Fast Kalman Processing of the GPS Carrier-Phases for Mobile Positioning and Atmospheric Tomography

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SUMMARY

Information on orbits and clocks of the satellites of Global Navigation Satellite Systems (GNSS) like GPS are obtained in Near Real-Time (NRT) from global and/or local computing centres. The carrier-phases are received from a local Geodetic reference network for most precise estimation of all involved calibration parameters in real-time by using the Helmert-Wolf blocking (HWb) method. The distributions of Integrated Water Vapour (IWV) of the troposphere and Total Electron Content (TEC) of the stratosphere need also be evaluated for improving accuracy and reliability of mobile positioning and navigation. Large moving windows of the carrier-phases and meteorological data must be used in order to maintain Kalman's observability condition. Optimal statistical, physical and mathematical modelling is crucial for satisfying Kalman's controllability condition. The residual error variances of carrier-phase data are obtained from C. R. Rao's (1972) Minimum Norm Quadratic Unbiased Estimation (MINQUE) theory. The instantaneous quality of each carrier-phase is thus derived from their observed internal consistency. The theory of optimal Kalman filtering requires that an exact error covariance matrix must also be found. Though the error covariance matrix of all estimated parameters is exceedingly large it, fortunately, can be inverted in a block by block manner using outcomes of the HWb method. The generalized Canonical Correlation Analysis (gCCA) is the method of choice for decorrelating those physical dependencies that can be taken into account only statistically in operational applications. Thus, those problems of optimality that stem from correlated errors can be overcome with methods based on the MINQUE theory as computationally feasible solutions are found by fast block by block inversions. In fact, the Normal Equations and the Quadratic Equation systems are solved semi-analytically by exploiting the simple and most effective inversion formula of Frobenius and Issai Schur (1875 - 1941). Ongoing research and development efforts cover the fastest possible computation of most reliable accuracy estimates for ultra-reliable mobile positioning and navigation whereas 3-dimensional Water Vapour (3WV) tomography is also necessary for some environmental engineering applications.

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1. INTRODUCTION

The patented Fast Kalman Filter (FKF) processing method that is briefly described here relates to satellite geodesy and meteorology but more particularly to estimating rapid fluctuations of the tropospheric water vapour and the ionospheric total electron content in order to adjust the carrier-phase measurements made by a precision GPS receiver for improving accuracies of navigation, mobile positioning, crustal motion estimation, sea level and wave-height measurements as well as for issuing alerts of meteorological hazards like tornados, thunderstorms, fog, ice formation on slippery roads etc. under the very general context of Global Monitoring of Environment and Security (GMES).

2. FAST KALMAN PROCESSING

2.1 Helmert-Wolf blocking

The linearised joint regression equation system of all raw measurements from the Geodetic and Meteorological observing instruments can be written out in the Canonical Block-Angular (CBA) form outlined first by F. R. Helmert (1880) as follows:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \\ \end{bmatrix} = \begin{bmatrix} X_1 & & & & G_1 \\ & X_2 & & & G_2 \\ & & \ddots & & \vdots \\ & & & X_K & G_K \\ & & & & \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_K \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_K \\ \end{bmatrix} \qquad (1)$$

where vectors $\mathbf{y}_1$, $\mathbf{y}_2$,…, $\mathbf{y}_K$ and $\mathbf{e}_1$, $\mathbf{e}_2$,…, $\mathbf{e}_K$ represent the measurements and their errors, respectively, from the different observation series (k = 1, 2,..., K) during an observing period t (t = 1, 2,...). Vectors $\mathbf{b}_1$, $\mathbf{b}_2$,…, $\mathbf{b}_K$ represent the unknown regression (state) parameters to be estimated for each data block k. Those regression parameters that are common to several blocks are represented here by a vector $\mathbf{c}$. Matrices X_1, X_2,…, X_K and G_1, G_2,…, G_K are the Jacobians that are related to these parameters, respectively.

The Wolf formulas (1978) for computing adjustments of all the state parameters are:

$$\mathbf{b}_k = (X'_k X_k)^{-1} X'_k (\mathbf{y}_k - G_k \mathbf{c}) \qquad (2)$$
$$\mathbf{c} = (\sum G'_k R_k G_k)^{-1} \sum G'_k R_k \mathbf{y}_k$$

where
$\mathbf{b}_k$ = vector of the state parameters for block k of observations
$\mathbf{c}$ = vector of the common state parameters
X_k = Jacobian matrix for the state parameters for block k

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G_k = Jacobian matrix for the common adjustments for block k
$\mathbf{y}_k$ = vector of the observations for block k
$\sum$= summation where the index k runs over all blocks of observations
$R_k = I - X_k(X'_k X_k)^{-1} X'_k$ = residual operator for block k.

These formulas above yield the Best Linear Unbiased Estimates (BLUE) of the state parameters if the covariance matrices of all errors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_K$ were first transformed into identity matrices. This can usually be done by physical modeling of observations and/or by using the statistical method of generalized Canonical Correlation Analysis (gCCA).

2.2 Linearised Equations for the Carrier-Phases

The **Observation Equations** for a moving data-window of length L are obtained from the phase measurements $\varphi_{i,j,k,t}$ of a receiver as follows:

$$y_{i,j,k,t-l} = \varphi_{i,j,k,t} - \rho_{i,k,t} = \tau_{k,t} + \gamma_{j,t} + \mathbf{g'}_{j,k,t}\,\mathbf{w}_t + h_{i,j,t}\,c_t + e_{i,j,k,t} \qquad (3)$$

for i =1,2,…,m, j=1,2,…,n, k=1,2,…,K, l=0,1,2,…,L-1 and t=L, L+1,L+2,…,∞

where
y = difference of the total carrier-phase from the distance between a satellite and a receiver
i = index of the signals (L1, L2, L3,… , G1,… , E1,… , etc.)
j = index of the satellites (GPS, Glonass, Galileo and Beidou, etc.)
k = index of the receivers (or receiver sites)
l = local index of epochs for a moving data window of length L at epoch t
t = index of the epoch times (t=1, 2, 3,…)
φ = total phase of the reconstructed carrier of the i^{th} signal at epoch t
ρ = propagation distance [phase] in dry air from the j^{th} satellite to the k^{th} receiver at epoch t
τ = clock correction of the k^{th} receiver at epoch t
γ = clock correction of the j^{th} satellite at epoch t
$\mathbf{g}$ = vector of the slant-path 3WV refractivity values of voxels from the j^{th} satellite to the k^{th} receiver at epoch t (see Slant-delay models on pages 39-49 of Kleijer (2004))
$\mathbf{w}$ = vector of the 3WV values of voxels at epoch t
h = slant-mapping of the TEC refractivity for the i^{th} signal from the j^{th} satellite to the receiver network(s) at epoch t
c = the TEC value of the receiver network(s) at epoch t
e = random measurement error at epoch t; and,
m, n, K and V = the number of signals, satellites, receivers and voxels, respectively.

There are four **System Equations** that are as follows:

$$\begin{aligned} \tau_{k,t} &= \tau_{k,t-1} + \zeta_{k,t} \\ \gamma_{j,t} &= \gamma_{j,t-1} + \eta_{j,t} \\ \mathbf{w}_t &= (A_t + dA_t)\mathbf{w}_{t-1} + \boldsymbol{\nu}_t \\ c_t &= c_{t-1} + \xi_t \end{aligned} \qquad (4)$$

where

$\zeta_{k,t}$, $\eta_{j,t}$, $\mathbf{v}_t$ and ξ_t = the random walk terms; respectively
$\mathbf{w}_t$ = vector $[w_{1,t}, w_{2,t},\ldots, w_{V,t}]'$
$\mathbf{v}_t$ = vector $[v_{1,t}, v_{2,t},\ldots, v_{V,t}]'$
A_t = state transition matrix describing advection of 3WV values in the air-mass
dA_t = matrix of the state transition errors to be adjusted by adaptive Kalman Filtering.

Matrix A is a tangent-linear approximation of the Numerical Weather Prediction (NWP) model that is applied in the data assimilation of the 3WV values at epoch t for obtaining them from their previous values at epoch t-1 (see Equations (26-29) on pages 12-13 in PCT/FI93/00192 (WO 93/22625) of Lange (1999)). Matrix dA is approximated by a vector **r** that is estimated by adaptive Fast Kalman Filtering (FKF) (see Equations (22-24) on pages 12-13 in PCT/FI96/00621 (WO 97/18442) of Lange (1999)).

2.3 Adaptive Kalman filtering

The semi-analytical computing method of FKF provides the possibility of processing huge amounts of input data in real-time with ultra-reliable internal Statistical Calibration (Lange, 1999) and accuracy estimation (Rao, 1972). Different sensors like radars, transponders, profilers and GPS- receivers usually operate far from each other and consequently opportunities for immediate maintenance and/or precise physical calibration are very sparse. Thus, this Helmert-Wolf blocking method is being implemented in the GAMIT/GLOBK GPS software of the Massachusetts Institute of Technology (MIT).

2.4 GPS reference receiver network

There currently are more than 100 geodetic dual-frequency GPS-receivers in Finland which all are connected with ASDL-lines trough the internet.

Figure 1. The GPS antenna of SuomiNet Station SG40 on the roof-top of FMI.

The GPS network is operated in real-time for Virtual Reference Station (VRS) surveys:

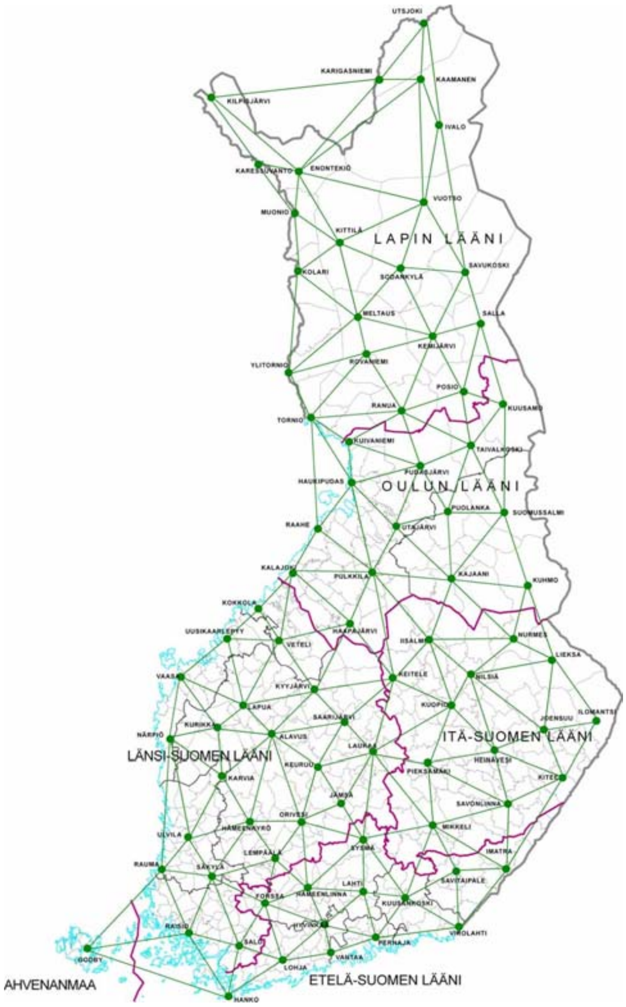

Figure 2. The national VRS network of Geotrim Ltd., Finland.

A low-cost receiver chip was developed from the Vaisala GPS-radiosonde, see Fig. 3:

Figure 3. Vaisala GPS Receiver iTrax03 measures the Carrier-Phase of L1.

3. CONCLUDING REMARKS

The water vapour tomography will develop fast both in its horizontal accuracy and vertical resolution because of increasing number of available signals from the GNSS satellites of GPS, Glonass, Galileo and Beidou as well as dense reference receiver networks. Reliable quality control of each of the GNSS signals can be based on their observed internal consistency by using advanced statistical inference.

The theory of optimal Kalman filtering provides the stable method for updating repeatedly various instrumental calibration drifts, systematic modeling errors, environmental parameters and receiver coordinates. The proposed fast Kalman processing makes it possible to fully exploit all input data in real-time. Reliable measurements on all observed parameters like signal propagation times, bearings, Doppler-shifts and amplitudes can be obtained. High-precision ultra-reliable navigation will improve automatic control and piloting all sorts of vehicles.

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