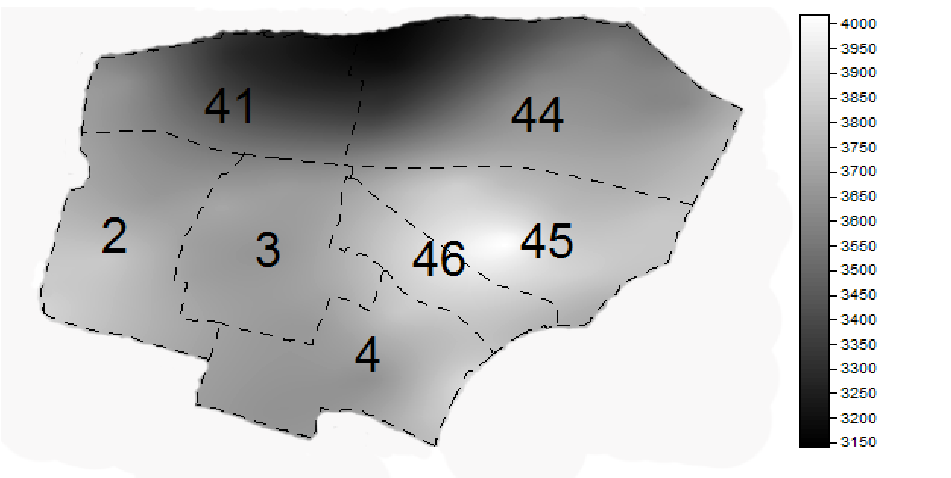
AKADEMIA GÓRNICZO-HUTNICZA
IM. STANISŁAWA STASZICA W KRAKOWIE

Marek Kulczycki, Marcin Ligas

Interpolation and 3D Visualization of Geodata

University of Science and Technology, AGH
Department of Geomatics

FIG Working Week, Eilat 2009



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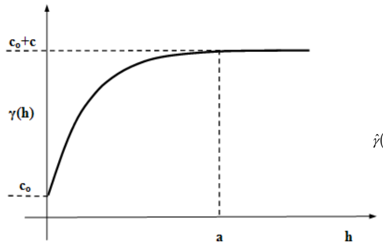

$$E(Z(s)) = \mu$$
$$Cov[Z(s), Z(s+h)] = C(h)$$

$$E[Z(s+h) - (Z(s))] = 0$$
$$V[Z(s+h) - (Z(s))] = 2\gamma(h)$$

$$P(Z(s_1) < z_1, Z(s_2) < z_2, \ldots, Z(s_n) < z_n) =$$
$$= P(Z(s_1+h) < z_1, Z(s_2+h) < z_2, \ldots, Z(s_n+h) < z_n)$$

$$\hat{\gamma}(h) = \frac{1}{2|N(h)|} \sum_{N(h)} [Z(s+h) - Z(s)]^2$$

$$2\hat{\gamma}(h) = \frac{\left[\frac{1}{|N(h)|} \sum_{N(h)} |Z(s+h) - Z(s)|^{\frac{1}{2}}\right]^4}{\left(0.457 + \frac{0.494}{|N(h)|}\right)}$$

$$\gamma(h,\theta) = \begin{cases} 0 & h = 0 \\ c_o + c\left[\frac{3}{2}\frac{h}{a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right] & 0 < h \le a \\ c_o + c & h > a \end{cases}$$

$$\frac{1}{2}\sum_{i=1}^{n} \frac{N(h_i)}{[\gamma(h_i,\theta)]^2} [\hat{\gamma}(h_i) - \gamma(h_i,\theta)]^2 = \min$$

No overburden with complicated formulas

$$E\left[(\hat{m} - m)^2\right] = Var(\hat{m}) + Var(m) - 2Cov(\hat{m}, m) =$$
$$= Var(\boldsymbol{\lambda}^T \mathbf{Z}(\mathbf{s})) = \boldsymbol{\lambda}^T \mathbf{C}(h)\boldsymbol{\lambda}$$

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$$E(\hat{m} - m) = E(\boldsymbol{\lambda}^T \mathbf{Z}(\mathbf{s}) - m) = \boldsymbol{\lambda}^T E(\mathbf{Z}(\mathbf{s})) - m =$$
$$= \boldsymbol{\lambda}^T \mathbf{m} - m = m(\boldsymbol{\lambda}^T \mathbf{1} - 1) = 0$$

$$E[p(\mathbf{Z}, \mathbf{s}_o) - Z(\mathbf{s}_o)]^2 = E[m + \boldsymbol{\lambda}^T \boldsymbol{\varepsilon}(\mathbf{s}) - m - \varepsilon(s_o)]^2 =$$
$$= E[\boldsymbol{\lambda}^T \boldsymbol{\varepsilon}(\mathbf{s}) - \varepsilon(s_o)]^2 = Var(\boldsymbol{\lambda}^T \boldsymbol{\varepsilon}(\mathbf{s})) + Var(\varepsilon(s_o)) -$$
$$- 2\operatorname{cov}(\boldsymbol{\lambda}^T \boldsymbol{\varepsilon}(\mathbf{s}), \varepsilon(s_o)) = \boldsymbol{\lambda}^T \mathbf{C}(h)\boldsymbol{\lambda} + \sigma_o - 2\boldsymbol{\lambda}^T \boldsymbol{\sigma}$$

$$\begin{cases} \frac{\partial \Psi(\boldsymbol{\lambda}, \mu)}{\partial \boldsymbol{\lambda}} = 2\mathbf{C}(h)\boldsymbol{\lambda} - 2\mu\mathbf{1} = 0 \\ \frac{\partial \Psi(\boldsymbol{\lambda}, \mu)}{\partial \mu} = -2\boldsymbol{\lambda}^T \mathbf{1} + 2 = 0 \end{cases}$$

$$\begin{bmatrix} c_{11}(h) & \cdots & c_{1n}(h) \\ \vdots & \ddots & \vdots \\ c_{n1}(h) & \cdots & c_{nn}(h) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{bmatrix}$$

$$\begin{bmatrix} c_{11}(h) & \cdots & c_{11}(h) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ c_{n1}(h) & \cdots & c_{nn}(h) & 1 \\ 1 & 1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ -\mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\sigma^2(\mathbf{s}_o) = E[p(\mathbf{Z}, \mathbf{s}_o) - Z(\mathbf{s}_o)]^2 = \boldsymbol{\lambda}^T \mathbf{C}(h)\boldsymbol{\lambda} + \sigma_o - 2\boldsymbol{\lambda}^T \boldsymbol{\sigma} =$$
$$= \boldsymbol{\lambda}^T (\boldsymbol{\sigma} + \mu\mathbf{1}) + \sigma_o - 2\boldsymbol{\lambda}^T \boldsymbol{\sigma} = \boldsymbol{\lambda}^T \boldsymbol{\sigma} + \mu\boldsymbol{\lambda}^T \mathbf{1} + \sigma_o - 2\boldsymbol{\lambda}^T \boldsymbol{\sigma} =$$
$$= \sigma_o - \boldsymbol{\lambda}^T \boldsymbol{\sigma} + \mu$$

$$\begin{bmatrix} c_{11}(h) & \cdots & c_{11}(h) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ c_{n1}(h) & \cdots & c_{nn}(h) & 1 \\ 1 & 1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ -\mu \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_n \\ 1 \end{bmatrix}$$

$$\sigma^2(\mathbf{s}_o) = E\left[(\hat{m} - m)^2\right] = Var(\hat{m} - m) = \boldsymbol{\lambda}^T \mathbf{C}(h)\boldsymbol{\lambda} =$$
$$= \boldsymbol{\lambda}^T \mu\mathbf{1} = \mu\boldsymbol{\lambda}^T \mathbf{1} = \mu$$

No overburden with complicated formulas

$$E\left[(\hat{m}-m)^2\right] = Var(\hat{m}) + Var(m) - 2Cov(\hat{m},m) = $$
$$= Var(\boldsymbol{\lambda}^T \mathbf{Z}(\mathbf{s})) = \boldsymbol{\lambda}^T \mathbf{C}(h)\boldsymbol{\lambda}$$

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$$\mathbf{Z}(\mathbf{s}) = \mathbf{m}(\mathbf{s}) + \boldsymbol{\varepsilon}(\mathbf{s}) = \mathbf{F}(\mathbf{s})\boldsymbol{\beta} + \boldsymbol{\varepsilon}(\mathbf{s})$$
$$Z(\mathbf{s}_o) = m(\mathbf{s}_o) + \varepsilon(\mathbf{s}_o) = \mathbf{f}^T(\mathbf{s}_o)\boldsymbol{\beta} + \varepsilon(\mathbf{s}_o)$$

$$E[p(\mathbf{Z},\mathbf{s}_o) - \mathbf{Z}(\mathbf{s}_o)] = E\left[\boldsymbol{\lambda}^T\mathbf{Z}(\mathbf{s}) - \mathbf{Z}(\mathbf{s}_o)\right] = $$
$$= E\left[\boldsymbol{\lambda}^T(\mathbf{F}(\mathbf{s})\boldsymbol{\beta} + \boldsymbol{\varepsilon}(\mathbf{s})) - (\mathbf{f}^T(\mathbf{s}_o)\boldsymbol{\beta} + \varepsilon(\mathbf{s}_o))\right] = $$
$$= \boldsymbol{\lambda}^T\mathbf{F}(\mathbf{s})\boldsymbol{\beta} - \mathbf{f}^T(\mathbf{s}_o)\boldsymbol{\beta} = 0$$

$$\begin{cases} \dfrac{\partial\Psi(\boldsymbol{\lambda},\boldsymbol{\mu})}{\partial\boldsymbol{\lambda}} = 2\mathbf{C}(h)\boldsymbol{\lambda} - 2\boldsymbol{\sigma} + 2\mathbf{F}(\mathbf{s})\boldsymbol{\mu} = \mathbf{0} \\ \dfrac{\partial\Psi(\boldsymbol{\lambda},\boldsymbol{\mu})}{\partial\boldsymbol{\mu}} = 2\left(\mathbf{F}(\mathbf{s})^T\boldsymbol{\lambda} - \mathbf{f}(\mathbf{s}_o)\right) = \mathbf{0} \end{cases}$$

$$E[p(\mathbf{Z},\mathbf{s}_o) - \mathbf{Z}(\mathbf{s}_o)]^2 = E\left[\boldsymbol{\lambda}^T\mathbf{Z}(\mathbf{s}) - \mathbf{Z}(\mathbf{s}_o)\right]^2 = E\left[\boldsymbol{\lambda}^T\boldsymbol{\varepsilon}(\mathbf{s}) - \boldsymbol{\varepsilon}(\mathbf{s}_o)\right]^2$$
$$= Var(\boldsymbol{\lambda}^T\boldsymbol{\varepsilon}(\mathbf{s})) + Var(\boldsymbol{\varepsilon}(\mathbf{s}_o)) - 2\operatorname{cov}(\boldsymbol{\lambda}^T\boldsymbol{\varepsilon}(\mathbf{s}),\boldsymbol{\varepsilon}(\mathbf{s}_o)) = $$
$$= \boldsymbol{\lambda}^T\mathbf{C}(h)\boldsymbol{\lambda} + \sigma_o - 2\boldsymbol{\lambda}^T\boldsymbol{\sigma}$$

$$\begin{bmatrix} c_{11}(h) & \cdots & c_{1n}(h) & 1 & f_1(s_1) & f_2(s_1) & \cdots & f_k(s_1) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{n1}(h) & \cdots & c_{nn}(h) & 1 & f_1(s_n) & f_2(s_n) & \cdots & f_k(s_n) \\ 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \\ f_1(s_1) & \cdots & f_1(s_n) & 0 & 0 & 0 & \cdots & 0 \\ f_2(s_1) & \cdots & f_2(s_n) & 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ f_k(s_1) & \cdots & f_k(s_n) & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \cdots \\ \lambda_n \\ \mu_1 \\ \mu_2 \\ \cdots \\ \cdots \\ \mu_k \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \cdots \\ \sigma_n \\ 1 \\ f_1(s_o) \\ f_2(s_o) \\ \cdots \\ f_k(s_o) \end{bmatrix}$$

$$\sigma^2(\mathbf{s}_o) = E[p(\mathbf{Z},\mathbf{s}_o) - \mathbf{Z}(\mathbf{s}_o)]^2 = \boldsymbol{\lambda}^T\mathbf{C}(h)\boldsymbol{\lambda} + \sigma_o - 2\boldsymbol{\lambda}^T\boldsymbol{\sigma}$$

No overburden with complicated formulas

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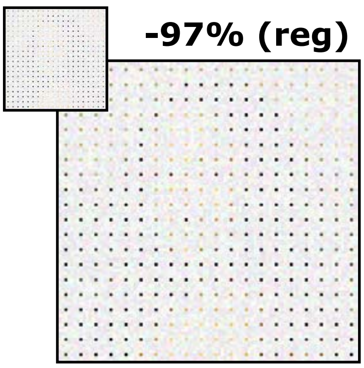
A riddle, what can you see?

-97% (reg)



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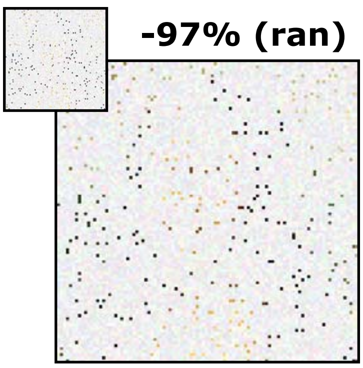
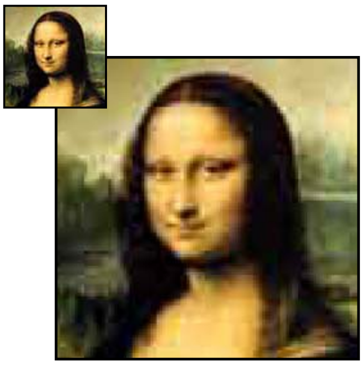


Solution



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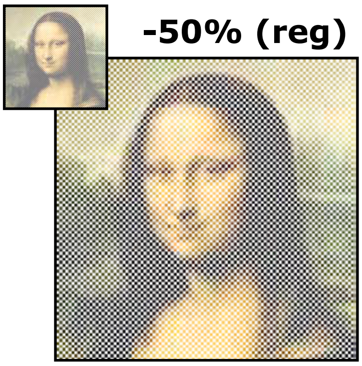

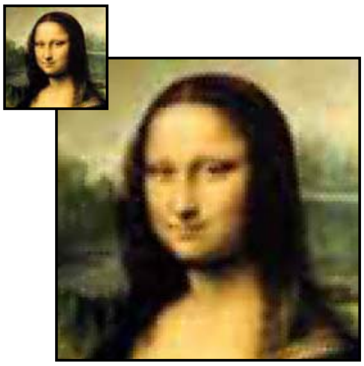

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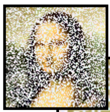

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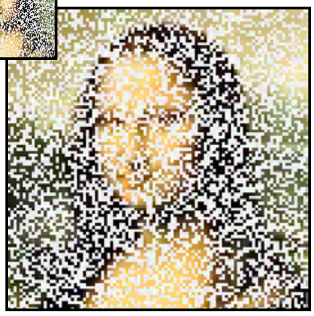

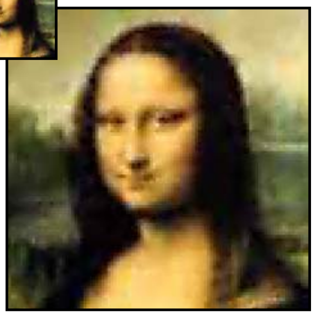

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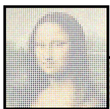

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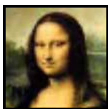

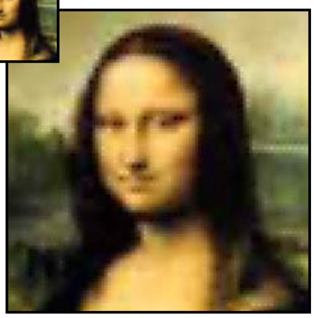

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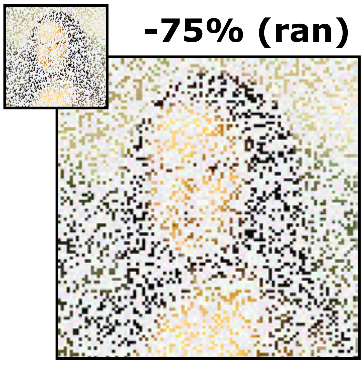

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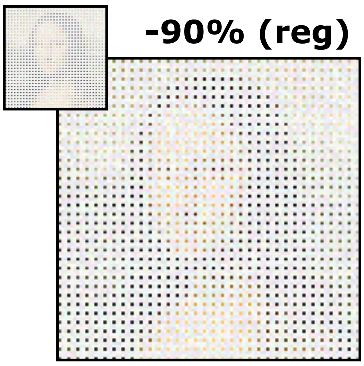

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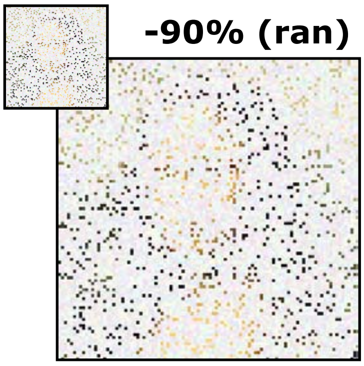

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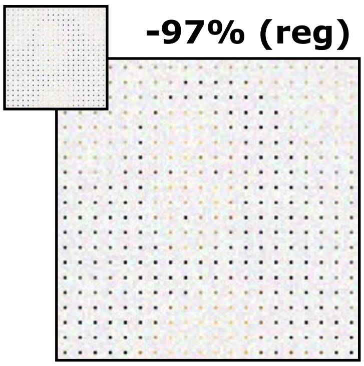

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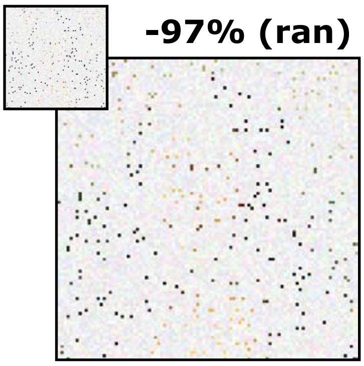

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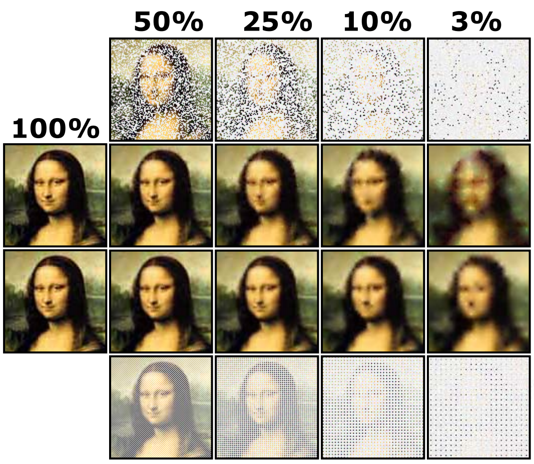
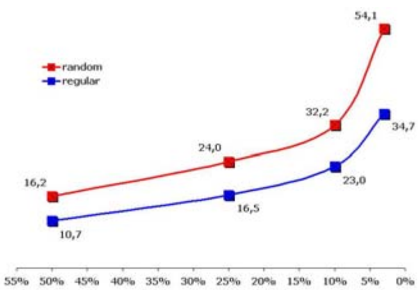
Comparison





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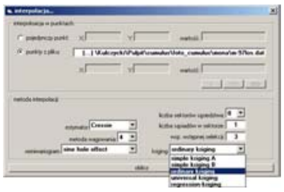
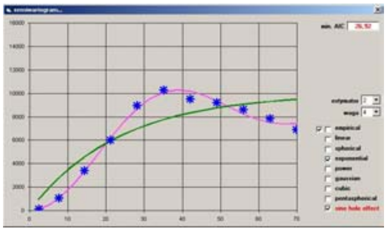
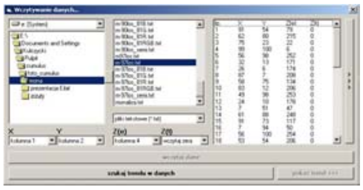


m&m_Cumulus





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AKADEMIA GÓRNICZO-HUTNICZA
IM. STANISŁAWA STASZICA W KRAKOWIE

Thank you for your attention

Marek Kulczycki, Marcin Ligas

**University of Science and Technology,
Department of Geomatics**

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