IAG/FIG Commission 5/ICG Technical Seminar

Reference Frame in Practice

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Session 1.3: Worked examples of Terrestrial Reference Frame Realisations

Determining Coordinates in a Local Reference Frame from Absolute ITRF Positions: A New Zealand Case Study

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Overview

- Realization of a reference frame over a limited area (tens to hundreds of kilometres) is the domain of the surveyor
- Transformation between reference frames using standard transformations
- Transformation between epochs using a velocity model
- Concepts illustrated through a worked example

Key Concepts

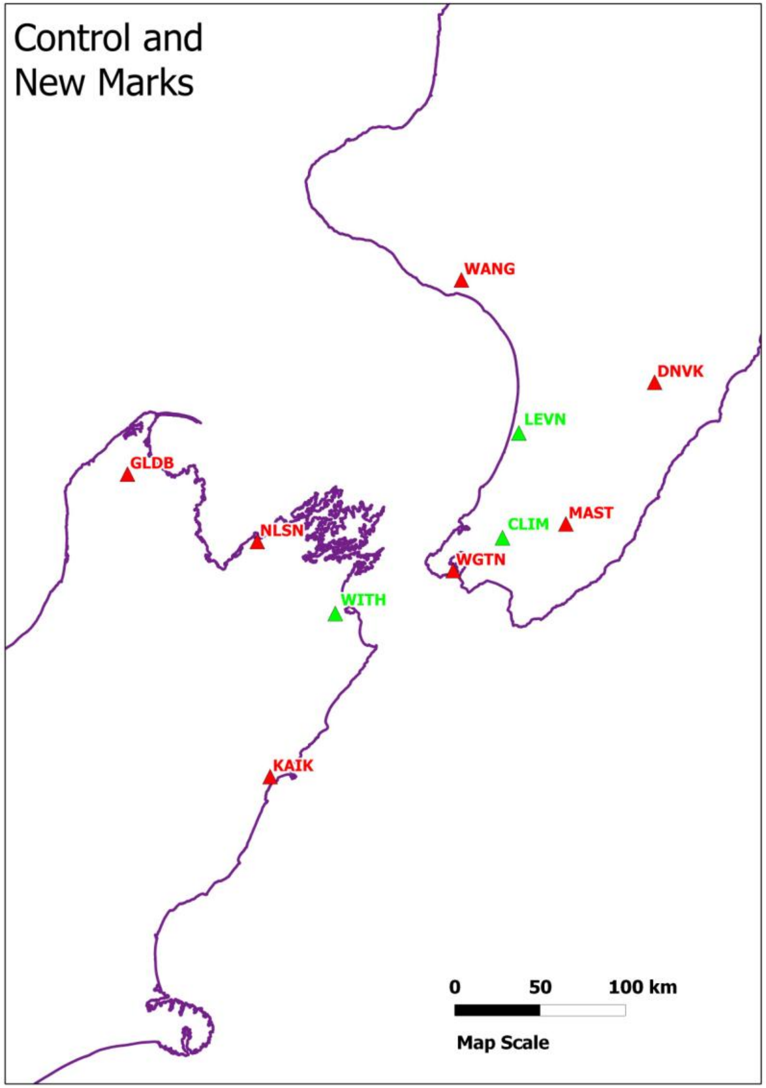
- Local, project-specific reference frame realizations can be made by the surveyor
- Incorporating velocities may be new, but the calculations are simple
- It is vital to check the accuracy of your realization
- A concise but clear description of how the coordinates were generated is needed
- Government geodetic agencies need to support surveyors as they transition to using dynamic datums

Why is this important?

- Getting precise coordinates in the latest ITRF realization has been greatly simplified through the provision of online GNSS processing services. Many of these provide absolute positions
- But most countries do not use the latest version of ITRF as their official datum
- So we need to be able to transform coordinates from ITRF to the local datum
- We could always just make relative connections to control provided by the national geodetic agency, but this is not always the most efficient method
- Both coordinates may be required: ITRF for maximum precision and global consistency and local coordinates to meet regulatory requirements and ensure consistency with local datasets

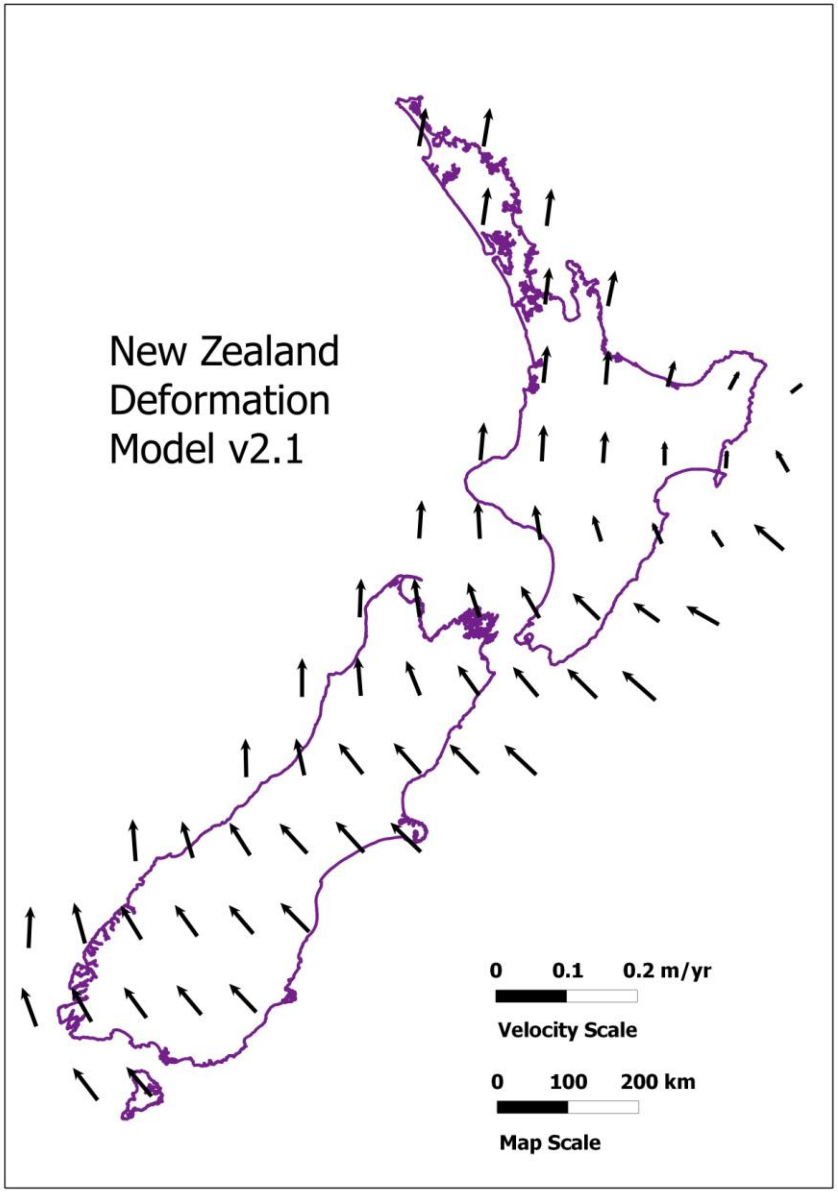
Scenario

- Client has requested control for a large project in New Zealand
- They are a global company, and hold all of their data in the latest ITRF realization. Therefore need ITRF2008 coordinates
- To meet regulatory requirements, data must also be provided in the official datum. Therefore need NZGD2000 coordinates
- Client also requires a means of transforming between the two sets of coordinates
- Seven control stations (GLDB, NLSN, KAIK, WGTN, MAST, DNVK, WANG)
- Three new stations (CLIM, LEVN, WITH)



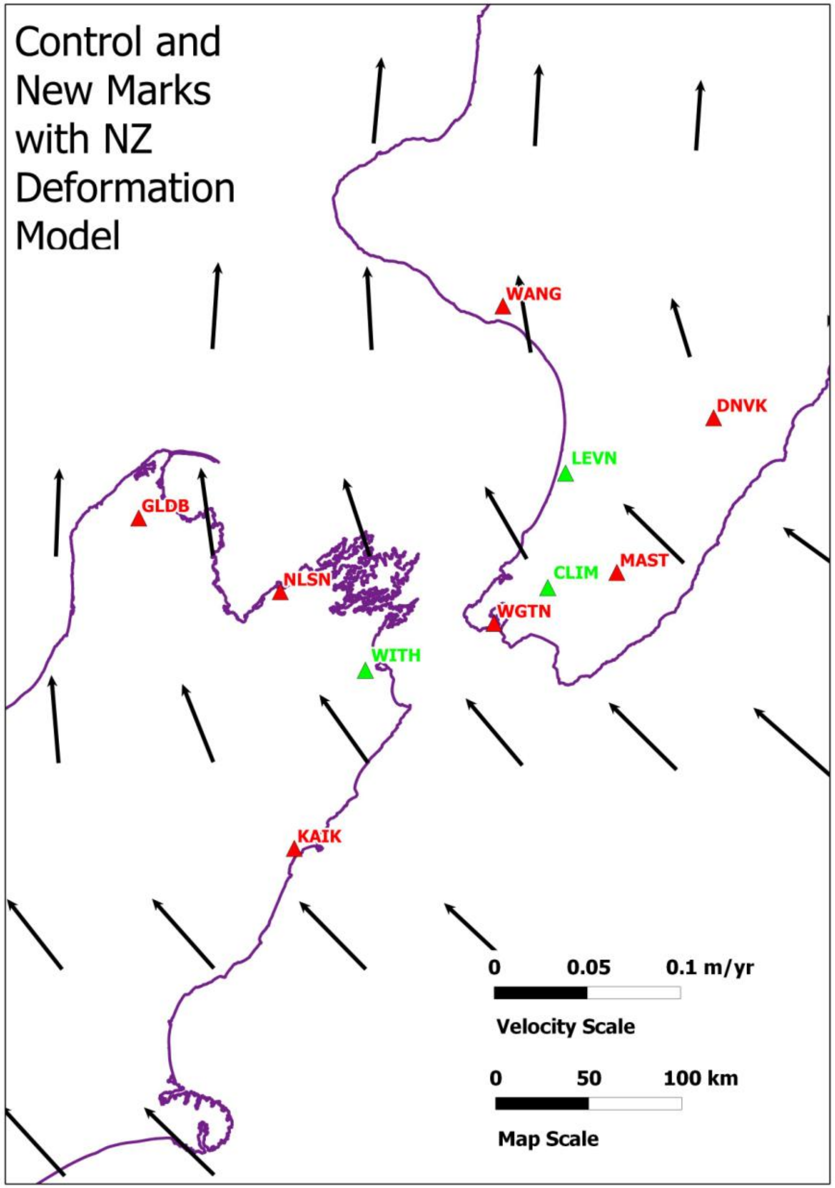
Background

- The official datum is New Zealand Geodetic Datum 2000 (NZGD2000)
- Defined as ITRF96 at epoch 2000.0
- New Zealand is at the boundary of the Australian and Pacific plates
- Even over small distances, marks can be moving at different velocities. Cannot assume a static Earth
- Includes a deformation model which can be used to generate coordinates at other epochs
- Official, highly accurate coordinates are published at CORS stations, and other passive marks



Deformation over Project Area

- Our project area is about 300km x 300km
- Station velocities vary significantly over this area



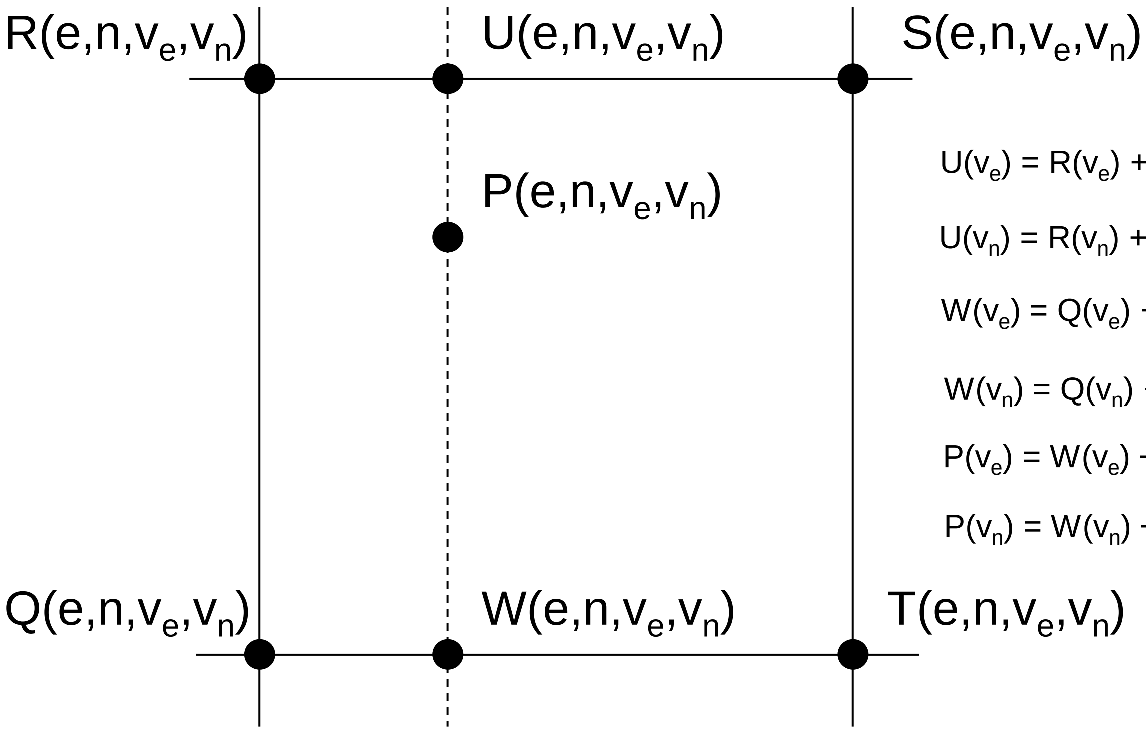
ITRF2008

- We do all our processing in the more accurate reference frame, and then transform to any other desired reference frame and epoch
- Generation of high precision ITRF coordinates usually requires scientific GNSS processing software, not used by most surveyors
- Therefore choose to use an online processing service (in this case JPL precise point positioning)
- This will give us ITRF2008 coordinates, in terms of the reference frame used by the IGS orbital products (IGS08).
- Process 24-hour sessions
- We end up with IGS08 coordinates at observation epoch, which is 2012 Julian Day 60 (2012.16)

Transforming Coordinates

- Throughout, we are working in Cartesian (geocentric) coordinates. Any other transformations, such as to a mapping projection, are made at the end
- Step 1: Identify stations at which coordinates are available in both the desired reference frames
- Step 2: Use velocities at each station to obtain coordinates at a common epoch in the two reference frames
- Step 3: Calculate *appropriate* transformation parameters, using least squares. This will usually be three translation/rotation parameters, or three translation/rotation parameters plus one scale parameter over small portions of the Earth's surface
- Step 4: Use the transformation parameters to convert coordinates between reference frames

Bilinear Interpolation



$U(v_e) = R(v_e) + [(U(e) – R(e))/(S(e) – R(e))][S(v_e) – R(v_e)]$

$U(v_n) = R(v_n) + [(U(e) – R(e))/(S(e) – R(e))][S(v_n) – R(v_n)]$

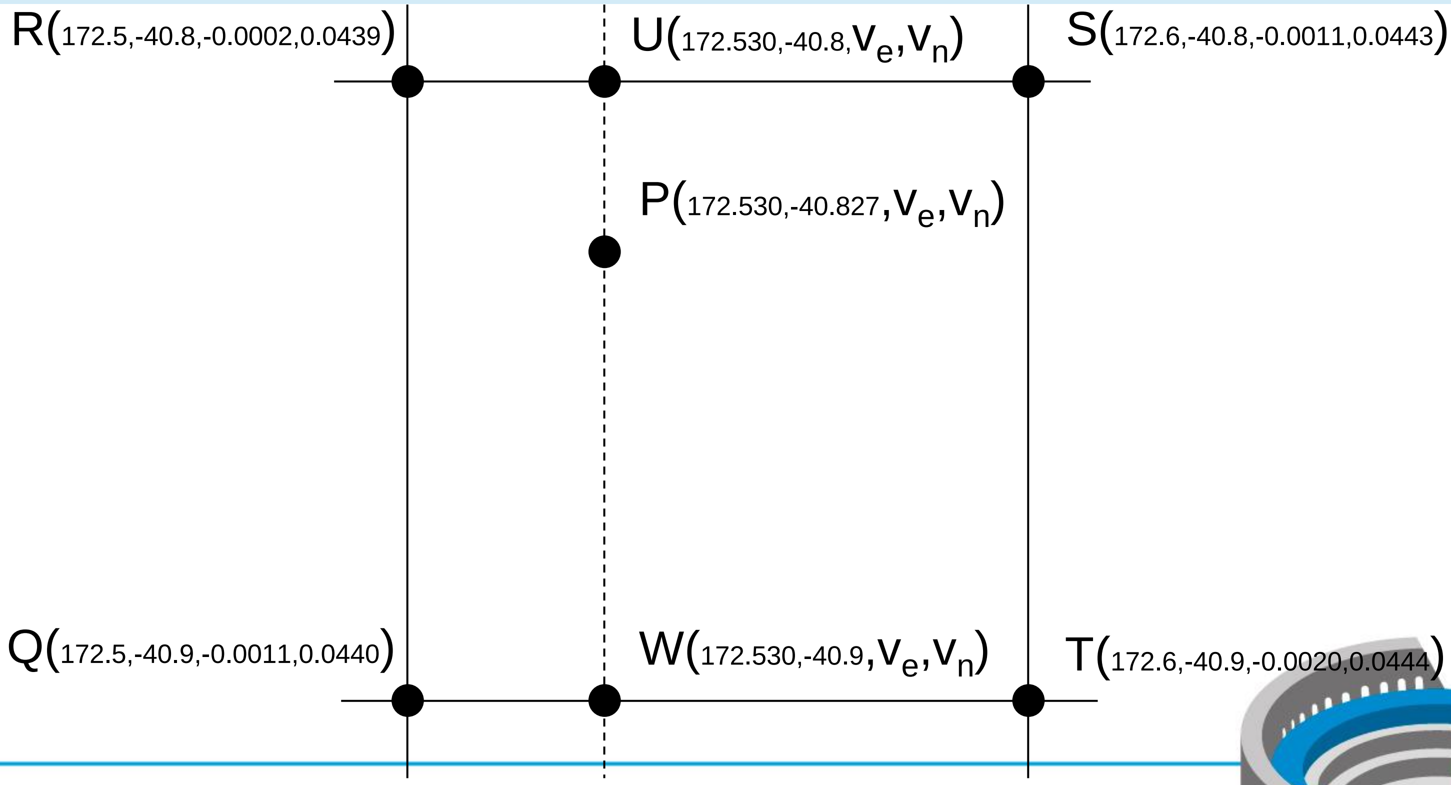
$W(v_e) = Q(v_e) + [(W(e) – Q(e))/(T(e) – Q(e))][T(v_e) – Q(v_e)]$

$W(v_n) = Q(v_n) + [(W(e) – Q(e))/(T(e) – Q(e))][T(v_n) – Q(v_n)]$

$P(v_e) = W(v_e) + [(P(n) – W(n))/(U(n) – W(n))][U(v_e) – W(v_e)]$

$P(v_n) = W(v_n) + [(P(n) – W(n))/(U(n) – W(n))][U(v_n) – W(v_n)]$

Calculating Velocity – Station GLDB



Calculating Velocity – Station GLDB

$U(v_e) = -0.0002 + [(172.530 – 172.5)/(172.6 – 172.5)][-0.0011 – -0.0002] = -0.0005$

$U(v_n) = 0.0439 + [(172.530 – 172.5)/(172.6 – 172.5)][0.0443 – 0.0439] = 0.0440$

$W(v_e) = -0.0011 + [(172.530 – 172.5)/(172.6 – 172.5)][-0.0020 – -0.0011] = -0.0013$

$W(v_n) = 0.0440 + [(172.530 – 172.5)/(172.6 – 172.5)][0.0444 – 0.0440] = 0.0441$

$P(v_e) = -0.0013 + [(-40.827– -40.9)/(-40.8 – -40.9)][-0.0005 – -0.0013] = -0.0007$

$P(v_n) = 0.0441 + [(-40.827– -40.9)/(-40.8 – -40.9)][0.0440 – 0.0441] = 0.0441$

Transforming Velocities to Cartesian Reference Frame

- Recall that we are always working in Cartesian (XYZ) coordinates, so need XYZ velocities. Call this column vector $\mathbf{v}_{XYZ}$
- But the velocity model is topocentric (ENU). Call this column vector $\mathbf{v}_{ENU}$
- We can convert between the two using the geocentric to topocentric rotation matrix, $\mathbf{R}_{GT}$, for the point's latitude (ϕ) and longitude (λ)
- $\mathbf{v}_{ENU} = \mathbf{R}_{gt}\mathbf{v}_{XYZ}$
- $\mathbf{v}_{XYZ} = \mathbf{R}_{gt}^{-1}\mathbf{v}_{ENU}$

$$R_{gt} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}$$

Transforming Velocities to Cartesian Reference Frame – Station GLDB

- $\mathbf{v}_{XYZ} = \mathbf{R}_{GT}^{-1}\mathbf{v}_{ENU}$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -0.130 & -0.992 & 0 \\ -0.648 & 0.085 & 0.757 \\ -0.750 & 0.098 & -0.654 \end{bmatrix}^{-1} \begin{bmatrix} -0.0007 \\ 0.0441 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.0285 \\ 0.0045 \\ 0.0333 \end{bmatrix}$$

Calculating NZGD2000 Epoch 2012.16 Coordinates – Station GLDB

- $\mathbf{x}_{\text{NZGD Epoch 2012.16}} = \mathbf{x}_{\text{NZGD2000 Epoch 2000.0}} + 12.16\mathbf{v}_{XYZ}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{201216} = \begin{bmatrix} -4792405.831 \\ 628416.781 \\ -4148068.669 \end{bmatrix} + 12.16 \begin{bmatrix} -0.0285 \\ 0.0045 \\ 0.0333 \end{bmatrix} = \begin{bmatrix} -4792406.177 \\ 628416.835 \\ -4148068.263 \end{bmatrix}$$

Calculating Transformation Parameters

- Use least squares to obtain the best solution, as we have more observations than parameters
- Functional model: **At** = **b**, where **A** is the design matrix, **b** = Calculated (IT96) minus observed (IGS08) and **t** is the matrix of unknown transformation parameters
- Stochastic model: **W** = **I**, in this case we choose to weight all coordinates equally
- So $\mathbf{t} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$, the standard least squares solution
- And $\mathrm{Cov}(\mathbf{t}) = \sigma_0^2(\mathbf{A}^T\mathbf{A})^{-1}$
- The Aposteriori Standard Error of Unit Weight is $\sigma_0^2 = (\mathbf{A}^T\mathbf{t}-\mathbf{b})^T(\mathbf{A}^T\mathbf{t}-\mathbf{b})$/(degrees of freedom)
- This is a linear problem, so no need to iterate
- Note: if you wish to weight your coordinates: $\mathbf{t} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{b}$

$$
\begin{array}{ll}
GLDB & x \\ & y \\ & z \\
NLSN & x \\ & y \\ & z \\
KAIK & x \\ & y \\ & z \\
WGTN & x \\ & y \\ & z \\
MAST & x \\ & y \\ & z \\
DNVK & x \\ & y \\ & z \\
WANG & x \\ & y \\ & z
\end{array}
\quad
A=\begin{bmatrix}
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\
1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1
\end{bmatrix}
\quad
b=\begin{bmatrix}
-4792406.177 \\ 628416.835 \\ -4148068.263 \\
-4775888.435 \\ 549740.211 \\ -4177981.109 \\
-4685479.521 \\ 531055.197 \\ -4280819.034 \\
-4777269.652 \\ 434270.387 \\ -4189484.267 \\
-4801933.943 \\ 370789.222 \\ -4167752.305 \\
-4860760.939 \\ 325692.752 \\ -4103646.312 \\
-4888073.52 \\ 443004.771 \\ -4060015.325
\end{bmatrix}
-
\begin{bmatrix}
-4792406.117 \\ 628416.851 \\ -4148068.23 \\
-4775888.398 \\ 549740.2 \\ -4177981.061 \\
-4685479.471 \\ 531055.245 \\ -4280819.009 \\
-4777269.602 \\ 434270.406 \\ -4189484.221 \\
-4801933.888 \\ 370789.24 \\ -4167752.257 \\
-4860760.892 \\ 325692.771 \\ -4103646.255 \\
-4888073.493 \\ 443004.775 \\ -4060015.31
\end{bmatrix}
=
\begin{bmatrix}
-0.06 \\ -0.016 \\ -0.033 \\
-0.037 \\ 0.011 \\ -0.048 \\
-0.05 \\ -0.048 \\ -0.025 \\
-0.05 \\ -0.019 \\ -0.046 \\
-0.055 \\ -0.018 \\ -0.048 \\
-0.047 \\ -0.019 \\ -0.057 \\
-0.027 \\ -0.004 \\ -0.015
\end{bmatrix}
$$

$$
Cov(X)=\begin{bmatrix}
3.22\times10^{-5} & 0 & 0 \\
0 & 3.22\times10^{-5} & 0 \\
0 & 0 & 3.22\times10^{-5}
\end{bmatrix}
$$

Three Parameter Transformation Results

- SEUW = 0.015 m
- t_x = -0.046   0.006 m
- t_y = -0.016   0.006 m
- t_z = -0.039   0.006 m
- Note: In this case least squares simply gives us the average of the coordinate differences, so we could have avoided the matrix algebra, but would not get the precision information so easily

Four Parameter Transformation Results

- SEUW = 0.015 m
- t_x = -0.103   0.211 m
- t_y = -0.011   0.021 m
- t_z = -0.088   0.183 m
- s = -1.19 10^{-8} 4.40 10^{-8}
- None of the parameters is significant, so this is not the best transformation

Calculate IT96 Epoch 2012.16 for CLIM

- $\mathbf{x}_{\text{NZGD Epoch 2012.16}} = \mathbf{x}_{\text{IGS08 Epoch 2012.16}} + \mathbf{t}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NZGD2000\,201216} = \begin{bmatrix} -4793404.120 \\ 407108.010 \\ -4175081.520 \end{bmatrix} + \begin{bmatrix} -0.046 \\ -0.016 \\ -0.039 \end{bmatrix} = \begin{bmatrix} -4793404.167 \\ 407107.994 \\ -4175081.559 \end{bmatrix}$$

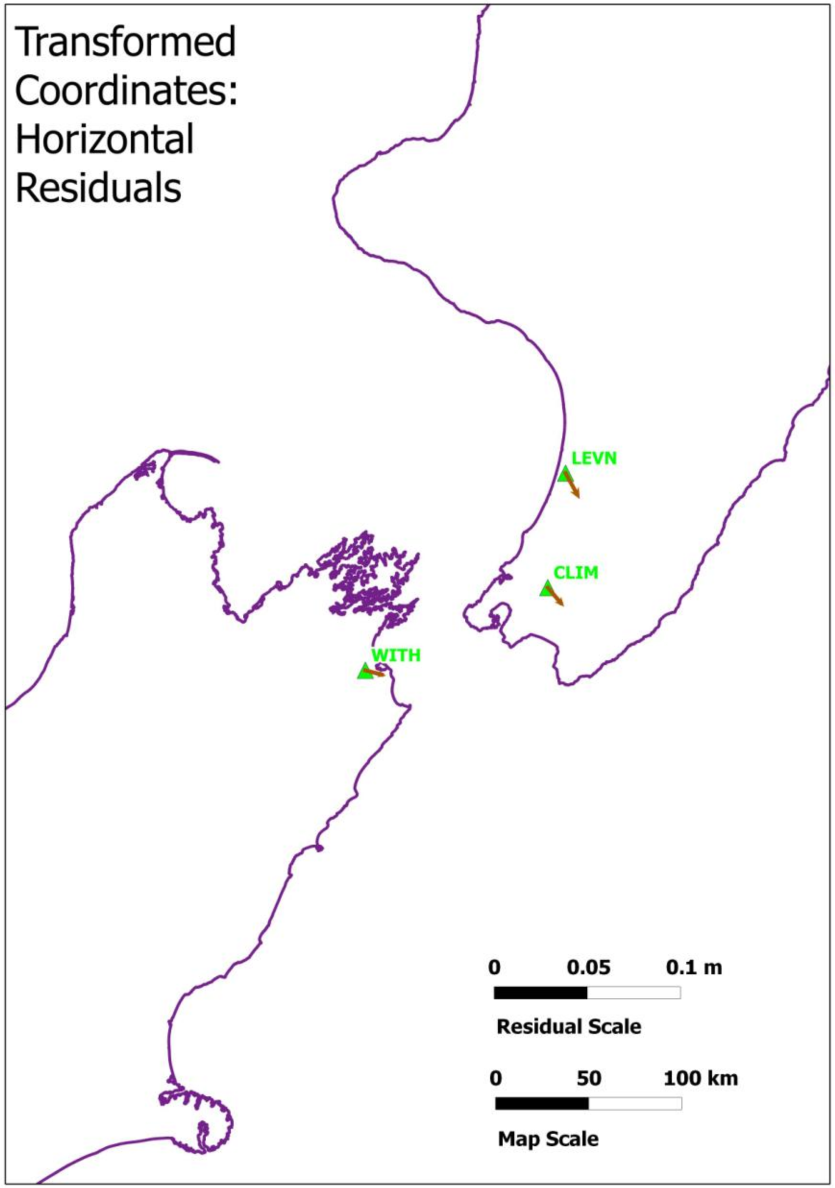
Calculate IT96 Epoch 2000 for CLIM

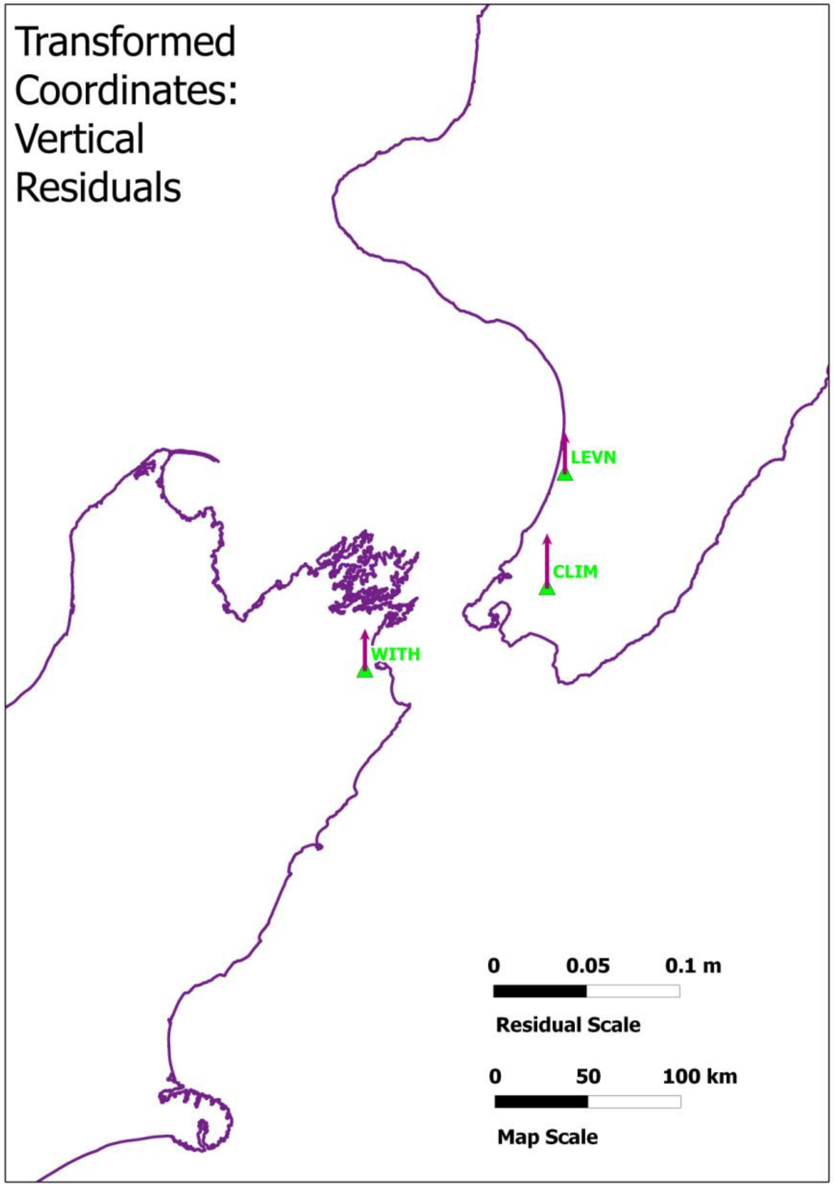
- $\mathbf{x}_{\text{NZGD Epoch 2000}} = \mathbf{x}_{\text{NZGD2000 Epoch 2012.16}} - 12.16\mathbf{v}_{xyz}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NZGD2000} = \begin{bmatrix} -4793404.167 \\ 407107.994 \\ -4175081.559 \end{bmatrix} - 12.16 \begin{bmatrix} -0.0196 \\ 0.0277 \\ 0.0250 \end{bmatrix} = \begin{bmatrix} -4793403.928 \\ 407107.657 \\ -4175081.864 \end{bmatrix}$$

Calculate IT96 Epoch 2000 for CLIM, LEVN, WITH

Station	IGS08 Epoch 2012.16 (XYZ)	Velocity (ENU)	NZGD2000 Epoch 2000.0 (observed)	NZGD2000 Epoch 2000.0 (GDB)	Difference (ENU)
CLIM	-4793404.120	-0.026	-4793403.928	-4793403.914	0.007
	407108.010	0.0333	407107.657	407107.663	-0.008
	-4175081.5204	0	-4175081.864	-4175081.841	0.025
LEVN	-4833775.0621	-0.0164	-4833774.861	-4833774.854	0.006
	402451.2374	0.0335	402451	402451.006	-0.011
	-4127913.8068	0	-4127914.155	-4127914.134	0.018
WITH	-4753506.3677	-0.0195	-4753506.156	-4753506.143	0.009
	500939.4145	0.0352	500939.133	500939.14	-0.002
	-4209496.456	0	-4209496.815	-4209496.801	0.018


Transformed Coordinates: Horizontal Residuals
LEVN
CLIM
WITH
0 0.05 0.1 m
Residual Scale
0 50 100 km
Map Scale


Transformed Coordinates: Vertical Residuals
LEVN
CLIM
WITH
0 0.05 0.1 m
Residual Scale
0 50 100 km
Map Scale

Summary

- Absolute positioning is readily available, and its use will increase
- These positions are in terms of the satellite orbit reference frame (latest IGS realization of current ITRF)
- Software to convert to a local reference frame may not exist, or may need to be tested
- This conversion can be done by the surveyor using a spreadsheet and the procedure outlined in this presentation
- Worked examples are very useful to aid understanding of reference frame and epoch transformations. Government agencies should make these more readily available

Questions and References

- http://apps.gdgps.net/ (JPL PPP service)
- http://apps.linz.govt.nz/gdb/index.aspx (LINZ Geodetic Database)

- For any questions please contact:

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