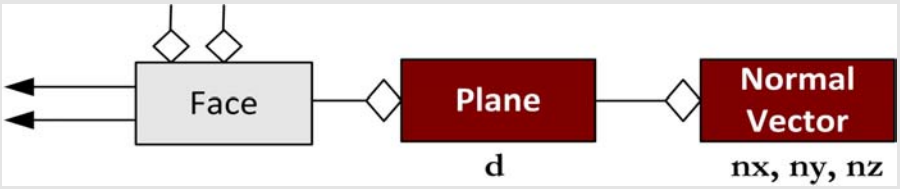
Geometric Normalization

Planarity

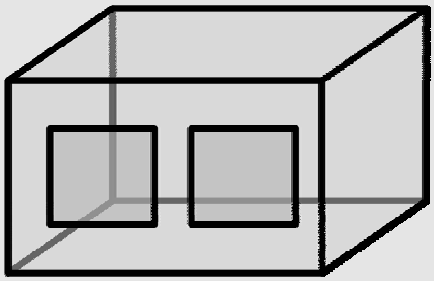
Geometric normalization



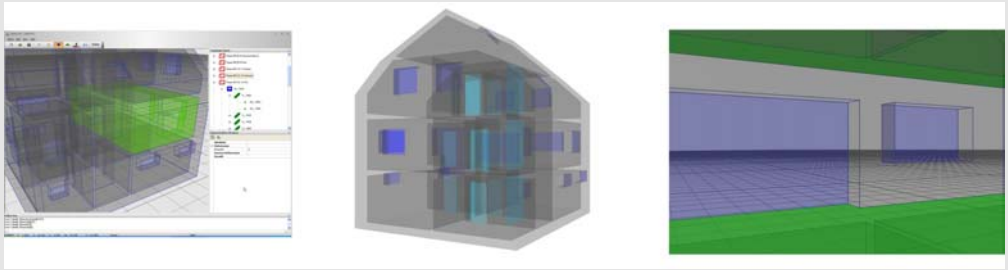
Geometric normalization



- Geometric Normalization
- Planarity
- Co-planarity
- Paralellism



Geometric normalization



Point based parameterization:
412 nodes -> 1236 coordinate values

Surfaced based parameterization:
104 d, 14 nv-values

Reduction: 91%

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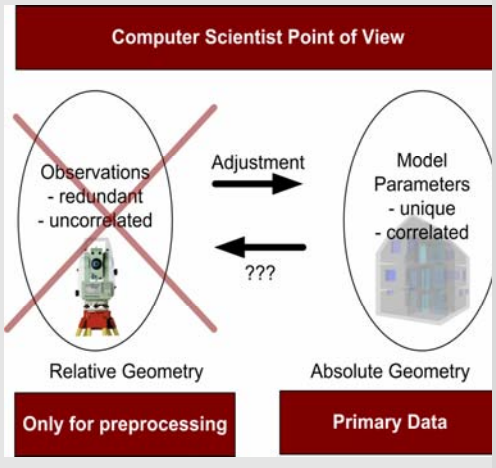
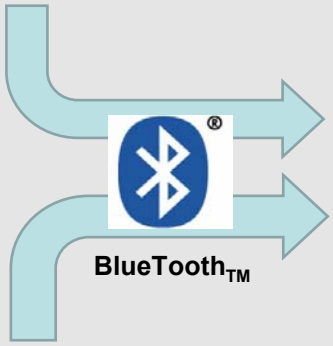
Adjustment

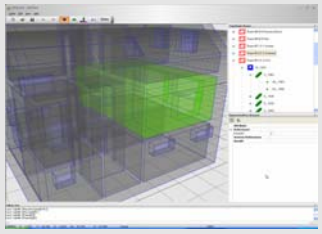


Functional model
observation equations: Distances between topological primitives, local spherical points on topological primitives, angular constraints (parallel, perpendicular)

Leica DISTO$_{TM}$ A6

Total station





Application

Functional model (observation equations, constraints): refer to the paper

$$l + v = d_j - d_i \quad \|\vec{n}\| = \langle \vec{n}, \vec{n} \rangle = \sqrt{n_x^2 + n_y^2 + n_z^2} = 1$$

$$l + v = \langle \vec{n_i}, \vec{p_{jk}} \rangle + d_i \ldots \vec{p_{jk}} = \left[\left(\vec{n_j}, \vec{n_k}, \vec{n_j} \times \vec{n_k} \right)^T \right]^{-1} \left(d_j, d_k, 0 \right)^T$$

$$l + v = \left\| \vec{p_{ij}} - \vec{p_{kl}} \right\| \ldots \vec{p_{ij}} = \left[\left(\vec{n_i}, \vec{n_j}, \vec{n_i} \times \vec{n_j} \right)^T \right]^{-1} \left(d_i, d_j, 0 \right)^T ; \vec{p_{kl}} = \left[\left(\vec{n_k}, \vec{n_l}, \vec{n_k} \times \vec{n_l} \right)^T \right]^{-1} \left(d_k, d_l, 0 \right)^T$$

$$\left\langle \vec{n_i} \times \vec{n_j}, \vec{n_k} \times \vec{n_l} \right\rangle = 0 \qquad \left. \begin{matrix} 0+v \\ 1+v \end{matrix} \right\} = \left| \left\langle \vec{n_i} \times \vec{n_j}, \vec{n_k} \times \vec{n_l} \right\rangle \right| \left\{ \begin{matrix} perpendicular \\ parallel \end{matrix} \right.$$

$$f(x, l+v) = \langle \vec{p_i}, \vec{n_j} \rangle - d_j = \left\langle \begin{pmatrix} (r_i + v_r)\sin(\theta_i + v_\theta)\cos(\phi_i + v_\phi + \omega_T) \\ (r_i + v_r)\sin(\theta_i + v_\theta)\sin(\phi_i + v_\phi + \omega_T) \\ (r_i + v_r)\cos(\theta_i + v_\theta) \end{pmatrix} + \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix}, \vec{n_j} \right\rangle - d_j = 0$$

Adjustment

Post Adjustment Statistics are applied in order to

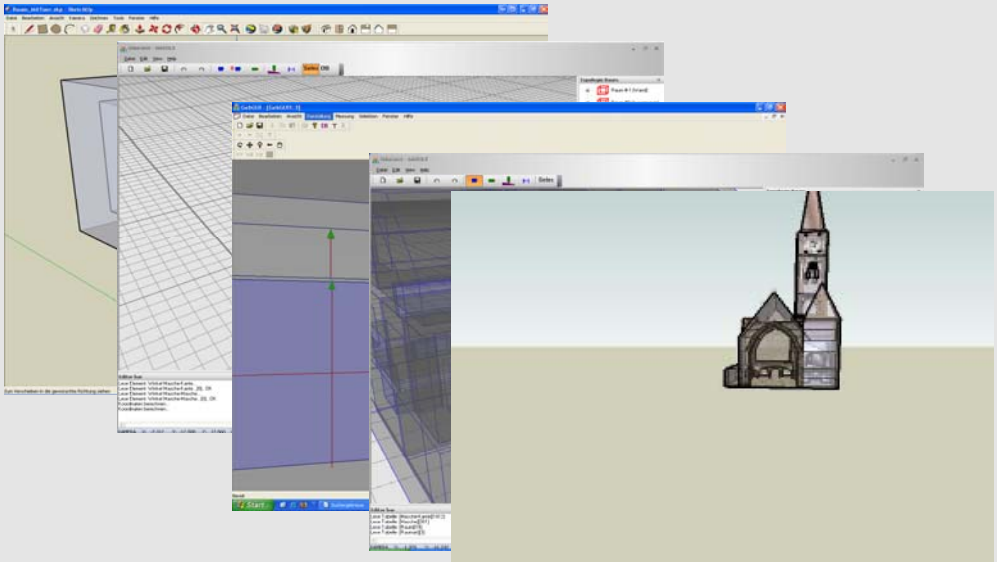
• test the general goodness of fit and detect whether mistakes occur in observations or the functional/statistical model. (-Test χ^2

• test if two normal vectors are stochastically equal. If true, the normal vectors would be unified by interchanging the references in the data base. (Student's t-test)

• test if two orthogonal distances *d* of parallel planes are stochastically equal. If true the two planes will be unified (Student's t-test).

• Test for outliers due to bad face-plane assignment. (z-Test on residuals v_i)

Workflow



Recap

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  - why topology ?
  - topological normalization
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  - Redundancy of point-based geometry
  - Advantages of surface-based geometry
- Adjustment
  - functional model
  - statistical testing

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