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Splines and Kriging

the use of two methods for shell structures shape analysis

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FIG Working Week, Marrakech, Morocco, 18-22 May 2011

Thin shell structures

A thin shell is defined as an object with a thickness which is small compared to its other dimensions. They are usually self – supporting which statics is conditioned by proper stress distribution in a shell **tending to preserve the state of minimal potential energy**. In their case very dengerous things are local and abrupt shape changes which distort the distribution of stresses. Therefore, approximating their shape, it is important to use mathematical tools that will illustrate these deformations properly. Splines are suitable choice because they are mathematical equivalent of beam being bent within its resilliency limit and **tend to preserve the state of minimal potential energy** what is connected with minimization of total curvature.

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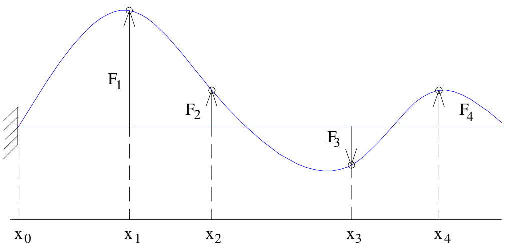
Fitting quadrics – drawbacks:

- give an ideal image of objects, saying nothing about local deformations. It is difficult to use them in order to compare 2 measurements carried out at different points.
  (splines allow local description)
- need to know the mathematical function by which the object was constructed. Approximation by improper function yields large errors.
  (splines do not carry this problem)
- repositioning a single point affects the entire quadrics parameters.
  (by using splines local impact is limited only to surrounding points).

Advantages:

Unrestricted sampling
Small number of points necessary to fit quadrics

How splines are derived...



Bernoulli – Euler formula

$$\frac{1}{\rho} = K = \frac{f''(x)}{[1+(f'(x))^2]^{3/2}} = \frac{M_z}{EJ}$$

The most popular and the most common form of splines - B - splines

$$S_i(t) = \sum_{i=0}^{n-m-1} d_i N_i^m(t) \qquad t_i = \{t_0,..,t_n\}$$

$$N_i^0(t) = \begin{cases} 1 & dla \quad t \in [t_i, t_{i+1}) \\ 0 & dla \quad t \notin [t_i, t_{i+1}) \end{cases}$$

$$N_i^k(t) = \frac{t - t_i}{t_{i+k} - t_i} N_i^{k-1}(t) + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+1}} N_{i+1}^{k-1}(t)$$

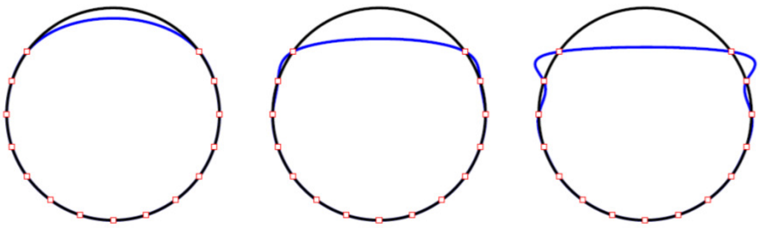
$$S_{i,j}(t,u) = \sum_{i=0}^{n-4} \sum_{j=0}^{r-4} d_{i,j} \cdot N_i^3(t) \cdot N_j^3(u) \qquad t_i = \{t_0,..,t_n\} \quad u_i = \{u_0,..,u_r\}$$

Key factors determining the quality of interpolation by splines:

Proper sampling – the more regular the better, as in case of all interpolation methods (global factor)

Parameterization (knot selection) (global factor)

Boundary conditions (local factor)



True Distance **Square root** **Constant distance**

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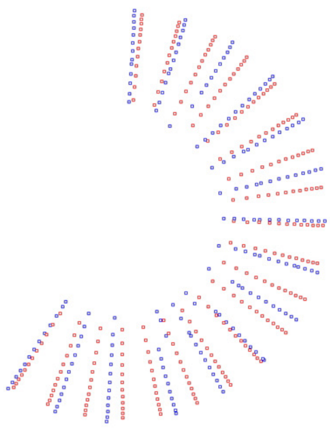
The dome of RMF radio in Nieporaz, Poland

How was it measured?
Reflectorless total station Leica TCRA 1102+

3 mm + 2 ppm for the range up to 80 m and 5 mm + 2 ppm for range exceeding 80 m. The distance from the stations to the object did not exceed 100 m. Acute incidence angles for upper part of the structure.



Points given (blue)
Points to be restored (red), top view

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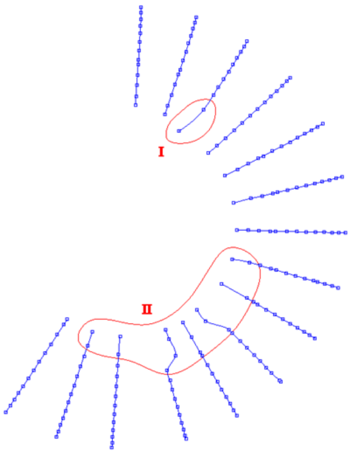

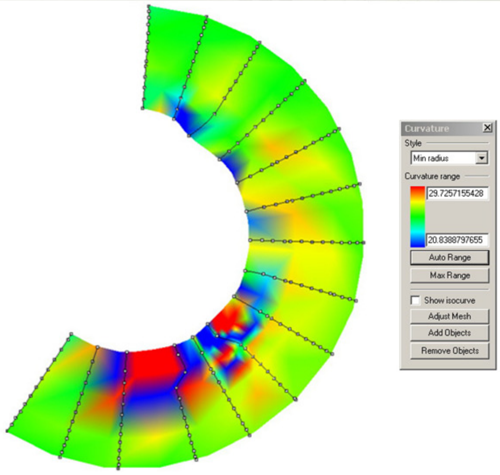

Spline curves created on the basis of given points.
Envelopes highlights areas with the greatest distortion of sampling.

The graph of curvature

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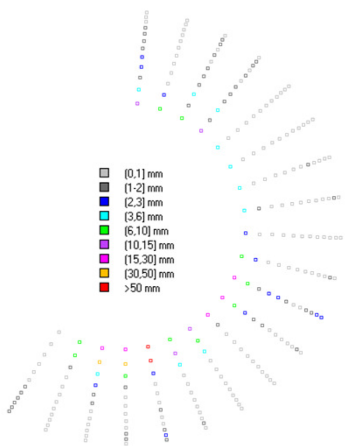

Spatial distribution of deviations obtained by spline interpolation

interval of deviations [mm]	number of points	Percentage [%]
(0,1]	148	61.7
(1,2]	46	19.2
(2,3]	13	5.4
(3,4]	4	1.7
(4,5]	3	1.3
(5,6]	3	1.3
(6,7]	2	0.8
(7,8]	3	1.3
(8,9]	3	1.3
(9,10]	2	0.8
(10,12]	1	0.4
(12,14]	2	0.8
(14,16]	1	0.4
(16,18]	0	0.0
(18,20]	0	0.0
(20,25]	2	0.8
(25,30]	3	1.3
(30,35]	1	0.4
(35,40]	1	0.4
(40,60]	1	0.4
(60,80]	1	0.4

The magnitude of deviations obtained by spline interpolation

The principles of kriging:
Unbiasedness of the predictor (estimator) i.e.:

$$E[\hat{Z}(s_0) - Z(s_0)] = 0$$

Minimization of the mean square error of prediction (estimation) i.e.:

$$E\left(\hat{Z}(\mathbf{s}_o) - Z(\mathbf{s}_o)\right)^2 \to \min$$

$$\hat{Z}(s_0) = \sum_{k=0}^{K} a_k f_k(s_0) + \sum_{i=1}^{N} \lambda_i \left[Z(s_i) - \sum_{k=0}^{K} a_k f_k(s_i) \right] = \sum_{i=1}^{N} \lambda_i Z(s_i) + \sum_{k=0}^{K} a_k \left[f_k(s_0) - \sum_{i=1}^{N} \lambda_i f_k(s_i) \right]$$

After some math... Universal kriging system

$$\begin{cases} \sum_{j=1}^{N} \lambda_j \gamma(s_i, s_j) - \sum_{k=0}^{K} \mu_k f_k(s_i) = \gamma(s_0, s_j), \ i = 1..N \\ \sum_{j=1}^{N} \lambda_j = 1 \\ \sum_{j=1}^{N} \lambda_j f_k(s_j) = f_k(s_0), \ k = 1..K \end{cases}$$

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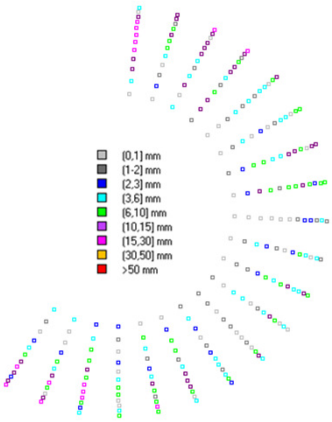

Spatial distribution of deviations obtained by kriging interpolation

interval of deviations [mm]	number of points	percentage [%]
(0,1]	45	18,8
(1,2]	36	15
(2,3]	22	9,2
(3,4]	11	4,6
(4,5]	23	9,6
(5,6]	10	4,2
(6,7]	8	3,3
(7,8]	8	3,3
(8,9]	12	5
(9,10]	13	5,4
(10,12]	19	7,9
(12,14]	13	5,4
(14,16]	10	4,2
(16,18]	3	1,3
(18,20]	3	1,3
(20,25]	3	1,3
(25,30]	1	0,4
(30,35]	0	0
(35,40]	0	0
(40,60]	0	0
(60,80]	0	0

The magnitude of deviations obtained by kriging interpolation

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Thank you for your attention

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