Usability of Cholesky Factorization Method in the Determination of Horizontal Deformations: A Case Study, Ermenek Dam

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Key words: Cholesky factorization, deformation, deformation analysis, MATLAB7.6.0

SUMMARY: The final and most important task of deformation analysis is evaluating data and interpretation of the results. Different methods may be used in evaluations of deformation measurements. In this study, Cholesky Factorization Method, which is one of the static evaluation methods used in the determination of deformations in the horizontal direction, is theoretically examined, using direction observations and ranging data measured in Ermenek Dam for two periods. Geodetic network consists of 13 reference points and 10 object points which were located on the crest. Evaluation was made separately both for direction observations and for direction observation + ranging data. With 95% statistical confidence, any deformation was not observed on reference points 4, 5, 6, 7, 8, 9, 13 and object points 104, 502 in the evaluation according to direction observations and on reference points 3, 6, 7, 8, 9 in the evaluation according to direction observation + ranging data. In the points exposed to deformation, movements were less than 6mm. In the computations, a program prepared MATLAB 7.6.0 Release 13.0 M-File was used.

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1. INTRODUCTION

One of the important task of geodesy is to determine deformation formed on the Earth and buildings. In the determination of deformation, geodetic and physical measurement methods may be used. Absolute deformations are determined with geodetic methods and relative deformations with physical methods and later the results are interpreted.

Deformation measurements obtained from different areas are analyzed with different methods. Generally, θ^2 Criteria, Relative Confidence Ellipse Method, Mierlo Method, Cholesky Factorization Method and S Transformation Method are used in the analysis.

In this study, the horizontal deformations on the Ermenek Dam have been theoretically and practically determined by using Cholesky Factorization Method. This method requires that the fixed and object points of network must be determined at the beginning.

In the computations, a program was prepared by MATLAB 7.6.0 Release 13.0 M-File for analysis with Cholesky Factorization Method.

2. DEFORMATION ANALYSIS BY CHOLESKY FACTORIZATION METHOD

In case object points and fixed points in the control network can be geometrically separable Cholesky Factorization Method is a method that can be used effectively. The following sequence is followed in the evaluation of deformation measurements.

-Showing the unknown coordinates vector of fixed points as x_F, measurements for t_1 and t_2 periods are separately adjusted according to the least squares method if the with a partial trace of unkown of fixed point are minimum ($\mathbf{x}_F^T\mathbf{x}_F = min$) (Ayan, 1983; Demirel, 1987)

- By using the unknowns for the fixed point (x_{1F}, x_{2F}) and the cofactor matrix of unknowns (Q_{1FF}, $\underline{Q}_{2FF}$) calculated by adjusting , the difference vector and cofactor matrix for fixed points,

$$\underline{\hat{d}}_F = \underline{\hat{x}}_{2F} - \underline{\hat{x}}_{1F}$$

$$\underline{Q}_{dF} = \underline{Q}_{1FF} + \underline{Q}_{2FF} \tag{1}$$

were calculated.

-To test whether the fixed points move or not, null hypothesis is formed as follow.

$$H_0 : E\{\hat{d}_F\} = 0 \quad (2)$$

In this hypothesis, the coordinate differences are tested by $\underline{d}_F$ squared test. Experimental variance for fixed points;

$$R_F = \underline{d}_F^T \underline{Q}_{dF}^+ \underline{d}_F \quad (3)$$

$$m_{01}^2 = \frac{R_F}{f_1} \quad , \quad f_1 = 2n_F \quad (4)$$

are obtained. Where n_F is the number of fixed points. Taking advantage of the sum of the squares of the adjustments that calculated separately, free adjustment results for both periods variance value that is common to both periods including $f_2 = f_{01} + f_{02}$ are calculated.

$$m_{02}^2 = \frac{\mathbf{v}_1^T \mathbf{P}_1 \mathbf{v}_1 + \mathbf{v}_2^T \mathbf{P}_2 \mathbf{v}_2}{f_{01} + f_{02}} \quad (5)$$

Here;

- $\mathbf{v}_1$: Correction vector that is calculated as a result of the adjusting of 1.period measurements
- $\mathbf{v}_2$: Correction vector that is calculated as a result of the adjusting of 2.period measurements
- f_{01} : The degree of freedom of measurements in the adjusting of 1. period measurements
- f_{02} : The degree of freedom of measurements in the adjusting of 2.period measurements

The degree of freedom of measurements, f_{01} and f_{02}, are calculated with $f_{01} = n_1 - u_1 + d$; $f_{02} = n_2 - u_2 + d$ equations.

In these equations, n_1 and n_2 are the number of measurements; u_1 and u_2 are the number of unknowns for first and second period respectively. Test value, by using the variances calculated with equations (4) and (5), is calculated as follow,

$$T_1 = \frac{m_{01}^2}{m_{02}^2} \quad (6)$$

T_1 value is compared with the F-table ($F_{f_1, f_2, 1-\alpha}$,,) value.

If $T_1 < F_{f_1, f_2, 1-\alpha}$,, there is no deformation in the fixed points.

If $T_1 > F_{f_1,f_2,1-\alpha}$, it is said that at least one fixed point has moved.

- In the case of deformation, the highest absolute value in the $\hat{\underline{\mathbf{d}}}_F$ vector is removed and the null hypothesis is re-established and tested. These operations are repeated until test value is smaller than F-table value (Yalçınkaya ve Tanır 2000).

- Fixed points determined to have moved at fixed point test are taken as object points in the next step. After the test of the fixed point, it is proceed to the testing of object points. For fixed points a pair of unknown coordinate and for object points two pairs of unknown coordinate are selected. A_I is coefficient matrix of fixed points, A_1 and A_2 are coefficient matrices for object points the first and second period respectively. Functional and stochastic models for mass adjustment are formed as follow,

$$\begin{bmatrix} \underline{v}_1 \\ \underline{v}_2 \end{bmatrix} = \begin{bmatrix} \underline{A}_I & \underline{A}_1 & 0 \\ \underline{A}_I & 0 & \underline{A}_2 \end{bmatrix} \begin{bmatrix} \underline{x}_F \\ \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} - \begin{bmatrix} \underline{l}_1 \\ \underline{l}_2 \end{bmatrix} \tag{7}$$

$$\underline{Q} = \begin{bmatrix} \underline{Q}_{FF} & \underline{Q}_{F1} & \underline{Q}_{F2} \\ \underline{Q}_{1F} & \underline{Q}_{11} & \underline{Q}_{12} \\ \underline{Q}_{2F} & \underline{Q}_{21} & \underline{Q}_{22} \end{bmatrix} \tag{8}$$

Where, matrix $\underline{Q}$ is cofactor matrix of mass adjustment. Difference vector ($\underline{d}$) and its cofactor matrix ($\underline{Q}_d$) are calculated as follow,

$$\underline{d} = \underline{x}_2 - \underline{x}_1 \tag{9}$$

$$\underline{Q}_d = \underline{Q}_{11} + \underline{Q}_{22} - \underline{Q}_{12} - \underline{Q}_{21} \tag{10}$$

Null hypothesis is formed as follow;

$$H_0: E\{\underline{d}\} = 0 \tag{11}$$

Experimental variance to test the null hypothesis m_{03}^2; n_B is the number of objects points and f_3 is the degree of freedom, including twice the number of object points is calculated with;

$$m_{03}^2 = \frac{\mathbf{d}^T \underline{\mathbf{Q}}_d^+ \mathbf{d}}{f_3} \qquad f_3 = 2n_B \tag{12}$$

T_2 Test value are calculated by using equations (5) and (12) and compared with F-table ($F_{f_3,f_2,1-\alpha}$) value.

$$T_2 = \frac{m_{03}^2}{m_{02}^2} \tag{13}$$

If $T_2 > F_{f_3,f_2,1-\alpha}$, object points have moved with s = 1-α statistics confidence.

If $T_2 < F_{f_3,f_2,1-\alpha}$, it should not be immediately decided that there is no deformation and further detail examination should be considered. Because deformations are roughly investigated up to here.

- Since vector of the coordinate differences in deformation points is already correlated, they don't subject to individual significance test. Therefore, elements of d vector must be converted into another uncorrelated vector. For that reason full weight matrix $\mathbf{P}_d$ of d vector;

$$\underline{P}_d = \underline{Q}_d^{-1} \tag{14}$$

and C is calculated to represent an upper triangular matrix as follow:

$$\underline{P}_d = \underline{C}^T \underline{C} \tag{15}$$

The upper triangular matrix C is calculated by means of a symmetrical P_d matrix as follows.

$$\underline{P}_d = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & P_{n3} & \cdots & P_{nm} \end{bmatrix}, C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & \cdots & C_{1n} \\ 0 & C_{22} & C_{23} & \cdots & C_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & C_{nm} \end{bmatrix} \tag{16}$$

Where, n is a value, which is dependent on the number of object points, and it is twice as many as the number of objects ($n = 2n_B$). Sub-indices of the matrix P represent the number of rows and columns. Elements of the C matrix were calculated from the following equations.

$$C_{11} = \sqrt{P_{11}} \tag{17}$$

$$C_{1i} = \frac{P_{1i}}{C_{11}} \qquad i = 1,2,3,\ldots,n \tag{18}$$

$$C_{ii} = \left(P_{ii} - \textstyle\sum_{k=1}^{i-1} C_{ki}^2\right)^{\frac{1}{2}} \qquad i = 1,2,3,\ldots,n \tag{19}$$

$$C_{ij} = \left(P_{ij} - \textstyle\sum_{k=1}^{i-1} C_{ki}C_{kj}\right)/C_{ii} \qquad i = 1,2,3,\ldots,n-1 \;\; ve \;\; j = i+1,\ldots,n \tag{20}$$

The quadratic form of vector was obtained by replacing the C matrix instead of P_d matrix;

$$q = \underline{d}^T \underline{P}_d \underline{d} = \underline{d}^T \underline{C}^T \underline{C}\, \underline{d} \tag{21}$$

This representation is shortened as so: (İnal, 2010; Ayan, 1993; Bektaş, 1998).

$$\underline{C}\, \underline{d} = \underline{r} \tag{22}$$

$$q = \underline{r}^T \underline{r} = r_{x1}^2 + r_{y1}^2 + \ldots + r_{xn}^2 + r_{yn}^2 \quad (23)$$

where, r is the number of object points. q values of each point is the sum of the squares r. q values is not correlated as vector d, it is a free function. q values for each object point are calculated with this equation and are individually subjected to significance test.

$$q_i = r_{xi}^2 + r_{yi}^2 \quad ; \quad i = 1,2,\ldots,n_B \quad (24)$$

r_{xi}^2, r_{yi}^2 , are square values corresponding the coordinate differences between x and y. If the test result shows that a point does not move, theoretically is necessary to add this point in to classes of the fixed points and repeat all steps of analysis starting from adjusting t_1, t_2 measurement groups separately. Such a situation increases the computing volume of deformation analysis. On other hand, an objective criteria that regulates the order of testing q_i must be developed. For this reason, during the reduction of q_i it is recommended that there should be a special pivot searching method assending order of q_i values automatically.

$$T = \frac{q_i}{2\, m_{02}^2} \quad (25)$$

For testing q value, a null hypothesis is formed as follow:

$$H_0 : E\{q_i\} = 2\, m_{02}^2 \quad (26)$$

If $T < F_{2,f_2,1-\overline{\alpha}}$, the hypothesis is not rejected. . If $T > F_{2,f_2,1-\overline{\alpha}}$, this point and following points are considered as replaced points. In such test methods, even if the null hypothesis is valid, the probability of being refuse enlarges in every step and if s=1-α is wanted to be valid for last q_i $\overline{\alpha} = 1 - (1-\alpha)^{1/k}$ should be in step k. New deformation vector is calculated with the following equation,

$$\overline{\underline{d}} = \overline{\underline{C}}^{-1} \overline{\underline{r}} \quad (27)$$

the $\overline{\underline{C}}$ ve $\overline{\underline{r}}$, are computed by removing rows and colums related to points where the movements are not proved from the C matrix and $\underline{r}$ vector (İnal, 2009; Yalçınkaya ve Tanır, 2001)

3. APPLICATION

The Ermenek Dam is located on the Göksu river in Ermenek(Karaman, Turkey) in 2002. The Görmel valley where the dam was built has highly steep cliffs. The dam is a thin concrete arch body –filling type. The volume of arch body is 272 000 m^3. and the height of the arch from the stream bed is 210.00 m. At normal water level, the lake volume is 4,582.00 $hm^3$and the lake area is 58.74 km^2. It is planned to produce 1.048 GWh energy annually with the power of 306 MW. Dam construction started in 2002 and began to collect water since August

10, 2009 (Vikipedi, 2012). Ermenek dam is 21th dam in the world and and 6th dam in Europe and first dam in Turkey in terms of body height, (Figure 1) (Bülbül, 2013).

Figure.1 The Wiew of Ermenek Dam from sky (Sezer, 2012)

In order to determine movement on the crest of the Ermenek Dam, 13 reference and 10 object points were used. Reference points are numbered as 1,2, .., 13. Object points are numbered as 501,502,503, 504.505 are the upstream side of the dam and 101,102,103,104,105 numbered points are at the downstream (Figure 2).

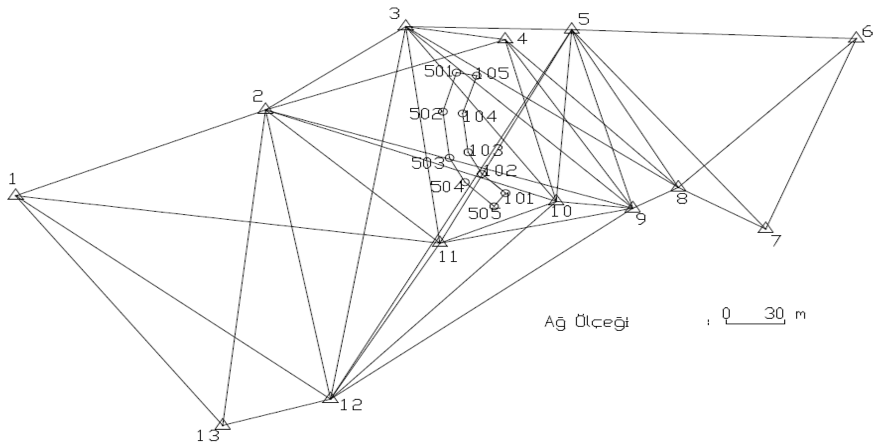

Figure.2 Horizontal Network, according to Reference points and Views

On the geodetic network, the reference points were constructed in form of pillar and the object points on crest were constructed in a way that it can be held reflectors.

In geodetic network, 4 series direction observations and ranging data were measured. In network, 166 direction observations, 128 ranging data were measured. Deformation research, using the ranging data + direction observations and the direction observations, were made separately and the results were compared.

3.1 Evaluation of Period Measurements

In this study, since the network can be geometrically divided, points 1-13 were taken as fixed-points and points 101-105 and points 501-505 placed on the crests as object points. Evaluation was made by using direction observations and direction observations + ranging data separately and the effect of changes in the measurement plans on the analysis result were investigated

In evaluation which was made by using both direction observations and direction observations + ranging data, difference vector and their cofactor matrix was calculated with the equation (1). For testing reference points, null hypothesis was formed according to equation (2), T_1 test value was calculated according to equation (6). The value of calculated T_1 test (1) was compared with F-table value. In both evaluations since T_1 was higher than F-table value ($T_1 > F$), it was concluded that there was a movement at reference points. Following this step, the localization of deformations was carried out by using equations (1) - (6). Reference points exposed to deformation were determined (Table 1).

Table 1. Determination of Reference points exposed to deformations

Measurement Plan	T_1	F-table	Result	Reference points, exposed to deformation
Direction	9.1896	$F_{26,220,0.95}$=1.5461	+	1,2,5,10,11,12,13
Direction + range	15.3178	$F_{24,469,0.95}$=1.5193	+	1,2,4,5,7,10,11,12,13

The fixed points that were determined to be subjected to deformation after running the test and object points on the crest were taken as deformation points. The test was run based on these points.

For testing the deformation points, mass adjustment were carried out according to equations (7) and (8). Difference vectors for object points and their cofactor matrix were calculated by means of equations (9) and (10) respectively. After then, T_2 test value computed with equation (13) was compared with F-table value. Since test gave result that, T_2 was higher than F-table value ($T_2 > F$), it was concluded that one or several of the object points had moved(Table 2).

Table.2 Global test for object Point

Measurement Plan	T_2	F-table	Results
Direction	70.5874	$F_{34,220,0.95}$=1.4830	+
Direction+range	94.5307	$F_{38,469,0.95}$=1.4307	+

Since the vectors of coordinate differences are correlated, they are not subjected to individual significance test. Therefore d vector elements must be converted into another uncorrelated vector. The weight matrix of d vector, with the help of the upper triangular matrix constituted according to equations (15)-(20), was calculated according to equation (21). q_i values computed by squaring (equation 23) r values calculated according to equation (22) were put in order from small to big. Then, T test value were calculated for each points separately according to equation (25). It was possibly 95% concluded that the points where T test value was higher than F-table value (T> F) were considered to be subjected to deformation with %95 possibility (Table 3).

Table.3 Determination of points, exposed to deformation

Analyses according to directions observations				**Analyses according to directions observations+ ranging data**			
NN	**T**	**F-table**	**Results**	**NN**	**T**	**F-table**	**Results**
1	20.5714	3.0369	+	1	19.6593	3.0149	+
2	45.1401	3.0369	+	2	68.1764	3.0149	+
				4	24.2672	3.0149	+
5	28.6424	3.0369	+	5	61.0242	3.0149	+
				7	**0.9296**	**3.0149**	**-**
10	14.2113	3.0369	+	10	23.4459	3.0149	+
11	76.6796	3.0369	+	11	163.3055	3.0149	+
12	29.8375	3.0369	+	12	92.0684	3.0149	+
13	**1.9745**	**3.0369**	**-**	13	22.369	3.0149	+
101	88.257	3.0369	+	101	82.1946	3.0149	+
102	55.6282	3.0369	+	102	65.0418	3.0149	+
103	59.4161	3.0369	+	103	76.8303	3.0149	+
104	**1.29E-08**	**3.0369**	**-**	104	34.9558	3.0149	+
105	6.7722	3.0369	+	105	4.7946	3.0149	+
501	19.8382	3.0369	+	501	30.4454	3.0149	+
502	**2.74E-09**	**3.0369**	**-**	502	167.1184	3.0149	+
503	138.832	3.0369	+	503	194.701	3.0149	+
504	126.077	3.0369	+	504	169.6504	3.0149	+
505	129.4477	3.0369	+	505	187.0865	3.0149	+

3.2 Introduction of the Program

In this study, a program performing the deformation analysis with Cholesky Factorization Method was prepared in the programming language MATLAB 7.6.0. Using the ranging data and directions observations measured in Ermenek Dam, the results have been interpreted. Before running the program a data file “measures.doc” was created by using Microsoft Office Excell to calculate the measurement, in this program primarily a file has been prepared in the format of the data to be received in as “measurement.doc”. In this file there are measurements of the ranging data, directions observations from the first and second periods, and approximated points coordinate which will be used in calculations (Figure 4).

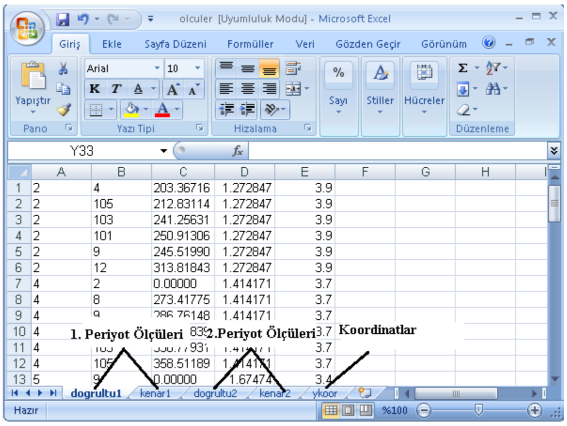

Figure.4 The screenshot of data in Excell file data

The codes which were necessary for the program to get data from this file were assigned. Then, the program takes this data sequentially and adjusts the first and the second period measurements with free adjustment and determines outlier measurements with Pope method. After then the partial trace minimum adjustment has been performed according to the fixed points(Figure 5).

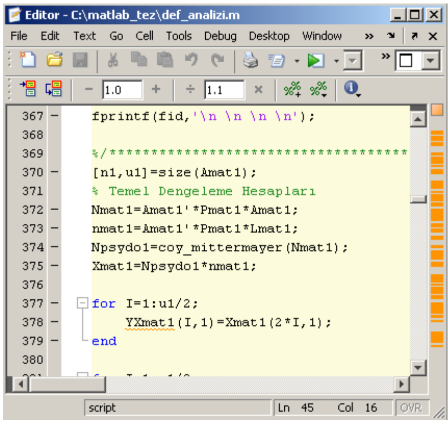

Figure 5. Screenshot of the code written in the basic adjustment operations

After completing the adjustment processes, a global test is first run for fixed points and the localization process for fixed points is carried out. Following the deformation analysis based on fixed point, mass adjustment was done for object points which were converted into fixed point due to their movement. Later, global test run for whether there is a deformation in the moving network points and object points and localization of deformed points is performed (Figure 6).

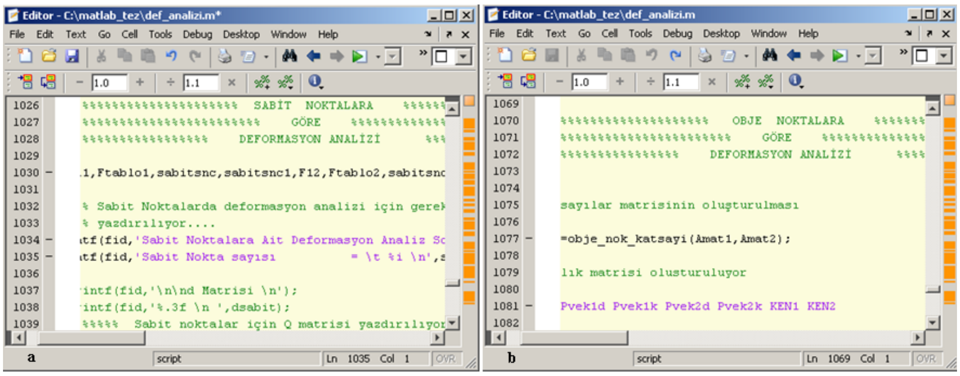

Figure 6. a) Deformation analysis according to Reference points, b) Deformation analysis according to Object points

After completing all the calculations and analysis, the program saves data from adjustments for first and second periods, global and localization test of fixed points, mass data adjustments, global test of object points and localization of deformation at the object points in a “txt” file called as “results.txt”. (Figure 7).

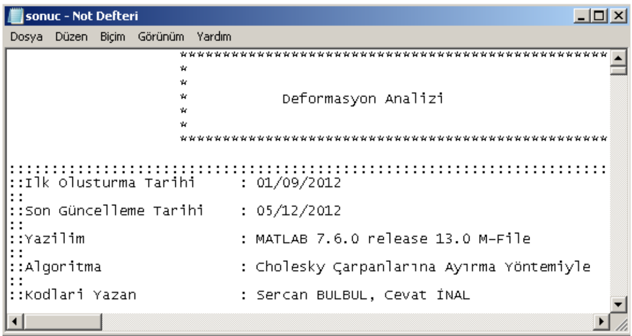

Figure 7. Screenshot of the results file

4. CONCLUSIONS

Different methods of analysis are used in the evaluation of deformation measurements. In this study, Cholesky Factorization Method which is one of the static evaluation methods used in the analysis of deformation is examined theoretically and directions observations and ranging data obtained from the Ermenek Dam for two periods in December 2010 and in June 2012 were separately evaluated according to directions observations and direction observations + ranging data separately and the results were compared.

If reference points and object points on the network are initially known, Cholesky Factorization Method can easily be applied. It is a suitable method for programming. Movement analysis can be made with uncorrelated difference vector.

When the measurements carried out on the Ermenek Dam in December 2010 and June 2012 were evaluated, with 95% statistical confidence, any deformation was not observed on reference points 4, 5, 6, 7, 8, 9, 13 and object points 104, 502 in the evaluation done according to direction observations and on reference points 3, 6, 7, 8, 9 in the evaluation according to direction observation + ranging data. At the points exposed to deformation, movements were less than 6mm. and it doesn't effect the result of movement analysis in the measurement plan. Using the MATLAB 7.0.6 Relase 13.0 M-File, a program developed by us and planned to male more professional in the future was used in calculations.

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