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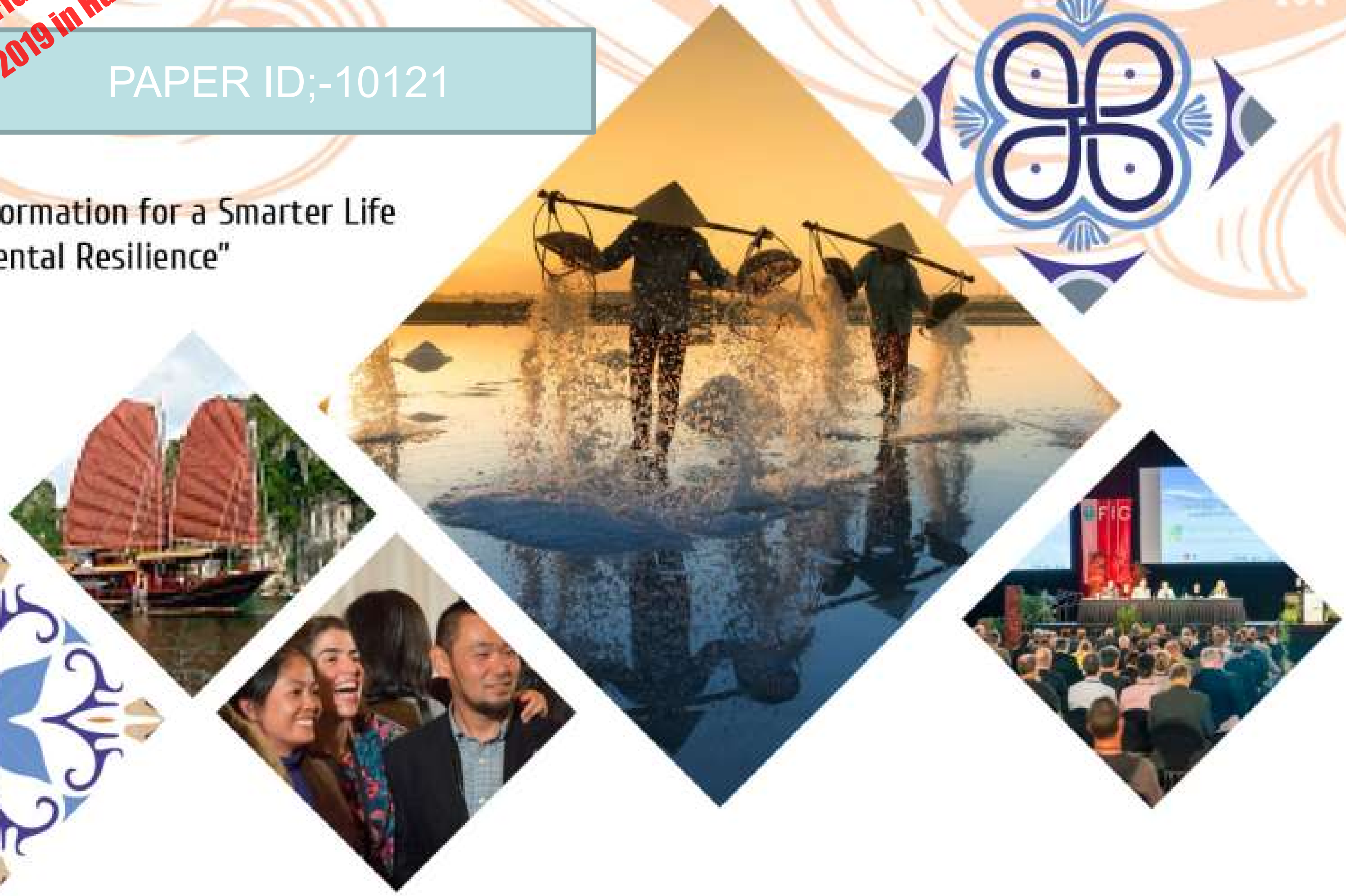
"Geospatial Information for a Smarter Life and Environmental Resilience"



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THEORETICAL PROCESS ON FORCES EXTENUATING THE EQUILIBRIUM TIDE: AN OVERVIEW

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INTRODUCTION

- The main goal in sea tidal elements and analysis is to determine a scientific form of the gravitational attraction on the global ocean caused by the presence of the moon and the sun (Haigh et al., 2011).

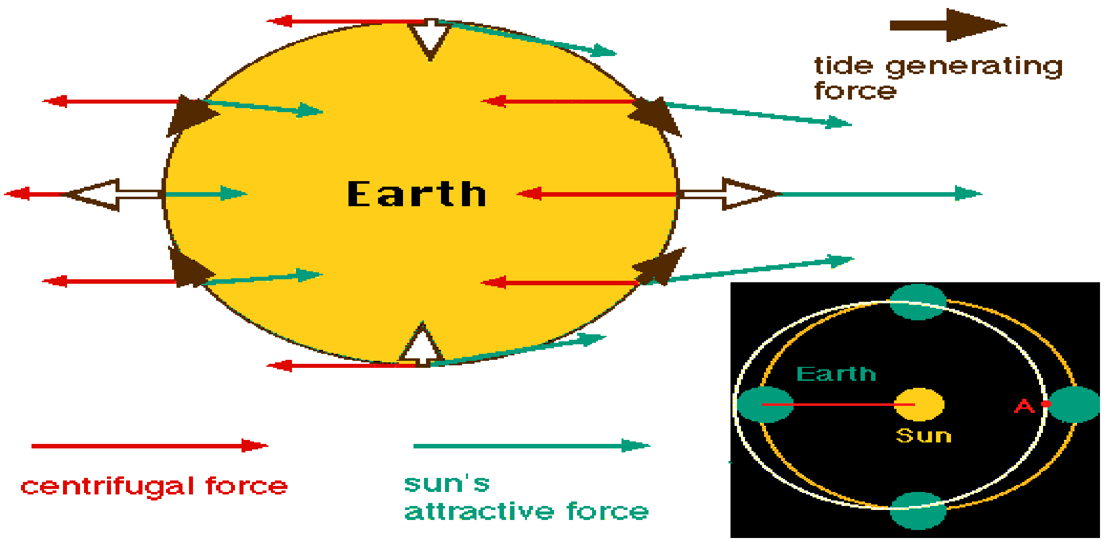
- The equilibrium hypothesis established by Newton (Coughenour *et al.*, 2009; Rahman, *et al.*, 2017; Zilrn *et al.*, 2017). Roosbeek, (1996) clarifies well the forces that generate the tides.

- The observed dominant semi diurnal periodicity of ocean tide was well clarified by Newton's equilibrium theory of tides (Chao and Ray, 1997; Bryden *et al.*, 2007).

TIDAL GENERATING FORCE

- The tidal force is the impact of gravity on a body, caused by the presence of a secondary body. On Earth, therefore the tidal force is caused by the Moon and the Sun, the tidal force and its potential are produced by the combined effect of the gravity of the earth and the moon (Matsuda et al., 2015)

TIDAL GENERATING FORCE



- The tide generating force
- Source: - http://oceanography.asu.edu/Oc_Nov14_pos.pdf (29/10/2018)

$$F = G\frac{m_e m_l}{R_l^2}$$

TIDAL GENERATING FORCE

- The gravitational force exerted by a celestial body (moon, sun or star) is directly proportional to its mass but inversely proportional to the square of the distance.

$$F = G\frac{m_e m_l}{R_l^2}$$

- were F is the gravitational force, G is the universal gravitational constant, m_e is the mass of the earth m_l is the mass of the moon and
- R_l is the distance between them.

TIDAL GENERATING FORCE

Difference (P1 – O) is the Tide generating Force (TGF) =

$2G\frac{mm_1 a}{(R_l)}$

At P2 the force away from the moon is = - [redacted]

While at P3 the force is directed towards O

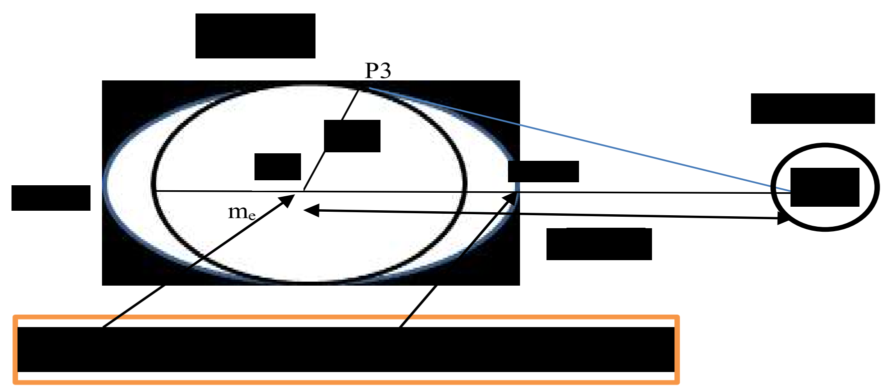


Figure 2.3 Position of the Earth - Moon System
Source http://www.incois.gov.in/documents/ITCOocean/ITC001/ppts/L3-Generation%20of%20Tides.pdf (10/20/2018)

TIDAL GENERATING FORCE

- Considering the earth – moon system a particle of Mass m at P1 on the surface of the earth, therefore, force toward the moon is given by

$$F_{P1} = \frac{Gm}{(R-a)^2}$$

- Where G is the gravitational universal constant, ($6.67 \times 10^{-11} Nm^2Kg^{-2}$).
- The centrifugal force which is the force at the center of the earth at point O is given by
- $$F_O = \frac{G\,m}{R^2}$$

TIDAL GENERATING FORCE

- The Difference Between the forces is the tide producing force at point P1,Therefore is,
- $$TPF = F_{P_1} - F_o$$

$$TPF = \frac{Gm}{(R-a)^2} - \frac{Gm}{R^2}$$

- Where TPF is Tide Producing Force, which gives the net force toward the moon.

$$TPF = \left[\frac{1}{(R-a)^2} - \frac{1}{R^2}\right] Gm$$

$$= \left[\frac{1}{\frac{(R-a)^2}{R^2}} - 1\right]\frac{Gm}{R^2} = \frac{Gm}{R^2}\left[\frac{1}{\left(1-\frac{a}{R}\right)^2} - 1\right]$$

TIDAL GENERATING FORCE

- The Expression

$$\left[\frac{1}{\left(1 - \frac{a}{R}\right)^2} \right]$$

- can be expanded using binomial theory, therefore the force at P1 becomes

$$TPF_{P_1} = \frac{2Gma}{R^3}$$

- Comparable thought demonstrates that, for a particle at P2, the gravitational attraction is excessively week to balance the centrifugal force, which gives rise to a net force away from the moon with the same strength as the force at P1 and is given by

TIDAL GENERATING FORCE

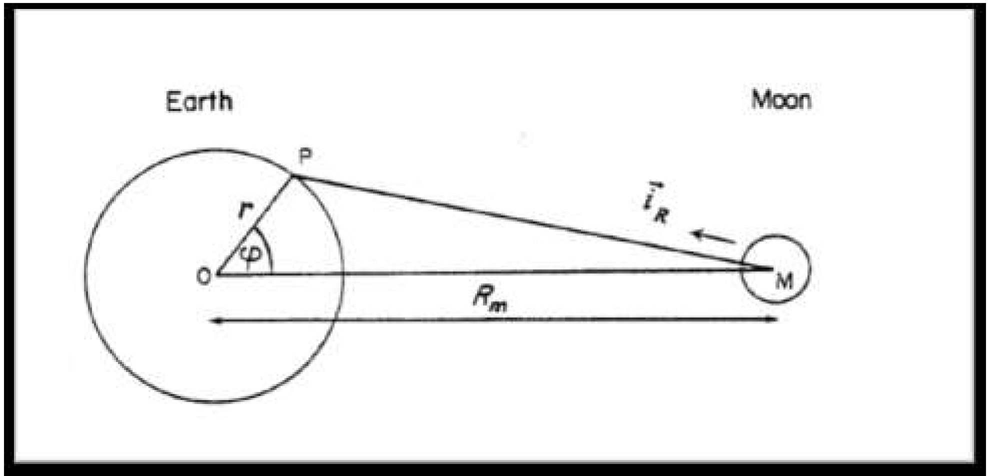
$$TPF_{P_2} = -\frac{2Gma}{R^3}$$

- It is also easy to prove that the net force at P3 is directed towards the center of the earth by making use of the approximation. $\sin(\emptyset) = \frac{a}{R}$

- the force at P3 is also given by $$TPF_{P_3} = \frac{Gma}{R^3}$$

TIDE GENERATING POTENTIAL

Geometry of calculation of the tidal potential



- Since the location of point P on the surface of the earth in the above, gravitational potential can be inscribed in terms of the lunar angle θ, the radius of the earth r and the distance Rm between the earth and the moon, using cosine rule we have the

$$PM^2 = r^2 + R^2 - 2rR\cos\theta$$

$$PM = \frac{R}{R}\left(R^2 + r^2 - 2Rr\,\mathrm{Cos}\,\theta\right)^{\frac{1}{2}}$$

$$V = \frac{Gm}{R}\left[1 + \frac{r}{R}\cos\theta + \frac{1}{2}\left(\frac{r}{R}\right)^2\left(3\cos^2\theta - 1\right) + \frac{1}{2}\left(\frac{r}{R}\right)^3\left(5\cos^3\theta - 3\cos\theta\right)\right]$$

$$\therefore V = \frac{Gm}{R}\left(1 + \frac{r}{R}L_1 + \left(\frac{r}{R}\right)^2 L_2 + \left(\frac{r}{R}\right)^3 L_3 \ldots\ldots\ldots\right)$$

Where Ln are Legendre polynomials given by

$$L_0 = 1$$

$$L_1 = \mathrm{Cos}\,\theta$$

$$L_2 = \frac{1}{2}\left(3\,\mathrm{Cos}^2\theta - 1\right)$$

$$L_3 = \frac{1}{2}\left(5\,\mathrm{Cos}^3\theta - 3\,\mathrm{Cos}\right)$$

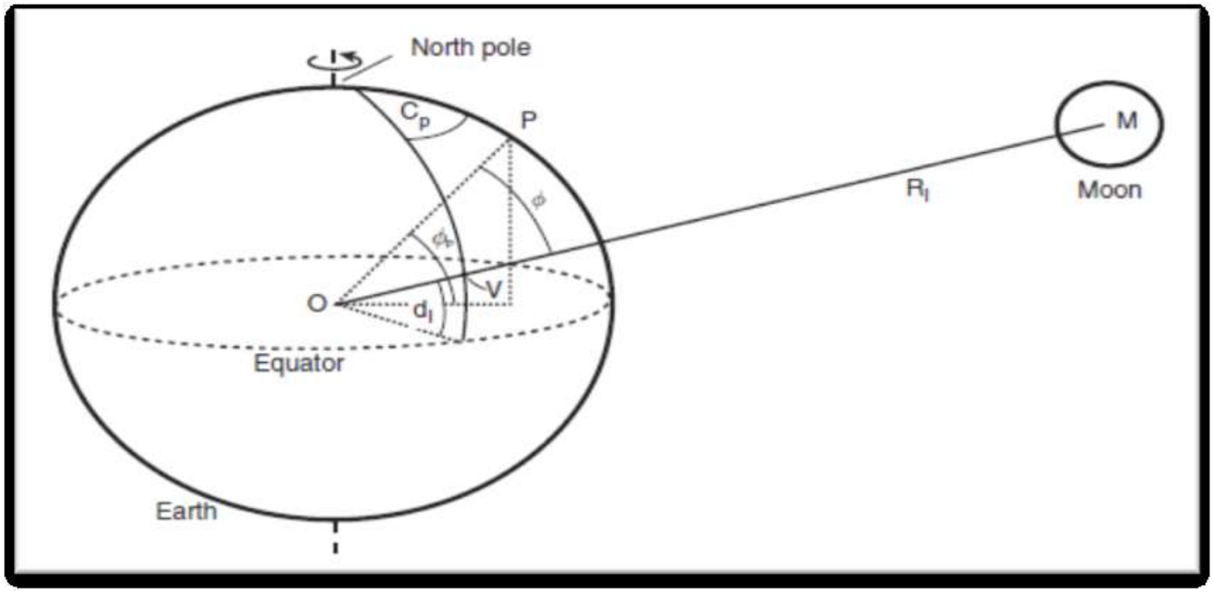
$$-\frac{\delta V}{\delta (r \cos \theta)} = G \frac{m}{R^2}$$

$$\Omega_p = -\frac{Gm}{2R^3} r^2 \left(3 \operatorname{Cos}^2 \theta - 1\right)$$

The Equilibrium Tide

- The Equilibrium Tide is defined as the elevation of the sea surface that would be in equilibrium with the tidal forces if the Earth were covered with water to such a depth that the response of the tidal generating forces would be instantaneous

The Equilibrium Tide



- The three-dimensional location of a point P on the Earth's surface relative to the sub-lunar position. Source

The Equilibrium Tide

$$\operatorname{Cos}\phi = \operatorname{Sin}\phi_p \bullet \operatorname{Sin} d_1 + \operatorname{Cos}\phi_p \operatorname{Cos} d_1 \operatorname{Cos} C_p$$

Late: - $\phi_p = \theta'$; $d_1 = \delta$; $C_p = \Upsilon$

- Substituting the value of Cos φ in above equation into $\Omega_p = -\frac{Gm}{2R^3} r^2 \left(3\operatorname{Cos}^2\theta - 1\right)$
- we have

$$= -\frac{3Gmr^2}{2R^3}\left(\left(\sin\theta \bullet \sin\delta + \cos\theta\cos\delta\cos\Upsilon\right)^2 - \frac{1}{3}\right)$$

$$-\Omega_p = \frac{3}{2} rGm\left(\frac{r}{R}\right)^3\left[\frac{3}{2}\left(\sin^2 d_1 - \frac{1}{3}\right)\left(\sin^2\phi_p - \frac{1}{3}\right) + \frac{1}{2}\sin 2d_1 \sin 2\phi_p \cos\Upsilon + \frac{1}{2}\cos^2 d_1 \cos^2\phi_p \cos 2\Upsilon\right]$$

- The above equation is used to compute equilibrium tide.

Conclusion

- In the above equation, the period associated to cos2θ is termed as semi – diurnal tides, Semidiurnal tides are those with a period of about half a day,
- while the term consists of sin2 θ is called the diurnal tides, diurnal tides are those with a period of about a day. And
- the term connected to 3sin2 θ -1 are long period tides. For the lunar tidal potential, those periods are nearly 12hours, 24 hours and 14 days respectively. For the solar tidal potential, they are close to 12 hours,24 hours, and 180 days respectively.
- .

Conclusion

- The equilibrium Tidal theory is a source for the definition of the harmonic frequencies by which the energy of the observed tides is distributed, and is also imperative as a reference for the observed phases variables. and amplitudes of the tidal constituents. Its practical usage is based on Doodson’s development and on the astronomical variables

THANK YOU !!!