



AKADEMIA GÓRNICZO-HUTNICZA  
IM. STANISŁAWA STASZICA W KRAKOWIE

*Marek Kulczycki, Marcin Ligas*

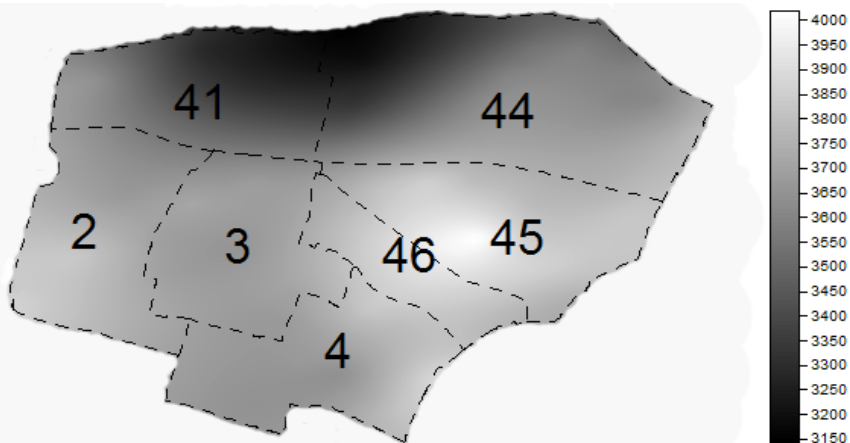
**Interpolation and 3D Visualization of Geodata**

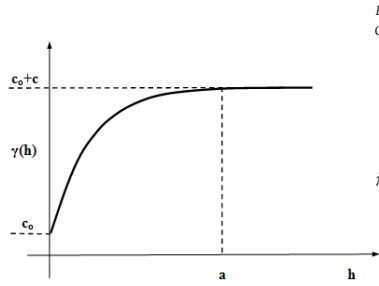
University of Science and Technology, AGH  
Department of Geomatics

FIG Working Week, Eilat 2009



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$$E(Z(s)) = \mu$$

$$\text{Cov}[Z(s), Z(s+h)] = C(h)$$

$$E[Z(s+h) - Z(s)] = 0$$

$$V[Z(s+h) - Z(s)] = 2\gamma(h)$$

$$P(Z(s_1) < z_1, Z(s_2) < z_2, \dots, Z(s_n) < z_n) =$$

$$= P(Z(s_1+h) < z_1, Z(s_2+h) < z_2, \dots, Z(s_n+h) < z_n)$$

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{s_i(s_j)} [Z(s_i+h) - Z(s_j)]^2$$

$$2\hat{\gamma}(h) = \frac{\left[ \frac{1}{N(h)} \sum_{s_i(s_j)} [Z(s_i+h) - Z(s_j)]^2 \right]}{\left( 0.457 + \frac{0.494}{N(h)} \right)}$$

$$\gamma(h, \theta) = \begin{cases} c_0 + c & h = 0 \\ c_0 + c \left[ \frac{3h}{2a} - \frac{1}{2} \left( \frac{h}{a} \right)^3 \right] & 0 < h \leq a \\ c_0 + c & h > a \end{cases}$$

$$\frac{1}{2} \sum_{s_i(s_j)} \frac{N(h_i)}{[V(h_i, \theta)]^2} [\hat{\gamma}(h_i) - \gamma(h_i, \theta)]^2 = \min$$

No overburden with complicated formulas

$$E[(\hat{m} - m)^2] = \text{Var}(\hat{m}) + \text{Var}(m) - 2\text{Cov}(\hat{m}, m) =$$

$$= \text{Var}(\lambda^T Z(s)) + \lambda^T C(h) \lambda$$



$$E(\hat{m} - m) = E(\lambda^T Z(s) - m) = \lambda^T E(Z(s)) - m =$$

$$= \lambda^T \mathbf{m} - m = m(\lambda^T \mathbf{1} - 1) = 0$$

$$E[(Z(s_0) - Z(s_0))^2] = E[m + \lambda^T \mathbf{e}(s) - m - \mathbf{e}(s_0)]^2 =$$

$$= E[\lambda^T \mathbf{e}(s) - \mathbf{e}(s_0)]^2 = \text{Var}(\lambda^T \mathbf{e}(s)) + \text{Var}(\mathbf{e}(s_0)) -$$

$$- 2\text{cov}(\lambda^T \mathbf{e}(s), \mathbf{e}(s_0)) = \lambda^T C(h) \lambda + \sigma_0 - 2\lambda^T \boldsymbol{\sigma}$$

$$\frac{\partial \Psi(\lambda, \mu)}{\partial \lambda} = 2C(h)\lambda - 2\boldsymbol{\mu} \mathbf{1} = 0$$

$$\frac{\partial \Psi(\lambda, \mu)}{\partial \mu} = -2\lambda^T \mathbf{1} + 2 = 0$$

$$\begin{bmatrix} c_{11}(h) & \dots & c_{1n}(h) \\ \vdots & \ddots & \vdots \\ c_{n1}(h) & \dots & c_{nn}(h) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{bmatrix}$$

$$\begin{bmatrix} c_{11}(h) & \dots & c_{1n}(h) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ c_{n1}(h) & \dots & c_{nn}(h) & 1 \\ 1 & 1 & \dots & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ -\mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\sigma^2(s_0) = E[(Z(s_0) - Z(s_0))^2] = \lambda^T C(h) \lambda + \sigma_0 - 2\lambda^T \boldsymbol{\sigma} =$$

$$= \lambda^T (\boldsymbol{\sigma} + \boldsymbol{\mu} \mathbf{1}) + \sigma_0 - 2\lambda^T \boldsymbol{\sigma} = \lambda^T \boldsymbol{\sigma} + \mu \lambda^T \mathbf{1} + \sigma_0 - 2\lambda^T \boldsymbol{\sigma} =$$

$$= \sigma_0 - \lambda^T \boldsymbol{\sigma} + \mu$$

$$\begin{bmatrix} c_{11}(h) & \dots & c_{1n}(h) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ c_{n1}(h) & \dots & c_{nn}(h) & 1 \\ 1 & 1 & \dots & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ -\mu \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_n \\ 1 \end{bmatrix}$$

$$\sigma^2(s_0) = E[(\hat{m} - m)^2] = \text{Var}(\hat{m} - m) = \lambda^T C(h) \lambda =$$

$$= \lambda^T \boldsymbol{\mu} \mathbf{1} = \mu \lambda^T \mathbf{1} = \mu$$

No overburden with complicated formulas

$$E[(\hat{m} - m)^2] = \text{Var}(\hat{m}) + \text{Var}(m) - 2\text{Cov}(\hat{m}, m) = \text{Var}(\lambda^T \mathbf{Z}(s)) - 2\lambda^T \mathbf{C}(h)\lambda$$



$$\begin{aligned} \mathbf{Z}(s) &= \mathbf{m}(s) + \mathbf{e}(s) = \mathbf{F}(s)\boldsymbol{\beta} + \mathbf{e}(s) \\ Z(s_n) &= m(s_n) + e(s_n) = \mathbf{f}^T(s_n)\boldsymbol{\beta} + e(s_n) \end{aligned}$$

$$\begin{aligned} E[p(\mathbf{Z}, s_n) - \mathbf{Z}(s_n)] &= E[\lambda^T \mathbf{Z}(s) - \mathbf{Z}(s_n)] = \\ &= E[\lambda^T (\mathbf{F}(s)\boldsymbol{\beta} + \mathbf{e}(s)) - (\mathbf{f}^T(s_n)\boldsymbol{\beta} + e(s_n))] = \\ &= \lambda^T \mathbf{F}(s)\boldsymbol{\beta} - \mathbf{f}^T(s_n)\boldsymbol{\beta} = 0 \end{aligned}$$

$$\begin{cases} \frac{\partial \Psi(\lambda, \boldsymbol{\mu})}{\partial \lambda} = 2\mathbf{C}(h)\lambda - 2\boldsymbol{\sigma} + 2\mathbf{F}(s)\boldsymbol{\mu} = \mathbf{0} \\ \frac{\partial \Psi(\lambda, \boldsymbol{\mu})}{\partial \boldsymbol{\mu}} = 2(\mathbf{F}(s)^T \lambda - \mathbf{f}(s_n)) = \mathbf{0} \end{cases}$$

$$\begin{aligned} E[p(\mathbf{Z}, s_n) - \mathbf{Z}(s_n)]^2 &= E[\lambda^T \mathbf{Z}(s) - \mathbf{Z}(s_n)]^2 = E[\lambda^T \mathbf{e}(s) - e(s_n)]^2 \\ &= \text{Var}(\lambda^T \mathbf{e}(s)) + \text{Var}(e(s_n)) - 2\text{cov}(\lambda^T \mathbf{e}(s), e(s_n)) = \\ &= \lambda^T \mathbf{C}(h)\lambda + \sigma_n - 2\lambda^T \boldsymbol{\sigma} \end{aligned}$$

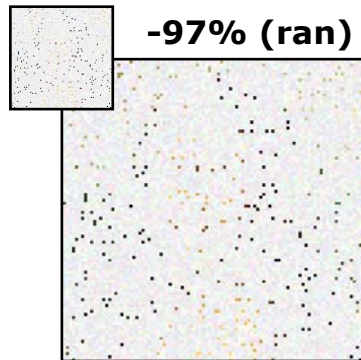
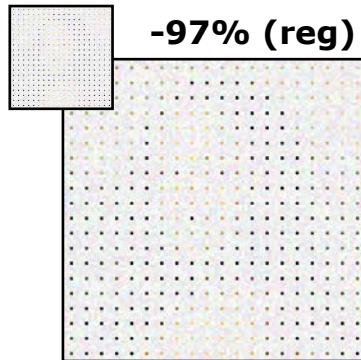
$$\begin{bmatrix} c_{11}(h) & \dots & c_{1n}(h) & 1 & f_1(s_1) & f_2(s_1) & \dots & f_1(s_n) & \lambda_1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_{n1}(h) & \dots & c_{nn}(h) & 1 & f_1(s_n) & f_2(s_n) & \dots & f_1(s_n) & \lambda_n \\ 1 & \dots & 1 & 0 & 0 & 0 & \dots & 0 & \mu_1 \\ f_1(s_1) & \dots & f_1(s_n) & 0 & 0 & 0 & \dots & 0 & \mu_2 \\ f_2(s_1) & \dots & f_2(s_n) & 0 & 0 & 0 & \dots & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ f_k(s_1) & \dots & f_k(s_n) & 0 & 0 & 0 & \dots & 0 & \mu_k \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \dots \\ \sigma_n \\ 1 \\ f_1(s_n) \\ f_2(s_n) \\ \dots \\ f_k(s_n) \end{bmatrix}$$

$$\sigma^2(s_n) = E[p(\mathbf{Z}, s_n) - \mathbf{Z}(s_n)]^2 = \lambda^T \mathbf{C}(h)\lambda + \sigma_n - 2\lambda^T \boldsymbol{\sigma}$$

No overburden with complicated formulas

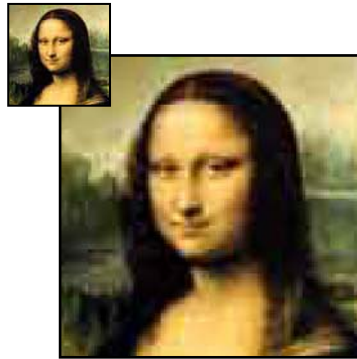


A riddle, what can you see?



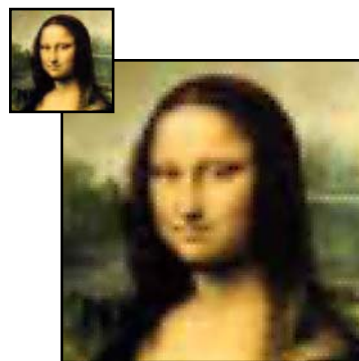
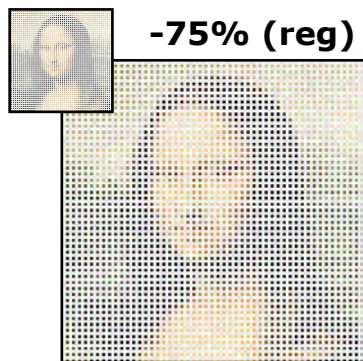
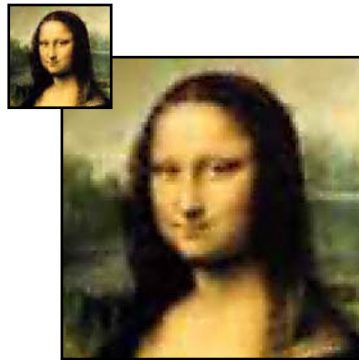
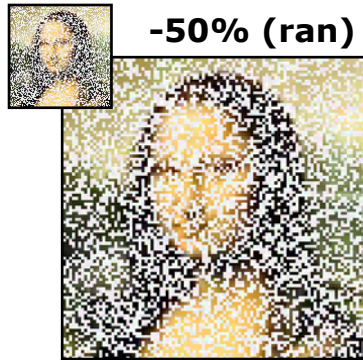


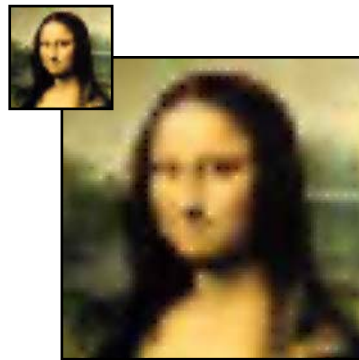
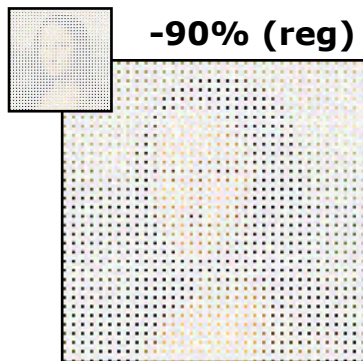
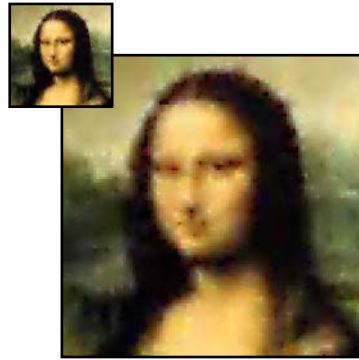
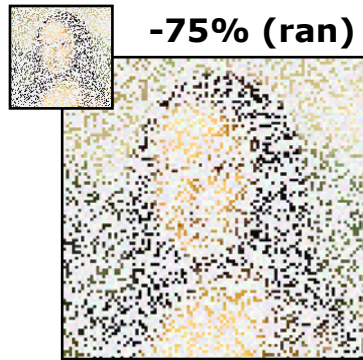
## Solution

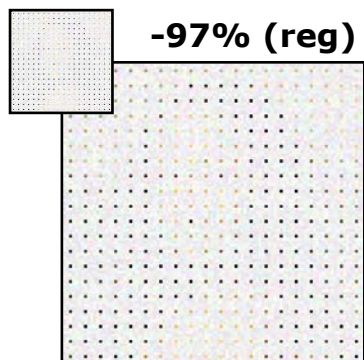
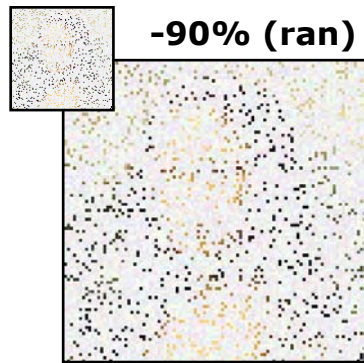


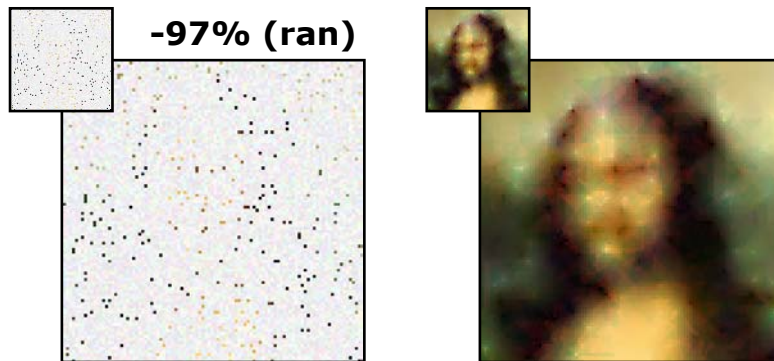
**-50% (reg)**



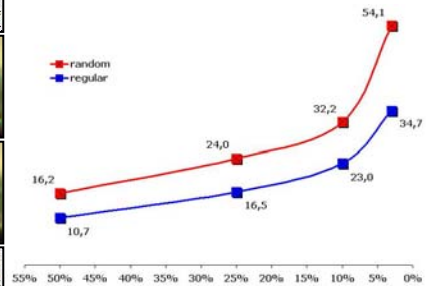
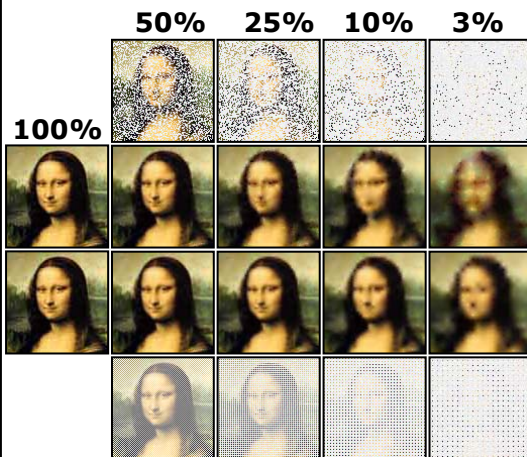








### Comparison







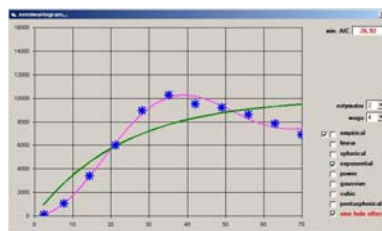
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id	x	y	z	azimuth	dip	strike	slip	type	date	time	depth	location	station	operator	notes
1	481	5	98	201	15	100	0	1	2000	10:00	100	100	100	M. Ligas	
2	147	601.26	676.47	87.37	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
3	1208	437.74	228.71	192.28	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
4	2296	574.25	497.52	570.25	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
5	282	421.14	802.89	408.25	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
6	362	421.14	409.11	801.83	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
7	412	421.14	374.65	702.88	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
8	412	421.14	395.19	821.28	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
9	4098	645.03	647.72	571.43	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
10	4171	426.49	805.29	821.14	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
11	4028	362.39	742.13	794.17	20	100	0	1	2000	10:00	100	100	100	M. Ligas	



**m&m\_Cumulus**



id	x	y	z	azimuth	dip	strike	slip	type	date	time	depth	location	station	operator	notes
1	481	5	98	201	15	100	0	1	2000	10:00	100	100	100	M. Ligas	
2	147	601.26	676.47	87.37	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
3	1208	437.74	228.71	192.28	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
4	2296	574.25	497.52	570.25	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
5	282	421.14	802.89	408.25	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
6	362	421.14	409.11	801.83	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
7	412	421.14	374.65	702.88	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
8	412	421.14	395.19	821.28	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
9	4098	645.03	647.72	571.43	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
10	4171	426.49	805.29	821.14	20	100	0	1	2000	10:00	100	100	100	M. Ligas	
11	4028	362.39	742.13	794.17	20	100	0	1	2000	10:00	100	100	100	M. Ligas	



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**Thank you for your attention**



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Department of Geomatics**

<http://geomatyka.agh.edu.pl>