Optimum Weight in Thin Plate Spline for Digital Surface Model Generation

Sina TAGHVAKISH and Jalal AMINI, Iran

Key words: weight, Thin Plate Spline, point density, Voronoi algorithm, bending, adjustment.

SUMMARY

The digital surface model (DSM) is used for several purposes in photogrammetry and remote sensing such as orthoimage production, contours derivation, extraction of high information such as buildings and trees, and used in GIS.
Creation of a DSM from 3-D sparse points that can be derived from stereo imagery, range data (e.g. laser data) can be done with several mathematical interpolation models. In this paper, thin plate spline is used for digital surface modeling. Determination of suitable weight is an important problem in thin plate function for a surface. The Voronoi algorithm has been proposed as a method for determination of the weights in thin plate spline. In this paper, we tested the method for different surfaces. The results shown that the thin plate spline can be independent of weight.

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1. INTRODUCTION

TPS is the abbreviation for the Thin Plate Spline. The mentioned concept was first introduced by J. Duchon in the 1976 [1]. He and J. Meinguet showed that the radial based functions admits a solution and only one (under the condition-always true-that the sample be counted at least on three non-aligned points) [2]. These days, TPS has been found for so many uses, as an example, geophysical applications in aeromagnetic & gravimetric surveying[3], modeling of fingerprints[4], medical researches[5], converting elevation contours to a grid[6][7]. Also it has been used for production of intermediate elevation contours [6]. Sjowal in his experiment on different methods of topographic information system in 1:10000 of Lithuanian for eliminating relief displacement of IKONOS satellite images uses TPS for constructing digital elevation model in contrary to polynomial methods used ERDAS software and proved that TPS is a more confidence method and proposed the using of PCI Geomatica OrthoEngine [12]. MicroStation Descartes 7.0 used TPS for rubber sheeting [3].
In other research Boztosun proved that TPS is compatible to Advection-Diffusion equations and is method with the unique answer for these kinds of differential equations.

In this article, section two, allocated to define mathematical concept and a solution for the establishment of the model, because of the mentioned solution employed, weight problems introduced, so one must allocated different section to discuss it [section 3]. In section 4, based on section 2 and 3, methods have been employed on elevations of two different zone-flat & mountainous – then the result analyzed. In section 5 summarized and the suggestions indicated.

2. THIN PLATE SPLINES MATHEMATICAL MODEL

The Thin plate spline is a physically based 2D interpolation which represents a thin metal sheet that is constrained not to move at the grid points, and is free from any external force relied upon control points, from this sight the bending energy in control points should have been minimized. This bending energy function is shown by nether formulation in 2D space [8],

$$I\left[f(x,y)\right]=\iint_{\Re^2} f_{xx}^2+2f_{xy}^2+f_{yy}^2\ . \qquad (1)$$

In equation number (1) f represents a height in 3D models. This equation is invariant under translation, rotation, or scaling of either set of control points[7][20], support second norm of the surface and when it is minima it means that the bending of the surface in control points have to become minimized and the slope variation reduced to minimum in the tangential plates.

From another point of view the Thin Plate Spline as an approximation for the real surface of the control points will enable user to get to the control points as near as possible. Therefore a constraint of the distance between the control points and the surface must also be minimized so L2 norm is used,

$$E_{TPS} = \sum_{i=1}^{n} \mu_i (f(x_i, y_i) - z_i)^2 + \iint_{\Re^2} f_{xx}^2 + 2 f_{xy}^2 + f_{yy}^2 \,. \qquad (2)$$

Solving of the bending part of the equation 2 is just like inhomogeneous biharmonic equation, $\nabla^4 f + \sum_{m=1}^{M} \lambda_m \delta(x - x_m) = 0$ [8][9]. Which has a unique answer in the polar coordinate system [7]. Duchon and Suber (1992) achieved the under gone answer for equation (1),

$$\begin{aligned} &\sum_i a_i \phi(r_i(x_j + y_j)) + b_0 + b_1 x_j + b_2 y_j = z_j ; (j = 1,...,n) \\ &\sum_j a_i = 0; \sum_j a_i x_i = 0; \sum_j a_i y_i = 0; \end{aligned} \qquad . \qquad (3)$$

By considering μ_i weights and adjustment equation [equation 2] upper answer can be rewritten as follows:

$$\frac{8\pi a_j}{\mu_j} + \sum_i a_i \phi(r_i(x_j + y_j)) + b_0 + b_1 x_j + b_2 y_j = z_j ; (j = 1,...,n) \,, \qquad (4)$$

ϕ components are radial base functions which can be in $\phi(x, x_j) = \phi(r_j) = r_j^{2m} \log(r_j)$ form and have C^{2m-1} continuity [10], selection of form of ϕ (order, m) is out of the argument of this article, therefore based on usual shape of the radial base functions in TPS for DTM, we assume m is 1; so ϕ have C^1 continuity and have $\phi(x, x_i) = \phi(r_i) = r_i^2 \log(r_i)$ shape; in which r is Euclidean norm in 2D space ($r_i(x_j, y_j) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$).

Duchon offered a straight solution for this equation series which they are linear system with n+ 3 equations,

$$\left(\begin{array}{cccc|ccc} 8\pi/\mu_1 & c_{21} & \cdots & c_{n1} & 1 & x_1 & y_1 \\ c_{21} & 8\pi/\mu_2 & \cdots & c_{n2} & 1 & x_2 & y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ c_{1n} & c_{2n} & \cdots & 8\pi/\mu_n & 1 & x_n & y_n \\ \hline 1 & 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \cdots & x_n & 0 & 0 & 0 \\ y_1 & y_2 & \cdots & y_n & 0 & 0 & 0 \end{array}\right) \left(\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_n \\ \hline b_0 \\ b_1 \\ b_2 \end{array}\right) = \left(\begin{array}{c} z_1 \\ z_2 \\ \vdots \\ z_n \\ \hline 0 \\ 0 \\ 0 \end{array}\right); \qquad . \qquad (5)$$

$$c_{ij} = \phi(r_i(x_j, y_j))$$

Which can be written as:

$$\begin{pmatrix} C & F \\ F^T & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} Z \\ 0 \end{pmatrix}.$$

Coefficient matrix $\begin{pmatrix} C & F \\ F^T & 0 \end{pmatrix}$ is a positive symmetrical matrix. C. Carasso in Baranger (1991) suggests solving the system (5) by QR factorization as follows:
Where Q is an n by n orthogonal matrix and R is n by 3 matrix in the following form:

$$R = \begin{pmatrix} U \\ 0 \end{pmatrix}; Dim[U] = 3 \times 3 . \qquad (6)$$

With U (3, 3) triangular superior matrix. F is of rank3, since there are at least three non-aligned points, therefore R is of rank 3, and thus U is invertible,

$$\begin{pmatrix} C & F \\ F^T & 0 \end{pmatrix} = \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} Q^T & 0 \\ 0 & I \end{pmatrix};$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \qquad (7)$$

By replacing equation (7) in equation (5) we have:

$$\begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} Q^T & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} Z \\ 0 \end{pmatrix} \Rightarrow$$

$$\left(\begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix}^{-1} \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} Q^T a \\ b \end{pmatrix} = \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix}^{-1} \begin{pmatrix} Z \\ 0 \end{pmatrix} \right) \wedge (Q^{-1} = Q^T) \Rightarrow . \qquad (8)$$

$$\begin{cases} \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} d \\ b \end{pmatrix} = \begin{pmatrix} Q^T Z \\ 0 \end{pmatrix} \\ a = Qd \end{cases}$$

And by supersede $Q = (Q_{1_{n\times 3}} Q_{2_{n\times n-3}})$ and $d = \begin{pmatrix} d_{1_{3\times 1}} \\ d_{2_{(n-3)\times 1}} \end{pmatrix}$ one can write:

$$\begin{pmatrix} Q_1{}^T C Q_1 & Q_1{}^T C Q_2 & U \\ Q_2{}^T C Q_1 & Q_2{}^T C Q_2 & 0 \\ U^T & 0 & 0 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ b \end{pmatrix} = \begin{pmatrix} Q_1{}^T Z \\ Q_2{}^T Z \\ 0 \end{pmatrix}; . \qquad (9)$$

$$a = Q_1 d_1 + Q_2 d_2$$

One deduces $U^T d_1 = 0$, thus $d_1 = 0$, from equation $Q_2{}^T C Q_2 d_2 = Q_2{}^T Z$, one deduces d_2, from where $a = Q_2 d_2$; from the equation $Q_1{}^T C Q_2 d_2 + Ub = Q_1{}^T Z$, one can finally extract b.

It is necessary to notice that the C matrix is full. Therefore the matrix $Q_2{}^T C Q_2$ is also full. So, if we have more than 3 control points this method can never be singular, but weights μ_i in equation (4) are one of the weaknesses of these methods and answers touched by them, we will dicuss weights in TPS in the following section:

3. WEIGHT COMPARISON ON TPS

For simplicity of discussion, equation 2 can be rewriten as:

$$E_P = P\sum_{i=1}^{n} r_i (f(x_i, y_i) - z_i)^2 + I(f), \quad (10)$$

With $P = \sum_i \mu_i$ and $r_i = \frac{\mu_i}{P}$ that r_i is the relative weight of the point (x_i, y_i, z_i) and the P weight fixes the importance of the adjustment criterion,

$$e(f) = \sum_{i=1}^{n} r_i (f(x_i, y_i) - z_i)^2, \quad (11)$$

In relation to the curvature criterion $I(f)$.

3.1 Let us First Examine the Effect of the Weight P,

To $P > 0$, corresponds a unique thin plate spline surface, denoted f_P, minimizing, $E_P(f) = P \times e(f) + I(f)$; in the same way, to $Q > 0$ corresponds the surface f_Q, minimizing, $E_Q(f) = Q \times e(f) + I(f)$.

Because of the minimal character of $E_P(f_P)$ and of $E_Q(f_Q)$, one has,

$$E_Q(f_Q) \le E_Q(f_P)$$
$$E_P(f_P) \le E_P(f_Q) \quad (12)$$

Or else:

$$P \times e(f_P) + I(f_P) \le P \times e(f_Q) + I(f_Q) \quad (13)$$
$$Q \times e(f_Q) + I(f_Q) \le Q \times e(f_P) + I(f_P) \quad (14)$$

If $P > Q$ then $I(f_P) \ge I(f_Q)$ and $e(f_P) \le e(f_Q)$; it means that the surface f_Q is less bent than f_P, or “smoother”. Form another point of view, if P becomes arbitrarily too large, and becomes near infinite, f_P must have been perfectly adjusted to the sample.

3.2 We Suppose the P Weights henceforth Fixed, and we Consider the Weights r_i,

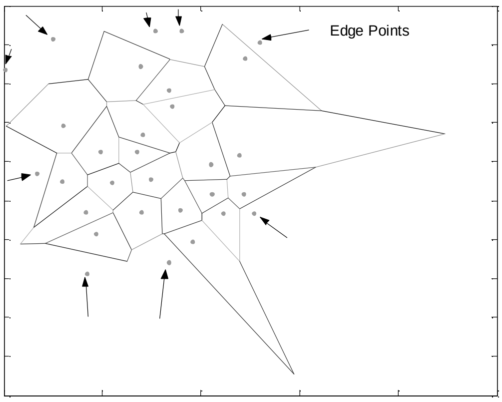


Figure 1:Voronoi Polygons

To The simplest choice is of course to give the same weight $r_i = 1/n$ (in this article we suppose, $r_i = 1$ so we don’t need to calculate P and P is equal to the number of the control points) but because of irregularity of points and differences in local density of control points one deduces using of Voronoi polygon areas for introducing weights[7][11]. Thus as we discuss this method briefly, we would emphasize on the point, which this article is based on.

Voronoi algorithm is a way to partition $\Re^2$ space. Those boundary conditions of these sets are bisector lines between neighbor control points.

$\Re^2$ Space has a dominion from $+\infty$ till $-\infty$ thus control points can never achieve the supremum or infimum of this space, therfore area of edge polygons must be separately computed (fig.1).

a) Normalized Voronoi with fixed P equal to one, μ_i in edge polygons are assumed to be zero.

In this method as you can see from its name, P is equal to one, r_i is a normalized area of Voronoi polygons and in edge polygons,

$$\mu_i = \begin{cases} r_i = \dfrac{A_i}{\sum_{j=1}^{n} A_j}; & if\,(i = 1,2,3,\ldots,n\ \&\ A_i \neq \infty) \\ 0; & if\,(A_i = \infty) \end{cases}, \tag{10}$$

From which A is Voronoi polygons area.

b) Normalized Voronoi with fixed P equal to one, μ_i in edge polygons area assumed to be one.

Weighs obeys the undergone relations:

$$\mu_i = \begin{cases} r_i = \dfrac{A_i}{\sum_{j=1}^{n} A_j}; & if\,(i = 1,2,3,\ldots,n\ \&\ A_i \neq \infty) \\ 1; & if\,(A_i = \infty) \end{cases}. \tag{11}$$

c) Normalized Voronoi with fixed weight P=100.

The surface constructed by normalized Voronoi, P=1 is a surface with softness more than contiguity of control points so based on what we said on the start of this section, P fixed brings itself forth. It is necessary to say P is a digit between 10^3 to 10^5. So, the four following equations analyse the versatility of P.

$$\mu_i = 100 \times \frac{A_i}{\sum_{j=1}^{n} A_j}; i = 1,2,3,\ldots,n\,. \tag{12}$$

d) Normalized Voronoi with fixed weight P=1000,

$$\mu_i = 1000 \times \frac{A_i}{\sum_{j=1}^{n} A_j}; i = 1,2,3,\ldots,n\,, \tag{13}$$

e) <u>Normalized Voronoi with fixed weight P=10000,</u>

$$\mu_i = 10000 \times \frac{A_i}{\sum_{j=1}^{n} A_j}; i = 1,2,3,...,n, \tag{14}$$

f) <u>Normalized Voronoi with fixed weight P=100000,</u>

$$\mu_i = 100000 \times \frac{A_i}{\sum_{j=1}^{n} A_j}; i = 1,2,3,...,n, \tag{15}$$

g) <u>Voronoi,</u>

This method is the comparison between, usual and normalized methods,

$$\mu_i = A_i; i = 1,2,3,...,n. \tag{16}$$

h) <u>Fixed weight equal to one,</u>

$$\mu_i = 1; i = 1,2,3,...,n \tag{17}$$

it means that P is the total number of control points, and r_i are equal to one.

4. EXPERIMENTAL RESULTS

In this project elevation data of a location with area of 910×804 m^2 includes 7330points taking to account. Based of thorium described in section 2 and proper weight decision a Matlab code written which some parts of the program can only be run under newer versions of it. This program after acquiring elevation data needs to select how much of data can be used by TPS model.

First of all, selection of points by their preliminary acquisitions order and not to regularize them, for this purpose one needs a start point number and a count of points must be used. Secondarily, regularize data and select a window of it based on construction of m × m network. For example first column of table 1 which named by "Data Location" represents these two methods of point selections. The differing on the last row, represent the first method, selecting points with point number 1 as start point and 1500 points data selected

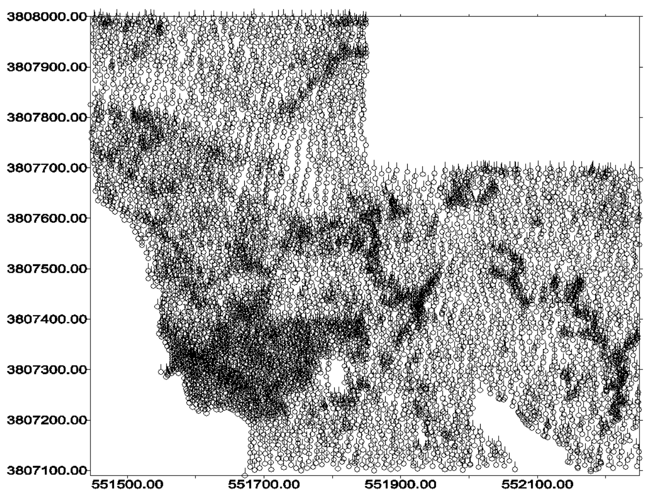


Figure 2: A view of scattering of original data in 2D

from original dataset. Or first row of this table is a selection of the window on the first row and the first column of the 5 × 5 network of datasets.

Table 1: Standard deviation of datasets shown in figure 4

Method → / Data Location	GCP no.	Norm. Voronoi (1at edge poly)	Norm. Voronoi (0 at edge poly.)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
5*5 section 11	**16**	**0.122**	**0.122**	**0.139**	**0.138**	**0.138**	**0.138**	**0.138**	**0.122**
5*5 section 12	**174**	**1.414**	**1.415**	**0.861**	**0.717**	**0.740**	**0.779**	**0.783**	**0.710**
5*5 section 13	**190**	**0.724**	**0.736**	**0.542**	**0.521**	**0.568**	**0.614**	**0.622**	**0.564**
5*5 section 21	**250**	**0.872**	**0.992**	**0.505**	**0.453**	**0.454**	**0.458**	**0.457**	**0.483**
5*5 section 22	**552**	**1.257**	**1.258**	**0.748**	**0.660**	**0.683**	**0.718**	**0.723**	**0.663**
5*5 section 23	**306**	**1.152**	**1.152**	**0.547**	**0.452**	**0.437**	**0.458**	**0.460**	**0.432**
5*5 section 33	**314**	**1.301**	**1.403**	**0.682**	**0.641**	**0.710**	**0.747**	**0.749**	**0.682**
5*5 section 43	**242**	**0.692**		**0.373**	**0.382**	**0.417**	**0.444**	**0.447**	**0.398**
5*5 section 44	**96**	**1.693**		**1.429**	**1.410**	**1.416**	**1.420**	**1.420**	**1.404**
From Pnt. 1to1500	**1000**	**1.401**		**0.867**	**0.654**	**0.536**	**0.507**	**0.510**	**0.504**

For comparison of weight defining methods mentioned in section 3, ten series of data with different dispersal selected from original data. After applying of methods mentioned on this article on every set of data for finding precision of every set, check points with ratio of 2 to 1 for control points selected have been chosen. Standard deviation and average of residuals for every weight definition methods computed on check points which you can find results on tables 1 and 2 also figure 4.

Table 2: Average of datasets shown in figure 4

Method → / Data Location	GCP no.	Norm. Voronoi (1at edge poly)	Norm. Voronoi (0 at edge poly.)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
5*5 section 11	**16.000**	**-0.032**	**-0.031**	**-0.045**	**-0.048**	**-0.049**	**-0.049**	**-0.049**	**-0.043**
5*5 section 12	**174.000**	**0.411**	**0.410**	**0.140**	**0.033**	**-0.034**	**-0.051**	**-0.051**	**-0.020**
5*5 section 13	**190.000**	**0.385**	**0.385**	**0.092**	**0.032**	**-0.009**	**-0.023**	**-0.025**	**0.001**
5*5 section 21	**250.000**	**0.123**	**0.124**	**0.056**	**0.040**	**0.040**	**0.041**	**0.041**	**0.035**
5*5 section 22	**552.000**	**0.140**	**0.142**	**0.088**	**0.042**	**0.021**	**0.008**	**0.006**	**0.016**
5*5 section 23	**306.000**	**-0.069**	**-0.068**	**0.003**	**0.005**	**0.017**	**0.019**	**0.019**	**0.012**
5*5 section 33	**314.000**	**0.177**	**0.239**	**-0.009**	**-0.054**	**-0.082**	**-0.093**	**-0.094**	**0.010**
5*5 section 43	**242.000**	**0.040**		**-0.022**	**-0.044**	**-0.068**	**-0.079**	**-0.080**	**-0.061**
5*5 section 44	**96.000**	**-0.390**		**0.112**	**0.133**	**0.139**	**0.143**	**0.143**	**0.117**
From Pnt. 1to1500	**1000.000**	**0.156**		**0.049**	**0.029**	**0.005**	**-0.006**	**-0.010**	**-0.013**

The notable point is the smoothness behavior of TPS in void areas is lake existence naught extrapolation in it[4](figure 3).

As it was expected, methods using normalized Voronoi polygon areas with P weight equal to one, because of greater bending effect than L2 norm discussed in equation 2 part 1 in section

3 in contiguity to control points relative to other methods showing a weakness and only in 16 point series that have a better answer. And it is because of lack of data and probably existence of unusual point in the set of data in comparison of these two methods (a,b). In fact participating edge points (a) have been a better response than not using them at all. Therefore in other methods we use weights equal to one for edge polygons.

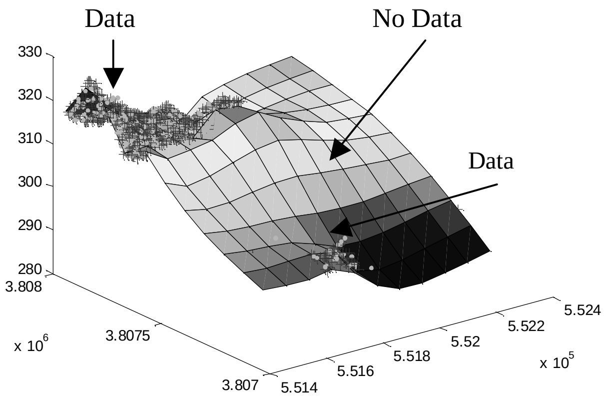


Figure 3: Smooth form of TPS surface

Voronoi methods with weights different from 1 are the best, but in figure 4 for the 242 point condition, 'c' have a less standard deviation than 'e', 'f' and 'd'. Therfore for every set of data the best P must be optimized (figure 4 methods f, e, d and c)

That is in the contrary with ideal best fitting described in ending of section 3.1 which P is leading to infinite.

Method 'g' have a better answer compared with 'b' and 'a', but it is not so much worse than 'c', 'd', 'e' and 'f'. so normal methods in TPS haven't any significant preferences.

Based on ten series as not expected, if μ_i considered fixed and equal to one (method h) there is no significant difference between its result and the best results of Voronoi methods. On the other hands Voronoi algorithm add a new kind of problems which the fixed weight of one doesn't. those problems are:

– The probability of existence of more than 3 control points on one circle and creation of embedded polygons(n points). Because of the existence of more than C_n^2 equitable perpendiculars, ambiguity in defining neighbor points come to existance.
– The problem of determination of these polygons created for these points is another problem of computer coding which is complex and time consuming
– Computing area of these irregular polygons is another problem.

Therefore revision necessities of indicated reasons for methods using Voronoi algorithm show their importance and make the writer to design another test for it.

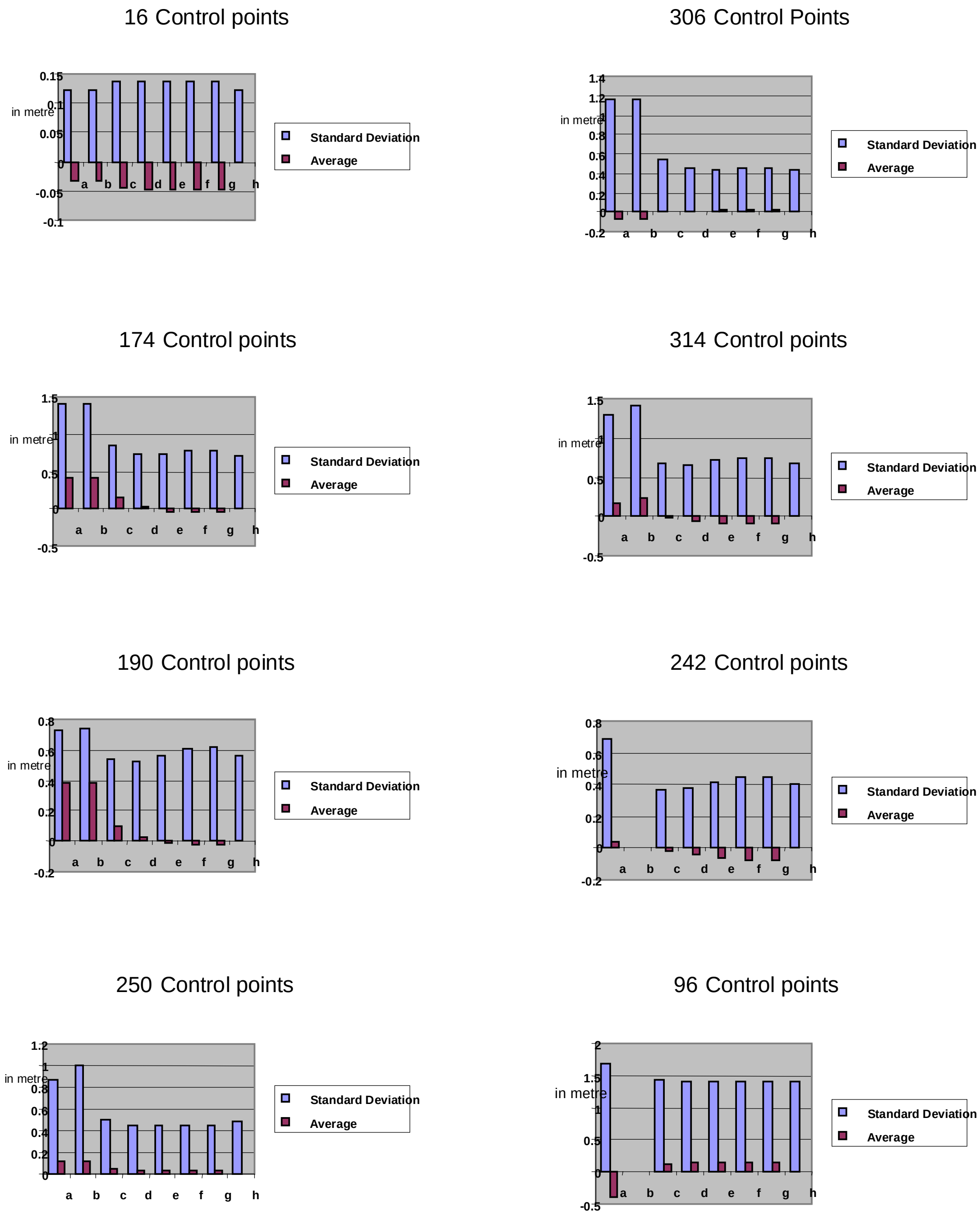


Figure 4: Standard deviation and average for 10 chosen series of data that, ”a” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and μ in edge polygons equals to zero, ”b” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and μ in edge polygons equals to one, ”c” is the Voronoi normalized areas method with fixed weight P equals to 100 (P=100), ”d” is the Voronoi normalized areas method with fixed weight P equals to 1000 (P=1000), ”e” is the Voronoi normalized areas method with fixed weight P equals to 10000 (P=10000), ”f” is the Voronoi normalized areas method with fixed weight P equals to 100000 (P=100000), ”g” is the method of Voronoi areas and ”h” is the fixed weight (equal to1) method.

the algorithm of Voronoi has purposed because of this reason:
this method is sensate of the density of control points, then in continuation ; has researched this significance in specific conditions of TPS and defined an experiment in this form that two adjusted areas of original data chosen, the first (I) has 454 points and second dataset (II) has 473 points. Control and check points have separated by 2:1 ratio till the created surface error by control points, to be determined by check points.

Density of control points in I, is 10.51 points per square kilometers and in data II is 10.37 points per km^2. Errors were computed for combination of two series, and results have been mentioned in table 3 and 4.
In the next stage, whereas the data of zone I not touched, zone II decreased to half and became 236 points. Again control and check points are separated in 2:1 ratio and the density of control points in II are decreased to 5.7 points per km^2 (i.e. less than half of II) results have been shown in row I+½II of tables 3and 4.

Table 3: Standard deveation calculated for analysing density effects

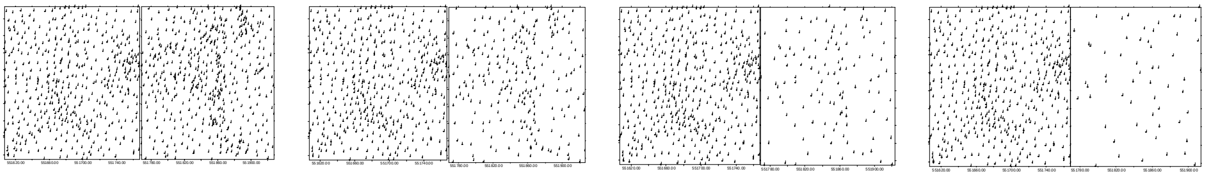
Method / Data Location	Norm. Voronoi (1at edge poly)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
I+II	**1.789**	**0.816**	**0.637**	**0.583**	**0.595**	**0.609**	**0.581**
I+½II	**1.308**	**0.685**	**0.623**	**0.634**	**0.634**	**0.635**	**0.615**
I+¼II	**2.638**	**1.004**	**0.648**	**0.648**	**0.622**	**0.638**	**0.620**
$I+\frac{1}{8}II$	**1.690**	**0.578**	**0.578**	**0.534**	**0.545**	**0.554**	**0.531**

In resumption while zone I still unchanged, dataset of ½II became half again and reached 119 points that is corresponds 2.58 points per km^2 density, in other word less than ¼ zone II. The results are shown in row I+¼II of table 3 and 4.

Table 4: Average calculated for analysing density effects

Method / Data Location	Norm. Voronoi (1at edge poly)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
I+II	**0.342**	**0.046**	**0.021**	**0.017**	**0.010**	**0.003**	**0.011**
I+½II	**0.110**	**-0.014**	**-0.013**	**-0.001**	**-0.001**	**-0.001**	**-0.009**
I+¼II	**0.412**	**-0.084**	**-0.090**	**-0.090**	**-0.081**	**-0.077**	**-0.082**
$I+\frac{1}{8}II$	**-0.045**	**-0.038**	**-0.038**	**-0.025**	**-0.017**	**-0.017**	**-0.026**

Now for last time data of zone ¼II becomes half and reaches 60 points and density of 1.32 points per km^2 which is one eights of zone II data. Results have been displayed in $I+\frac{1}{8}II$ row of tables 3 and 4.



(a) (b) (c) (d)

Figure 5: sensivity of TPS relative to control points density: a) I+II, b) I+½II, c) I+¼II, d) $I + \frac{1}{8} II$

In figure 5 a two dimensional diagram of the chain of density reduction described above have been illustrated, what was expected was because of the sensitivity of Voronoi methods compared with fixed weights methods, by reducing the density in a part of data, fixed weight methods must have shown weakness in final results.

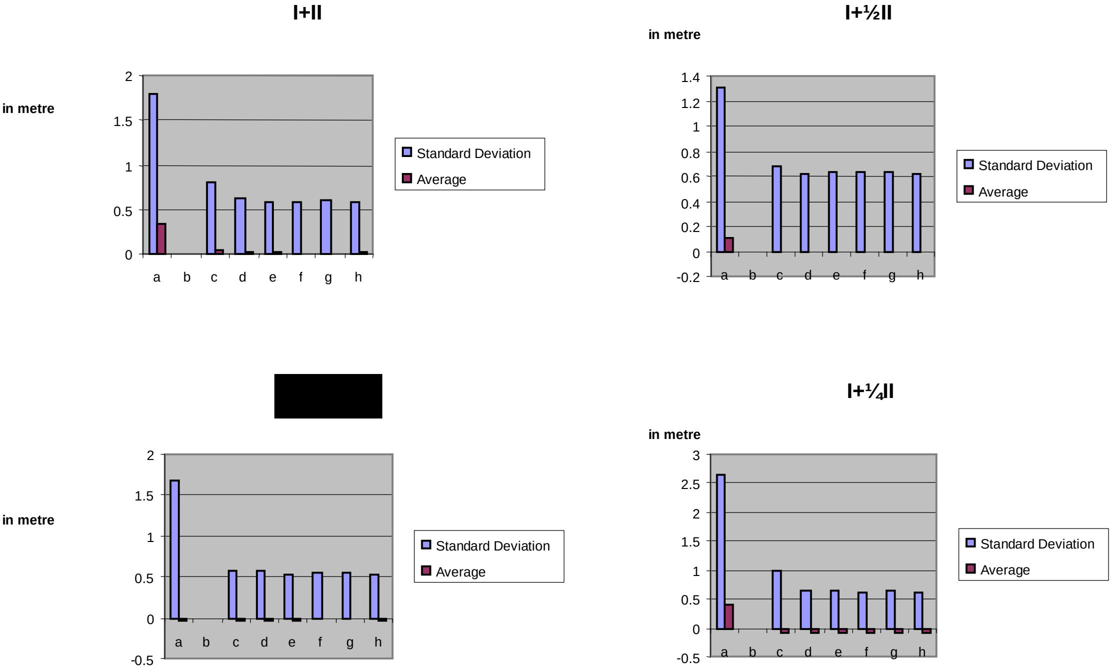


Figure 6: Standard deviation and average for density analysing datasets, ”a” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and μ in edge polygons equals to zero, ”b” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and μ in edge polygons equals to one, ”c” is the Voronoi normalized areas method with fixed weight P equals to 100 (P=100), ”d” is the Voronoi normalized areas method with fixed weight P equals to 1000 (P=1000), ”e” is the Voronoi normalized areas method with fixed weight P equals to 10000 (P=10000), ”f” is the Voronoi normalized areas method with fixed weight P equals to 100000 (P=100000), ”g” is the method of Voronoi areas and ”h” is the fixed weight (equal to1) method.

As you can see in figure 6 method ’h’ even when the density reaches one eighths in a part of original data, produces a comparable precision with Voronoi methods and will not weaken and will not adhere the expected pattern. In figure 7 behavior of every weight definition method except ’b’(because of lack of importance) is illustrated. This figure prove that with reduction of density even Voronoi methods will not follow a single pattern(p=100 till

100000, figure 7) and neither of results is strictly ascendant function nor strictly descendent function of density. Therefore one deduces by wonder that TPS with this solution is not sensitive to density of points.

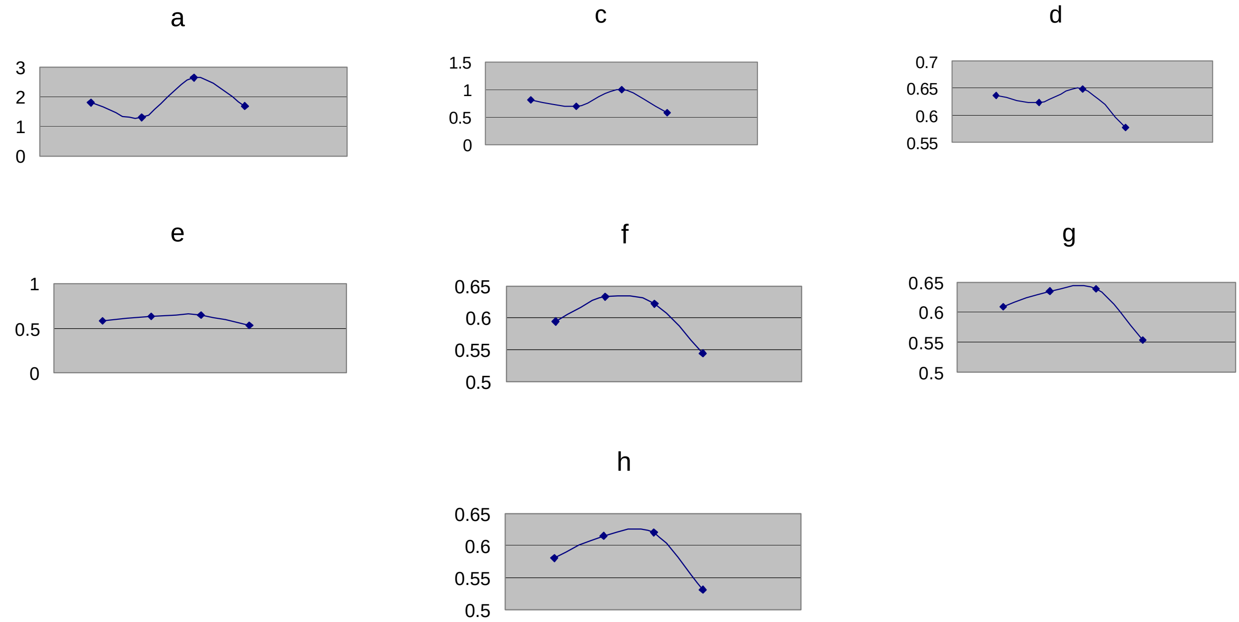


Figure 7: those 9 above diagrams are for showing behavior of every weight defenition metohd s relative to density variation, ”a” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and μ in edge polygons equals to zero, ”b” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and μ in edge polygons equals to one, ”c” is the Voronoi normalized areas method with fixed weight P equals to 100 (P=100), ”d” is the Voronoi normalized areas method with fixed weight P equals to 1000 (P=1000), ”e” is the Voronoi normalized areas method with fixed weight P equals to 10000 (P=10000), ”f” is the Voronoi normalized areas method with fixed weight P equals to 100000 (P=100000), ”g” is the method of Voronoi areas and ”h” is the fixed weight (equal to1) method.

What was discussed in this part illustrates that for this specific solution of thin plate Splines using of fixed weight is the best answer, need no complex calculations and make TPS as a unified method for miscellaneous datasets from which data density is not bothering.

5. CONCLUSION

TPS as a density independence method can be used for the construction of the surfaces in the new brand geomatics, because of its miscellaneous of data sources and differences in their densities, note that the weights are always of concern of the surveyors. On the other hand using TPS, without using the Voronoi algorithms, whereas it limits the number of control points and reduces the speed of operation, one can create a surface in contrary to other surface simulation methods such as BSplines all at once. Because of the density independence mathematical form of straigh method, one can use it in CAD/CAM softwares.

It is certain that the independency of density must to be of specifications of the thin plate spline, therefore other researches on other solutions of TPS (for example iterative method introduced by Frank 1982[14]) must be considered. Because at least one can prove that, in the dense and the bare zones can be used with comparable error results. By the end result of this article, we recommand the usage of the TPS method for mountainious-flat regions because variation of one zone doesn't affect other zones so much.

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BIOGRAPHICAL NOTES

Sina Taghvakish was graduated as Geomatical Engineer in department of survey engineering, Faculty of Engineering, University of Tehran, in 2002. He is about to obtain his Master degree in Remote Sensing at the mentioned university.

Dr Jalal Amini has degree of Doctor in Photogrammetry at University of Tehran. He had his Master degree in photogrammetry and Bachelor degree in survey engineering in Khajeh Nasir University.

CONTACTS

Mr. Sina Taghvakish
Home: Vali Asre Avenue, Vanaq square 1266 appartment.
Tehran
IRAN
Tel + 98 21 88800908
P.O. Code: 1435674135
Email: f_o_t_time@yahoo.com

Dr. Jalal Amini
Faculty of Engineering-University of Tehran
North Kargar Avenue, after Jalal Al Ahmad junction, Daneshkadeh Fanni.
Tehran
IRAN
Tel + 98 21 8008837
Fax + 98 21 8008837
P.O. Box: 11365-4563
Email: jamini@ut.ac.ir