The Meridian Arc Measurement in Peru 1735 – 1745

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Key words: Peru. Meridian. Arc. Triangulation.

ABSTRACT:

In the early 18th century the earth was recognised as having some ellipsoidal shape rather than a true sphere. Experts differed as to whether the ellipsoid was flattened at the Poles or the Equator. The French Academy of Sciences decided to settle the argument once and for all by sending one expedition to Lapland- as near to the Pole as possible; and another to Peru- as near to the Equator as possible. The result supported the view held by Newton in England rather than that of the Cassinis in Paris.

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1. BACKGROUND

The story might be said to begin just after the mid 17th century when Jean Richer was sent to Cayenne, S. America, to carry out a range of scientific experiments that included the determination of the length of a seconds pendulum. He returned to Paris convinced that in Cayenne the pendulum needed to be 11 lines (2.8 mm) shorter there than in Paris to keep the same time.

This result did not impress J.D. Cassini who considered the evidence to be too flimsy but soon other French scientists were obtaining results that suggested the pendulum required to be shortened as one went towards the equator.

Up until around this time the earth had been considered as a true sphere (i.e. could be defined by one measure be it radius or circumference), but Newton in England and the Cassinis in France were of the opinion that it was not truly spherical but flattened in some way. Unfortunately they had opposing theories – the Cassinis said that it was flattened at he Equator i.e. a prolate spheroid, whereas Newton said it must be flattened at the poles i.e. an oblate spheroid. The observations of Richer supported the theory of Newton.

The Cassini family embarked on the measurement of a series of meridian and parallel arcs in France and by 1734 five such arcs all gave the same conclusion - the earth was prolate. Such a wealth of proof could surely not be wrong but nevertheless the French Academy of Sciences was persuaded to try and settle the matter once and for all.

Arising from various suggestions it was decided to send one expedition as near to the Equator as possible and another as near to the Pole as possible. Such an arrangement should help to overcome the effects of instrumental error by virtue of being as far apart as sensibly possible in terms of latitude with one measure near the Equator and the other as near to the pole as possible. The French arcs had all been within a few degrees of one another and it was felt feasible that any small variations in the earth's shape might be swamped by instrumental and other errors over such a narrow range of latitude.

The expedition to Lapland was led by Pierre Maupertuis and included Anders Celsius (of thermometer fame) and Alexis Clairaut. They left Paris in April 1736 and returned there in August 1737 with results that again supported Newton.

The expedition to Peru left Paris in April 1735 and only arrived at the site in Peru in June 1736. They were to be there until 1743.

2. THE CAST

Before describing the work of the expedition it may be useful to give brief comments of the main cast.

Louis Godin. Age 31, son of a lawyer. He was a philosopher and astronomer. He became a member of the Academy of Sciences at 21 and edited the early volumes of its Memoirs. In 1733 he wrote a paper that discussed the shape of the earth.

Pierre Bouguer. Age 37, son of a Professor of Hydrography. He was an infant prodigy and succeeded his father as Professor at the age of 32. He invented the heliometer and constructed refraction tables. He was one of the first to use a knife edge support for a pendulum. While on the expedition he made the first observations into the effect of the attraction of mountain masses on the plumb line.

Charles Marie de La Condamine. Age 34, son of a wealthy tax collector. He took a deep interest in all things scientific.

After returning from Peru he took an active part in the establishment of an international standard of length. He favoured the seconds pendulum but died 20 years before the appearance of the metre which included the results of the Peru work in its determination.

Jorge Juan. Age 23, orphaned at 3 he was brought up by an uncle. He joined an elite naval organisation when he was 16 and later rose to Captain in the Spanish navy. He was elected to several scientific academies and in 1767 became Spanish Ambassador to Morocco.

Antonio de Ulloa. Age 19, son of an economist. He went to sea when he was 14 and later rose to Captain in the Spanish navy. On return from Peru he was captured by the English. After his release he was elected to several scientific academies and became Governor of part of Peru and later of Louisiana.

3. THE OUTWARD JOURNEY

The expedition to the Equator left Paris in April 1735 under the direction of the 3 Academicians Godin, Bouguer and La Condamine and when they reached Carthagena in November 1735 they were joined by the two Spanish Naval Officers Juan and Ulloa. This was a political arrangement since Peru was at that time under Spanish rule.

Wherever they called for provisions or a change of ship the time was spent doing local surveys or taking a variety of scientific observations. Even during the voyage they made numerous observations for magnetic variation, latitude and longitude; the last of which in places varied by as much as 40 from the existing charts.

From Carthagena there was a choice of route to Quito - one up river and over the mountains for some 900 miles, the other continuing by sea, crossing the isthmus of Panama and then sailing down the coast to a suitable disembarkation. Fear for the safety of the instruments led to their choosing the second route. They reached Guayaquil on 25th March 1736.

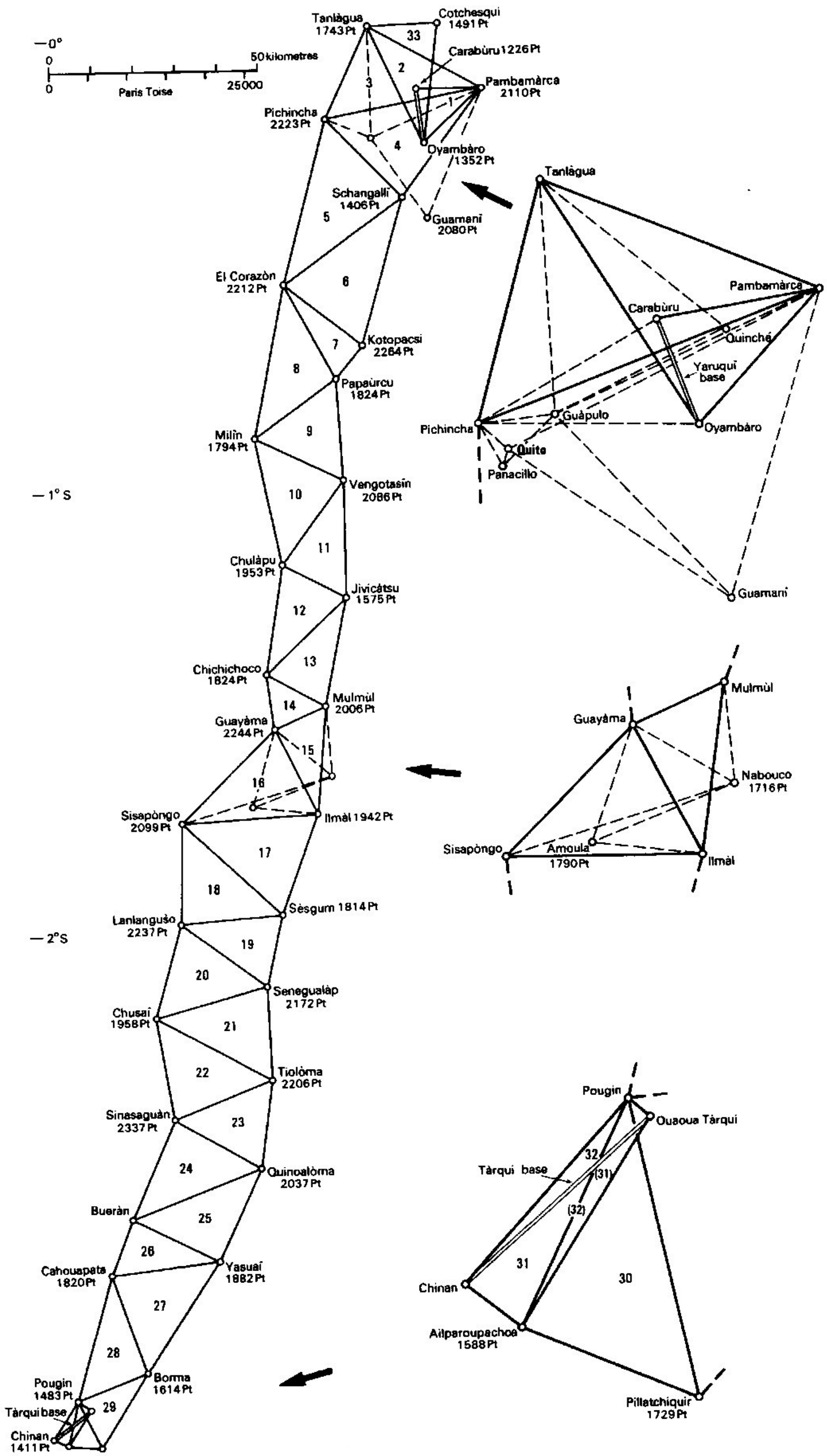
Among the observations they made a point of recording in the various publications were the temperatures. This they did in terms of the new Réaumer scale which at that time had 1000 as the freezing point and 1080 as the boiling point although it was not long before the thousand was dropped. Thus when they consistently recorded values in excess of 1025 (around 901 F) and you add in the heavy rains, numerous vicious insects, snakes, scorpions and the like it is not difficult to imagine the overall inhospitability of the area.

Before moving inland to the Andes they investigated the coastal area and mapped much of it by plane table. In particular they left an inscription on a rock at the Equator near Palmar.

From the coastal area Bouguer and La Condamine went separate ways towards Quito as did Juan and the others. All routes were extremely hazardous with dangerous rapids, frightening precipices, roads deep in bog and almost inpenetrable forest providing the ingredients of some of the worst travelling in the world. Even so ' scientific activities did not cease and La Condamine on rediscovering rubber was able to put it to good use. On the same journey he discovered platinum.

It was not until 10 June 1736 that the whole group was reunited in Quito. In climbing from sea level to Quito Bouguer found a change in his barometer of 8 inches while at the top of a nearby mountain it was down by a further 4 inches. From these and other barometric observations he derived a logarithmic relation between pressure and elevation.

Figure 1 below, shows the scheme of Bouguer and La Condamine


Tanlàgua 1743 Pt
Cotchesqui 1491 Pt
Carabùru 1226 Pt
Pambamàrca 2110 Pt
0°
50 kilometres
Paris Toise
25000
Pichincha 2223 Pt
Oyambàro 1352 Pt
Schangalli 1406 Pt
Guamani 2080 Pt
El Corazòn 2212 Pt
Kotopacsi 2264 Pt
Papaùrcu 1824 Pt
Milìn 1794 Pt
Vengotasin 2086 Pt
1° S
Chulàpu 1953 Pt
Jivicàtsu 1575 Pt
Chichichoco 1824 Pt
Mulmùl 2006 Pt
Guayàma 2244 Pt
Sisapòngo 2099 Pt
Ilmàl 1942 Pt
Sèsgum 1814 Pt
Lanlanguso 2237 Pt
2°S
Senegualàp 2172 Pt
Chusai 1958 Pt
Tiolòma 2206 Pt
Sinasaguàn 2337 Pt
Quinoalòma 2037 Pt
Bueràn
Cahouapata 1820 Pt
Yasuai 1882 Pt
Borma 1614 Pt
Pougin 1483 Pt
Tàrqui base
Chinan 1411 Pt
Tanlàgua
Pambamàrca
Carabùru
Quinché
Yaruqui base
Pichincha
Guàpulo
Oyambàro
Quito
Panacillo
Guamani
Mulmùl
Guayàma
Nabouco 1716 Pt
Sisapòngo
Amoula 1790 Pt
Ilmàl
Pougin
Ouaoua Tàrqui
Tàrqui base
Chinan
Ailparoupachoa 1588 Pt
Pillatchiquir 1729 Pt

4. THE TOPOGRAPHY

The high Andes, in survey terms gave extremely high survey stations - the lowest was at 2390 m and the highest at 4555 m. The survey scheme of Figure 1 shows the heights in Paris Toise, where 1 m = 1.95 Pt. Add to this that some were snow-covered, volcanic and subject to the severest of storms and they will be seen to be most uninviting. The tracks wended tortuous routes along precipitous mountain slopes where even the mules had difficulty finding footholds. On occasions a mountain would be occupied for a month to get an hour's observations. Food was scarce and the local Indian helpers unreliable.

5. BASE LINES

Survey started from a baseline at Yarouqui on the plains near Quito for which there had been 2 months reconnaissance. 6273 toise (12226 m) long it was divided into sections where the vertical angles were measured from one or other terminal to each intermediate point in turn.

After 8 days required to clear the line it was then marked with poles. The group divided into two measuring parties each with 3 wooden measuring rods of 20 ft length, tipped with copper. The rods were of different colours to ensure that they were always used in the same sequence. Daily comparisons were made against a field standard calibrated against the toise brought from France, and which came to be known as the 'Toise du Perou”. To keep the rods in correct alignment a cord was stretched between successive poles. A plumb line was used to ensure correct positioning of the bars if horizontal contact was not possible between them. The terminals were marked with millstones.

Agreement between the two measuring parties was to 3 inches. An alternative reduction procedure involved relating the intermediate points to the sloping line between the terminals. The measured lengths were then corrected to give the length of the slant line between the terminals. The two methods of reduction agreed to 0.012 t (23 mm).

The second baseline to the South was measured at Tarqui near Cuença. The method was similar except that for part of its length it was across a shallow pool and here the rods were floated into contact and tied to stakes to keep them in place. When reduced to the level of the Northern baseline it was 5259 toise (10 250 m).

With regard to temperature changes little is said in the printed documentation. Mean temperatures were taken as 10.5ºR at the Northern base and 16 – 17º R at the Southern one. When meaned they were the same as the temperature at which the toise bar had been standardised so no temperature corrections were deemed necessary.

6. ANGLES

The triangulation angles were measured by means of quadrants. These ranged from 21 inches (0.53 m) to 3 feet (0.91 m) in radius and were all on heavy cast iron stands. They were among the first such instruments to have telescopes attached and hence an essential requirement was meticulous verification that they related correctly to the arc. The graduations on the arc were marked by manual methods and similarly needed extensive checking procedures. La Condamine devised an interesting method for this check.

He set out a straight line, of say, 1500 toise (2925 m) and treated this as the diameter of a circle. The quadrant was correctly centred at the centre of the circle. The semi circle was subdivided into different numbers of equal parts e.g. 3, 4, etc so that successive checks could be made on 6º, 45º etc. For the smaller angles he set out a distance of 500 toise (975 m) from the quadrant and stretched a cord at right angles at the end. On this he put marks to indicate the tangents of successive degree values. In this way individual graduations could be checked to a few seconds of arc.

It must be remembered that a quadrant measured the angle in an inclined plane and as such had to be reduced to the horizontal. This was the first use of spherical trigonometry in geodesy.

For the astronomical observations zenith sectors of 12 ft (3.7 m) radius were used. Their limbs were limited to just a few degrees in keeping with the parameters of the chosen stars. Because of the extensive observing programme at each terminal the equipment was housed in a makeshift observatory. When transported to Peru the sectors had limbs of 30º but this was too cumbersome for convenience in observing so an ingenious method was devised to allow shorter limbs of only 3º or 4º to be built on the spot. Normal division by dividers and scale would have been too difficult to obtain acceptable accuracy on site. To decide on an acceptable size of limb they started from the intended range of zenith distances. For Tarqui this was 3º 22'. Now an angle of 15" less than this was equivalent to a chord of 1/17 of the radius. Thus for an arc of (8 inch 6 line) x 17 = 12 ft 6 inch. From an arc of 8 inch 6 line it was possible to step off this amount 17 times to determine the centre of the arc. The sector was fitted with a micrometer where 3 divisions were approximately 1 second.

Verification of the graduations both of the sector and the micrometer divisions was achieved in a manner similar to that for a quadrant.

When ready the sector was set up on a stable base in the observatory with the limb kept accurately in the meridian plane. It was possible to reverse (change face) as a means of eliminating some of the error sources. Temperature changes would have caused considerable problems so was kept as constant as possible.

Bouguer made a tenuous connect of the heights to sea level as illustrated in Figure 2. Isle de L' Inca was an island in a river near to the sea and Niguas was.the only other intervening point visible. Much of the difficulty with this connection was the continuous fog that blanketed the area between the mountains and the sea.

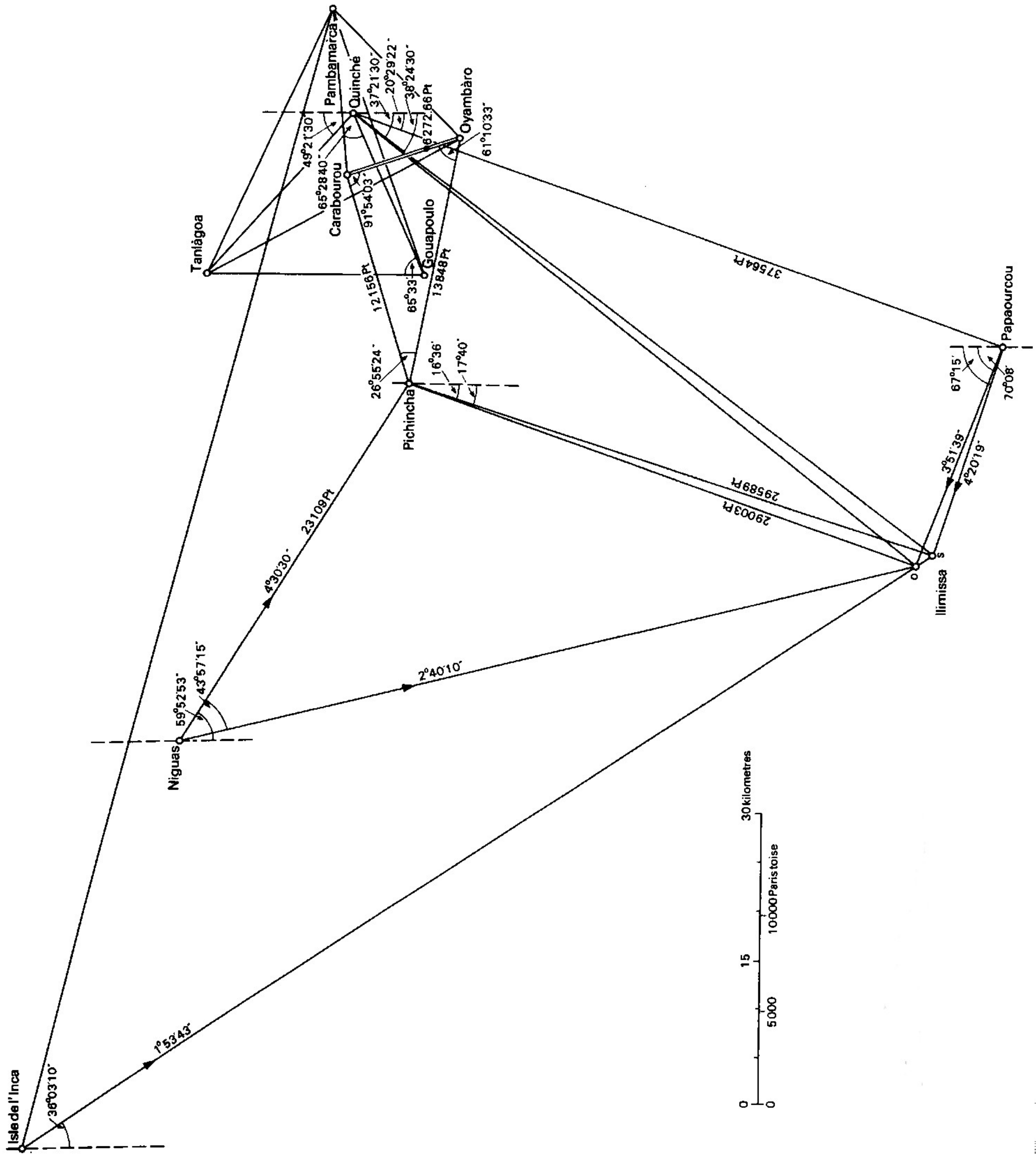


Figure 2. The connection of height values to sea level

7. RESULTS

Arc length	176 950 toise
Arc amplitude	3º 07' 01"
Whence 1º	56 749 toise at sea level.

These only form one set of results from several that were computed. This arose because:-

a) both Bouguer and La Condamine made separate calculations by slightly different routes and using slightly different values for some angles.
b) both Ulloa, and Juan with Godin, extended the arc somewhat and produced their own results.

The outcome of the long sojourn in S America was that the shape of the earth as suggested by Sir Isaac Newton was the correct one even if the actual values of the two minor axes would be refined over many more years and by virtue of many more arc measures. Whilst Newton derived a figure of 1:229 for the flattening of the ellipsoidal shape, a combination of the results from Lapland with those of Peru gave 1:310. Combining the Peru results with those in France gave 1:304; with USA arc of 39º parallel, 1:301; with Lake Erie arc of 42º parallel, 1: 289; with Lake Superior meridian arc, 1:289 and with eight other arcs, 1:278. Thus it will be seen that whilst there was agreement on the shape there was still a wide variation in the form of that shape. The results from further measures of arcs and by use of other methods gradually came together over the next 200 or so years to now be accepted to lie very near to 1:298.257

8. EXPERIMENTS

The survey work was supplemented with experiments in various related areas. The speed of sound was timed by use of a 4½ foot cannon and 8 pound cannon balls exploded on a mountain top. At two distant points - 5736 toise (11 180 m) and 6820 toise (13 290 m) the time was measured in half seconds to give a mean of 176.1 toise (343.22 m) per second.

Pendulum observations were made at various locations including Para near sea level and at Quito. The 28 inch pendulum was found to vibrate 31 times more often per day at Para than at Quito. La Condamine concluded that at the Equator if a 1000 pound weight was raised from sea level to the top of Mt Pitchincha, it would appear to lose 1 pound. From all this it was found that the length of a seconds pendulum at sea level at the Equator was 3 ft and 7.173 lignes.

The investigation that was of particular importance was the first attempt to determine the effect of a mountain mass on the plumb line. Bouguer took observations in the locality of Mount Chimborazo, Figure 3, and his name is retained today in relation to gravity anomalies. He chose 2 stations more or less on the same parallel one 1753 toise (3 415 m) from the centre of the mountain and the other at 4 572 toise (8 190 m). Meridian height observations on separate days to several stars, balanced north and south gave results that indicate an attraction of 7½".

He acknowledged that these results were not particularly good because the value for attraction was of the same magnitude as the accuracy of his observing procedures and thus neither proved nor disproved his theories.

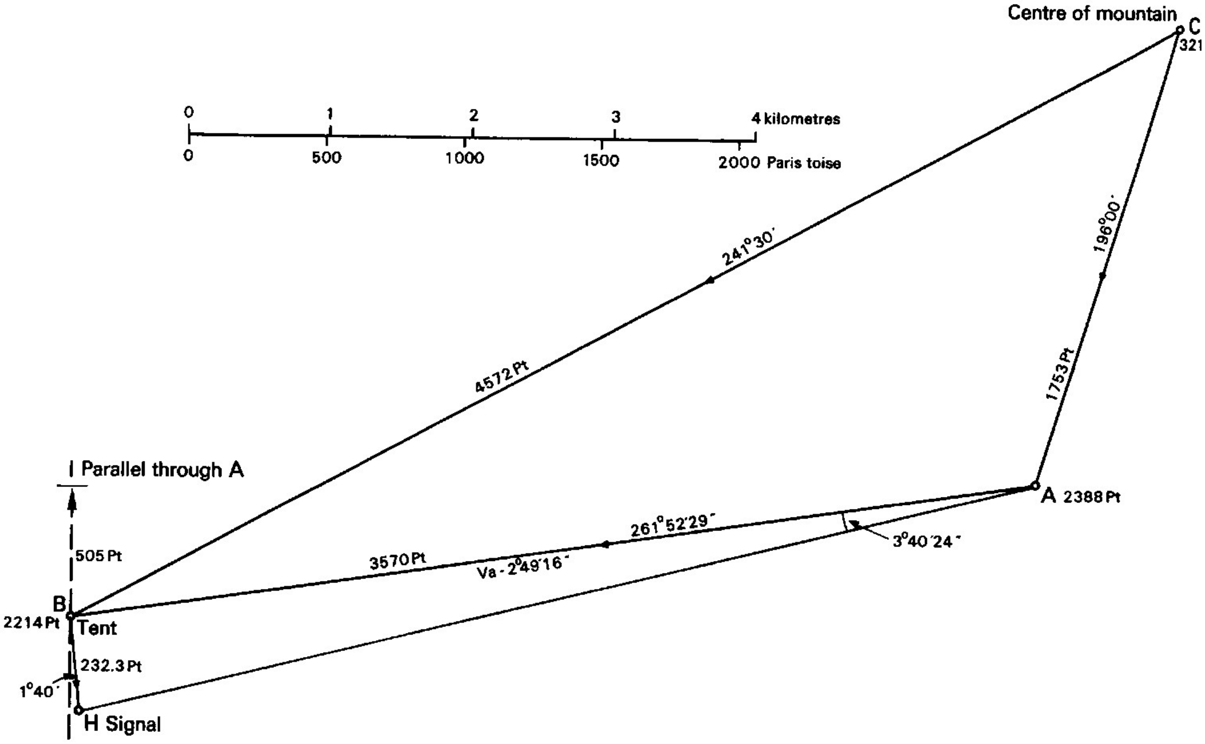


Figure 3. Points in the experiments at Chimborazo

Much barometric work was done during the expedition particularly for deriving the heights of the stations. The range of values from sea level to the summits of the Andean peaks ranged from around 28 inch of mercury at sea level to below 16 inches on the summit of El Coraçon. This led to the formulation of the first relationship between difference of height and change of pressure. The difference of height from a to d where the height values for a and b are known was given as:

Diff ht = Ht. A $(L_a – L_d)/(L_a – L_b)$

Where L= Log. of the barometric height reading

A = initial known height of point A in toise

a, b, d = barometric height readings at A, B and D in twelfths of lignes (12 lignes to 1 inch).

To evaluate this expression the Academicians used 10 figure logarithmic tables.

When the group travelled to S America they took with a variety of standard bars made of different materials. They experimented on these to find the various expansions for a rise of 10ºR in temperature. The ½ toise of iron changed by 0.132 ligne whilst the ½ toise of copper changed by 0.192 ligne. That in brass, by 0.200 ligne. From all these observations when Bouguer finally computed the length for 1º of the meridian he applied a correction of +7 toise for the variation of the metal toise as opposed to the wooden rods.

It will be seen that not only was this an expedition to measure the length of a degree of a meridian arc but also one that had to push forward the frontiers of the science of the time to get the best possible results.

9.COMMEMORATION

A commemorative plaque was fixed to the wall of the Jesuit Church in Quito and two pyramid shaped structures were built at the ends of the Yarouqui base but these led to considerable controversy. The basic cause of the trouble was the wording and decoration that was used. In particular the Spanish were very upset since they thought it slighted them and numerous alternative wordings were considered. Soon after their construction however the pyramids were destroyed on the orders of the Spanish Government. La Condamine was furious and raised further objections when he returned to Paris. In 1836 they were restored but in different positions. Various other plaques and monuments have appeared and disappeared over the years.

Particular mention should be made however of an epitaph in the Church at Cuença to the memory of Dr Jean Senierques, doctor to the expedition, who was murdered in the town in 1739. The moral is -do not dabble in the problems of a jilted lady.

10. THE RETURN

The members of the group split up and made their separate ways back to Europe. Three however are worthy of particular mention, La Condamine rather than return the way he had come, decided to travel down the Amazon from the Andes to the river mouth. Despite the extreme hazards he continued to make all sorts of scientific measurements as he went. The adventures were numerous but some 4½ months after leaving Tarqui he reached Para on the north coast. After waiting 6 months for a ship he finally reached Paris on 23rd February 1745.

The Spanish officers returned in separate ships after sailing round Cape Horn. Leaking like sieves they made slow progress towards Europe but unfortunately the ship with Ulloa on board was captured by the English. He was imprisoned for some while in Louisburg and then in England before returning to Madrid on 25th July 1746.

One member of the expedition, Jean Godin, and his wife Isabela, did not return to Paris until 1773, but that is another story.

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BIOGRAPHICAL NOTES

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1958-1960 Surveyor with the National Coal Board Opencast Executive
1960-1962 Surveyor with Hertfordshire County Council
1962-1964 Provincial Surveyor with Western Nigerian Government
1964-1968 Experimental Officer at Mapping & Charting Establishment
1968-1989 Principal lecturer in Land Surveying at Portsmouth Polytechnic (later University)

Member of Royal Institution of Chartered Surveyors (RICS) since 1961
On RICS Land Surveyors Divisional Council 1967-1996
Honorary secretary to the Land Surveyors Divisional Council 1968-1980
Secretary to FIG Commission 6, Engineering Surveying 1979-1987
British Standards Institution Working Groups related to surveying 1971-1996
International Standards Organisation Working Groups related to Surveying 1979-1996
Compiler of regular monthly page in *Chartered Surveyor* journal 1968-1980
Technical organiser of RICS Land Surveying Biennial Conferences 1958-1980
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Author of numerous technical papers and the following books:

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