Geoid Model Estimation without Additive Correction Using KTH Approach for Peninsular Malaysia

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Key words: Geoid, KTH approach, GPS, LSMS

SUMMARY

Geoid modeling provides vital information in the determination of orthometric height using Global Navigation Satellite System (GNSS) technology. Determination of precise geoid models is one of the main objectives of research around the globe. There are several approaches in the determination of geoid models. In this study, the KTH approach developed by the Royal Institute of Technology, Sweden is evaluated in the determination of a Peninsular Malaysia estimation gravimetric geoid model. The set of parameters: M=L= 180, $\Psi\_0=3.\ 0^o$, $\Psi=0.\ 4^o$ and $\sigma\_\Delta g= 5.0$ mGal was determined to provide the optimum values for the establishment of the desired Peninsular Malaysia geoid model using the KTH approach. The derived values for selected points from the estimated gravimetric geoid model are compared with the known values at benchmark (BM). It is found that, the Root Mean Square Error of the estimated gravimetric geoid model, without computed additive correction, is ±32.1cm. In general, the approach seems to be of high accuracy and potential in determining a precise geoid model for Peninsular Malaysia.

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1. INTRODUCTION

Determining geoid height with great precision is one of the main goals for geoscientists, especially geodesists, nowadays. Having the most accurate geoid height possible is important in order to derive a subsequent precise orthometric height on the Earth's surface from Global Positioning System (GPS) observation. In the last 15 years, a number of geoid models for Malaysia have been officially developed by the Department of Survey and Mapping Malaysia (DSMM). The latest geoid model developed is the Klang Valley geoid model under the Height Modernization System (HMS) project in the year 2008. The geoid model developed by DSMM used the Remove- Compute -Restore (R-C-R) approach by utilizing the Fast Fourier Transform (FFT) and Least Square Collocation (LSC) process (Forsberg & Strykowski, 2008)

Nowadays, in Malaysia, the determination of high resolution and precise geoid models is the top priority among the geodetic research community. The desired accuracy is 1cm (1 sigma) for flat and moderate topography; where as a level of 1 decimetre (10 cm) is used for mountainous areas. In this study, a computational scheme developed by the Royal Institute of Technology (KTH) (Sjöberg, 2011) known as Least Square Modification of Stokes's formula is used and explored in the determination of a geoid model with and without additive correction (LSMS). This scheme was successfully applied in the determination of geoid models in several countries such as: Zambia (Nsombo, 1996) Ethiopia (Hunegnaw, 2001), Sweden (Nahavandchi, 1998), the Baltic (Ellmann, 2004), Iran (Kiamehr, 2006), Tanzania (Ulotu, 2009), New Zealand (Abdalla & Tenzer, 2011) and Sudan (Abdalla & Fairhead, 2011). This proposed scheme utilised the least square approach based on modified Stokes's kernel (Sjöberg, 1991) whereby additive corrections such as: topography, downward continuation, atmosphere, and ellipsoidal correction are separated.

The main purpose of this study is to present the preliminary results from an ongoing research in collaboration with the Department of Surveying and Mapping Malaysia (DSMM). The main aim of the study is to compute a new seamless precise geoid model by using a least square modification approach with additive corrections. The main emphasis of this paper will encompass the computational aspects of estimating the geoid model for Peninsular Malaysia. This is carried out through a combination of geopotential model coefficients and surface gravity anomalies. A brief review on the adopted computational scheme using Least Square Modification of Stokes's (LSMS) formula without any additive correction will be the first agenda discussed in this study.

2. LEAST SQUARE MODIFICATION OF STOKES

The basic fundamentals of Stokes's formula assume that disturbing potentials are harmonic and occur outside the geoid. There are no masses outside the geoid, and all the existing masses must be removed. This assumption of forbidden masses outside the geoid is necessary when treating problems related to physical geodesy as a boundary value problem (Hofmann-Wellenhof & Moritz, 2005). The use of least square modification of Stokes's formula was proposed by Sjöberg (1984). The main goal in this computational scheme is to minimize the expected global mean square errors. The surface gravity anomaly and Global Geopotential Model is then used in the determination of the estimated geoid height. Hence, the associated corrections are then computed and added into the estimated geoid height separately as shown in Equation 1:

$$N = \widetilde{N} + \delta N_{comb}^{Topo} + \delta N_{dwc} + \delta N_{tot}^{atm} + \delta N_{tot}^{ell} , \quad (1)$$

where N is the geoid height, $\widetilde{N}$ is the estimated geoid from the combination of surface gravity anomaly and GGM, δN_{comb}^{Topo} is the combined topographic correction, δN_{dwc} is the downward continuation correction, δN_{tot}^{atm} is the combine atmospheric correction and δN_{tot}^{ell} is the ellipsoidal correction for the spherical approximation of the geoid in Stokes's formula.

The estimated geoid height ($\widetilde{N}$) can be computed using Equation 2 (Sjöberg, 2003):

$$\widetilde{N} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S_L(\psi)\, \Delta g^O d\sigma - \frac{R}{2\gamma} \sum_{n=2}^{M} (Q_n^L + s_n)\, \Delta g_n^{GGM}, \quad (2)$$

in which R is the mean Earth radius, γ is the normal gravity on the reference ellipsoid, Δg^O is the surface gravity anomaly, Q_n^L is the Molodensky truncation coefficient, s_n is the modification parameters, M is the maximum degree of GGM, Δg_n^{GGM} is the Laplace harmonics of degree n and it is derived from the GGM (Heiskanen & Moritz, 1980) and $S_L(\psi)$ is the modified Stokes's function to the modification limit L as computed using Equation 3:

$$S_L(\psi) = S(\psi) - \sum_{n=2}^{L} \frac{2n+1}{2} s_n P_n(\cos\psi) , \quad (3)$$

in which $S(\psi)$ is the original Stokes's function, ψ is the spherical distance for the computation point (ϕ, λ) to the block of σ_0 (the surface element) and $P_n(\cos\psi)$ is the Legendre polynomials.

The truncation coefficient is computed based on

$$Q_n^L = Q_n(\psi_0) - \sum_{n=2}^{\infty} \frac{2k+1}{2} s_k E_{nk}(\psi_0) , \quad (4)$$

in which Q_n is Molodensky's truncation coefficient and expressed as:

$$Q_n(\psi_0) = \int_{\psi_0}^{\pi} S(\psi) P_n(\cos\psi) sin\psi d\psi \quad (5)$$

While, E_{nk} is the Paul's coefficient (Sjöberg, 1984) and given as:

$$E_{nk} = E_{nk}(\psi_0) = \frac{2k+1}{2} \int_{\psi_0}^{\pi} P_n(\cos\psi)\, P_k(\cos\psi) sin\psi d\psi, \quad (6)$$

and the s_n is a modified parameter in which:

$$s_n = \begin{cases} s_n\ , & if\ 2 \le n \le L \\ 0\ , & otherwise \end{cases} . \quad (7)$$

The least square modification (LSM) parameters are then computed by using a linear system equation (Sjöberg, 2003)

$$\sum_{r=2}^{L} a_{kr} s_r = h_k, \qquad k = 2,3, \dots, L, \quad (8)$$

where

$$a_{kr} = \sum_{n=2}^{\infty} E_{nk} E_{nr} C_n + \delta_{kr}\, C_r - E_{kr} C_k - E_{kr} C_r \quad (9)$$

and

$$h_k = \frac{2\sigma_k^2}{k-1} - Q_k C_k + \frac{2k+1}{2} \sum_{n=2}^{\infty} \left(Q_n E_{nk} C_n - \frac{2}{n-1} E_{nk} \sigma_n^2 \right) \quad (10)$$

where

$$\delta_{kr} = \begin{cases} 1 & if\ k = r \\ 0 & otherwise \end{cases} \quad (11)$$

$$C_k = \sigma_k^2 \begin{cases} C_k dC_k / (C_k + dC_k) & if\ 2 \le k \le M \\ C_k & if\ k > M \end{cases} \quad (12)$$

$$E_{nk} = \frac{2k+1}{2} E_{nk}(\psi_0) \quad (13)$$

The modified parameter s_n varies depending on the quality of land gravity observations, the chosen radius of integration (Ψ_0), and the characteristics of the Global Geopotential Model. The system of equations in Equation 8 will involve inversion of the matrix A=[a_{kr}], whereas it will become an ill conditioned matrix with an increased size of L. Therefore, it cannot be solved by using standard methods. Investigations carried out by Ellmann (2004) and Ågren

(2004) show that Singular Value Decomposition (SVD) is an effective and efficient technique to overcome this problem. Moreover, Ågren (2004) stressed that an appropriate truncation of SVD will produce insignificant effects to the modified parameters.

Based on Equation 2, the geoid estimator is then computed based on a combination of land observed and global geopotential model gravity anomaly. The first phase of Equation 2 computes the value using the rough surface of the gravity anomaly while the second phase is computed using the Global Geopotential Model. As for the geoid estimator computation, adopting the GGM, as well as the modification parameter, is crucial. Based on the LSMS approach, the signal and error degree variance for both datasets need to be estimated. The signal degree variance from GGM is generated using the Tscherning & Rapp (1974) model. While, for the error degree variance of GGM, it is estimated using the standard errors of the GGM coefficients (Rapp & Pavlis, 1990).

For the estimation of errors for the degree variance associated with the land observed gravity anomalies, an isotropic error degree covariance function (Sjöberg, 1986) and an uncorrelated band limited white noise model was used. In this model, the constant degree-orders variance is applied (Jekeli & Rapp, 1980; Rummel, 1997).

3. DATABASE DEVELOPMENT

3.1 The gravity anomaly database

In this study, a total number of 3500 gravity points were provided by DSMM from throughout Peninsular Malaysia (Figure 1). To construct the surface gravity anomaly in gridded form, a combination of R-C-R with cross validation strategy is utilized. Figure 2 shows the steps undertaken to accomplish the task.

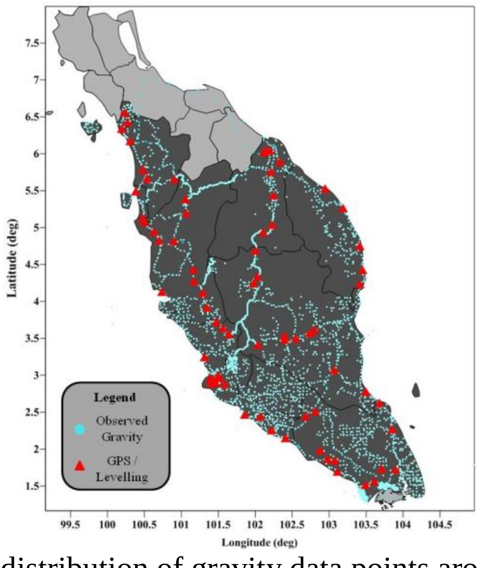

Figure 1. Geographical distribution of gravity data points around Peninsular Malaysia

Referring to Figure 2, the following descriptions illustrate the process;

i. Reduce the land observed gravity anomaly to a reference surface by removing long wavelength effects from the Global Geopotential Model (GGM) having a maximum degree and order (Amos & Featherstone, 2003).

ii. Next, remove the topographical effect. In this study, the RTM technique (Forsberg, 1984) utilizing the TC programming in GRAVSOFT (Forsberg & Tscherning, 2008) is used.

iii. Identify gross errors by using cross validation approach. In this study, any residual larger than 20 mGal declared as outliers are then removed (Sulaiman, Talib, Md Wazir, & Yusof, 2013). In this study, a total number of 56 out of 3224 gravity points were excluded.

iv. The reduced and cleaned surface gravity anomaly dataset are then gridded by using spatial interpolation technique (Kriging). Restore back the topographical effect to the cleaned, reduced surface gravity anomaly in grid form.

v. The resolution for the constructed Peninsular Malaysia geoid model chosen is 1 ' x 1' minute arc. It should be pointed out that, for the study area, the surface gravity anomaly varies from -32.896 to 115.936 mGal with the mean and the standard deviation values 21.489 and 16.744 mGal, respectively.

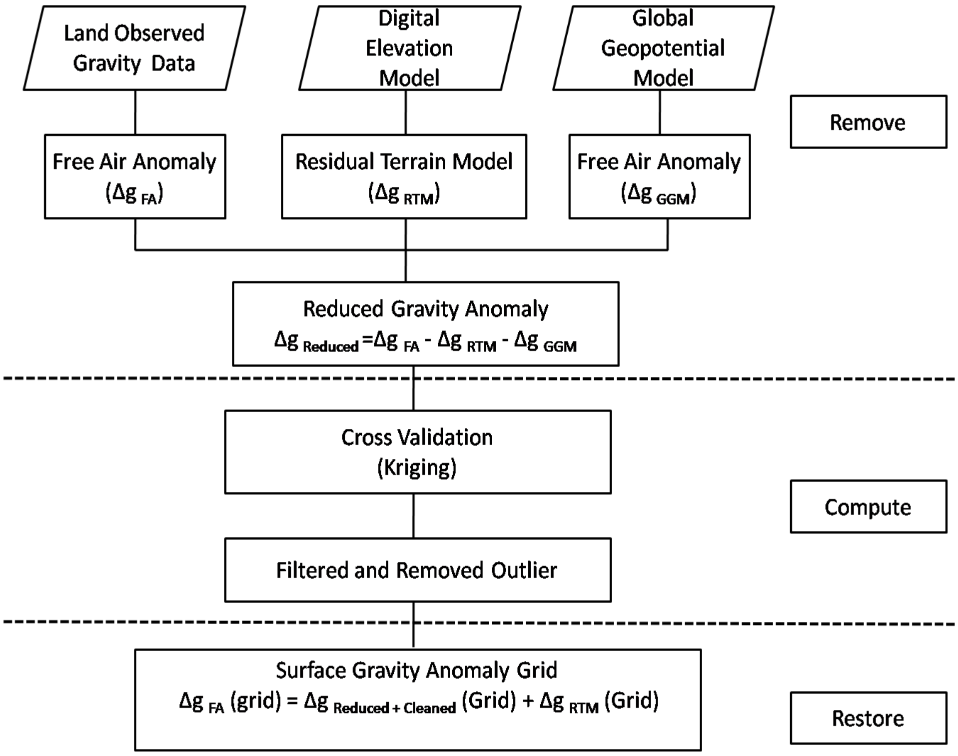

Figure 2. Schematic diagram for detecting developed surface gravity anomaly using combination R-C-R and Cross Validation

3.2 Global Geopotential Model

In the computation for estimating the geoid model using the least square modification of Stokes's approach, the most suitable global geopotential model needs to be chosen. The process of determining the best fit global geopotential model for Peninsular Malaysia is carried out by comparing the computed surface gravity anomaly from observed gravity anomaly and the derived from the global geopotential model. Furthermore, a comparison between the geoid height derived from the geopotential model and the existing regional model is used towards determining a suitable GGM. The most recent geopotential model from combined (i.e EGM2008, GGM03 and Eigen6C) and satellite dataset (i.e Goco01s, DGM-1s and Eigen5S) solutions is investigated in this study. Detailed information about GGMs can be found at http://icgem.gfz-potsdam.de/ICGEM.html. Recent studies carried out by Sulaiman et al. (2011 & 2013) found that the EGM2008 and GOCO01S are the most suitable global geopotential models for Peninsular Malaysia using the combined and satellite dataset mentioned. However, in this study, the combined solution will be used in preparing the surface gravity anomaly database, while the satellite dataset will be used for estimating the geoid height.

3.3 Digital Elevation Model

A Digital Elevation Model (DEM) represents the height of the Earth's surface digitally (Hirt, Filmer, & Featherstone, 2010; Kiamehr & Sjöberg, 2005). Normally, this model is represented in the grid form (e.g. 5 x 5 meter). A DEM is a main source of high-frequency gravity field signals (Forsberg, 1984). The DEM used in this study is a combination of 3 arc second DSMM Digital Terrain Elevation Data (DTED) and 30 second means from the Shuttle Radar Topography Mission (SRTM). The DTED data is produced using photogrammetric techniques while the SRTM data is based on a modified radar system that flew onboard the Space Shuttle Endeavor. The DSMM DTED data is used only in mountainous areas where the elevation is above 200 meters. This combined DEM is intended to be used for the interpolation of free air anomalies.

3.4 The GPS/Levelling Data

The Peninsular Malaysia Primary Geodetic Network was established in 1989, and consists of 238 points. The current coordinates are referred to as the Geodetic Datum Malaysia 2000 (GDM2000). In this study, 70 GPS levelling points (Figure 1) were used as external and independent tools in order to estimate the accuracy of computed geoid height in an absolute sense. Each GPS/levelling point was observed on the selected benchmark (BM). The overall accuracy for the ellipsoidal heights of GPS/levelling on the SBM and BM is estimated ±6 cm. The parameter adherence for the observation and processing is shown in Table 1.

Table 1. Field Observation Setting and Processing Strategy (Jamil, 2011)

Field Observation Setting		
No	**Items**	**Parameter**
1	Observation Technique	Static positioning
2	GPS Control	At least 3 stations

3	Observation Sessions	At least 2 independent sessions
4	Station Connections	At least 3 independent baselines
5	VDOP	Less than 6 (90% of the observation session)
6	Elevation Angle	Above 15
7	Satellite Tracking	At least 5 satellites with GDOP of less than 6
Processing Strategy		
1	General Procedure	Prescribed procedures as provided by manufacturer manual must be followed
2	Datum	GDM2000
3	Elevation Mask	15°
4	Ephemerides	Short baseline of less than 30 km: Broadcast
5		Long baseline: Precise
6	Baseline Processing	RMSE less than 2 cm
7	Quality	Maximum data rejection - less than 10 %
8		Ambiguity fixed solution
9	Adjustment	Least square adjustment should be used
10	Minimally Constrained Adjustment	One control station fixed in GDM2000 coordinates
11	Quality Indicator	Pass Chi-squares test at 95% confident region
12		All baselines must pass the local test
13	Over-Constrained Adjustment	At least 2 control stations must be fixed in the final adjustment

4. GRAVIMETRIC GEOID MODEL COMPUTATION

An explanation of mathematical algorithms and procedures for computing geoid height using Least Square Modification of Stokes's approach is mentioned in Section 2. In the KTH approach, modification coefficient parameters consisting of s_n and b_n need to be determined accurately (Sjöberg, 2003).In this study, the Least Square technique developed by KTH (Kiamehr & Sjöberg, 2012) is used in the determination of the modification coefficient parameters.

In KTH approach, the truncation errors in GGM and errors in observed land gravity data will be matched against each other in the least square sense. In addition, the integration cap(Ψ_0) computation point is limited to a few degrees. This is due to the lack of information on the observed land gravity data around the study area. The selection of upper limits (M) of the GGM and the upper bound of Stokes's function (L) is the crucial and vital part in the geoid modelling procedure in order to increase the computational efficiency. A higher degree of GGMs may significantly compensate the shortage information on the observed gravity data; however, the GGM's potential coefficient errors will also increase according to the increase of the degree of M.

Additionally, the error degree variance of the observed land gravity data also has an important role in the computation of a geoid model using KTH approach. The investigation done by Ågren & Sjöberg (2004), found that the reciprocal distance model produces more reliable results, and can be used in the determination of error degree variance of land observed gravity data. Based on the study, the reciprocal distance model will also be used in the determination of the error degree variance in the observed land gravity data.

Table 2. Condition evaluated in the determination of the Modified Coefficient Parameter

M=L	**Integration Cap (Ψ_0) (Deg)**	**Correlation Length (Ψ) (Deg)**	**Terrestrial Error ($\sigma_{\Delta g}$) (mGal)**
30			
60	0.1	0.05	0.40
120	0.5	0.10	1.00
150	1.0	0.20	5.00
180	2.0	0.30	10.00
max	3.0	0.40	20.00

In this study, for the computation of the estimation geoid model, there are few sets of conditions imposed (Table 2) in order to determine the modification coefficient parameters. Referring to Table 2, initially, one of the input condition parameters is replaced while the remaining condition parameters are fixed. Then, the second condition parameter will be replaced and the process is repeated. The optimum modification coefficient parameter will be determined by comparing the result of the computed Gravimetric Geoid Model with known GPS/levelling datasets used in this study.

5. RESULT AND ANALYSIS

In this study, it should be pointed out that, the modification coefficient parameters were computed based on various input condition parameters. An estimated gravimetric geoid model is then generated based on the coefficient parameters using KTH approach. The combination of GOCO01S and surface gravity anomaly data is also being used in computing the estimated geoid model. It should be noted that, for each condition parameter it will produce a different set of modification coefficient parameters. Then based on each set of modification coefficient parameters, it then estimates a gravimetric geoid model to be generated.

Table 3. Statistical analysis on the gravimetric geoid model computed using a different set of condition parameters

Step	**Parameter**	**Different Option**					
1	M=L	30	60	120	150	**180**	max
	Spherical Cap	3.0					
	Correlation Length	0.1					
	Terrestrial Error	0.4					
	Min	0.670	0.496	0.286	0.131	**0.121**	0.114
	Max	2.662	2.212	1.380	0.999	**0.939**	0.939

	Average	1.444	1.148	0.582	0.354	**0.338**	0.337
	RMSE	1.524	1.231	0.629	0.388	**0.368**	0.368
2	Spherical Cap		0.1	0.5	1.0	2.0	**3.0**
	M=L	180					
	Correlation Length	0.1					
	Terrestrial Error	0.4					
	Min		0.373	0.331	0.357	0.327	**0.121**
	Max		0.992	1.326	1.436	1.191	**0.939**
	Average		0.632	0.563	0.656	0.517	**0.338**
	RMSE		0.650	0.594	0.696	0.546	**0.368**
3	Correlation Length		0.05	0.10	0.20	0.30	**0.40**
	M=L	180					
	Spherical Cap	3.0					
	Terrestrial Error	0.4					
	Min		0.134	0.121	0.116	0.121	**0.116**
	Max		0.968	0.939	0.916	0.896	**0.882**
	Average		0.106	0.100	0.096	0.094	**0.092**
	RMSE		0.381	0.368	0.362	0.357	**0.356**
4	Terrestrial Error		0.40	1.00	**5.00**	10.00	20.00
	M=L	180					
	Spherical Cap	3.0					
	Correlation Length	0.4					
	Min		0.882	0.754	**0.689**	0.709	0.755
	Max		0.116	0.042	**-0.028**	-0.001	-0.050
	Average		0.330	0.305	**0.291**	0.289	0.285
	RMSE		0.356	0.330	**0.321**	0.324	0.335

Table 3 shows the statistical analysis of estimated gravimetric geoid models computed based on the different set of condition parameters. The estimated geoid models constructed will then be compared with the known benchmark (BM) values. Hence, the selection of optimum condition parameter values can then be identified as shown in Table 3. Based on Table 3, the optimum combination parameter is M=L= 180, Ψ_0=3.0^o, Ψ=0.4^o and $\sigma_{\Delta g}$= 5.0 mGal (lowest standard deviation). It also needs to be stressed that, the sequences of condition parameters do not carry any weight in the selection of optimum combination parameters.

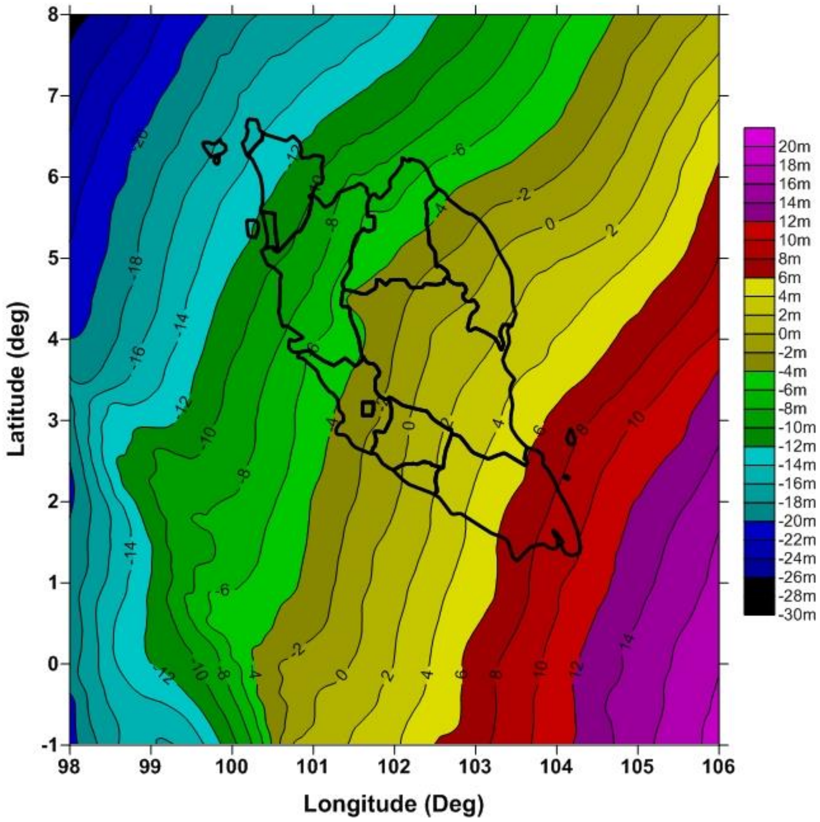

Figure 3. 1'x1' arc minute Gravimetric geoid model constructed

The gravimetric geoid model is then generated using KTH approached based on the optimum condition parameters without any additive correction as shown in Figure 3.

6. CONCLUSIONS

In this study, an estimated gravimetric geoid model of Peninsular Malaysia was developed using the KTH approached without additive corrections. The estimated geoid model of Peninsular Malaysia is computed based on a set of pre-determined modification coefficients. This derived estimated geoid model is a preliminary step towards obtaining a higher precision of geoid models for Peninsular Malaysia. Applying the additive correction will enhance the accuracy and precision of future gravimetric geoid models. However, more observed land gravity data is needed to fill in incomplete areas, in order to provide a more accurate representation of the Peninsular Malaysia gravity field.

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REFERENCES

Abdalla, A., & Fairhead, D. (2011). A new gravimetric geoid model for Sudan using the KTH method. *Journal of African Earth Sciences*, *60*(4), 213–221. doi:10.1016/j.jafrearsci.2011.02.012

Abdalla, A., & Tenzer, R. (2011). The evaluation of the New Zealand's geoid model using the KTH method. *Geodesy and Cartography*, *37*(1), 5–14. doi:10.3846/13921541.2011.558326

Ågren, J., & Sjöberg, L. E. (2004). Comparison Of Some Methods For Modifying Stokes' formula In The Goce Era. In *Proc. 2nd International GOCE user workshop "GOCE, The Geoid model and Oceanography."* Frascati, Italy: ESA-ESRIN.

Ågren, Jonas. (2004). *Regional Geoid Determination Methods for the Era of Satellite Gravimetry Synthetic Earth Gravity Models -(Numerical Investigations Using Synthetic Earth Gravity Models)*. Royal Institute of Technology (KTH).

Ellmann, A. (2004). *The geoid for the Baltic countries determined by the least squares modification of Stokes' formula*. *Technology*. Royal Institute of Technology (KTH), Department of Infrastructure.

Forsberg, R. (1984). A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modelling. *Scientific Report No.5 The Ohio State University*, 129.

Forsberg, R., & Strykowski, G. (2008). *Geoid Determination Update Peninsular Malaysia* (pp. 1–18).

Forsberg, R., & Tscherning, C. C. (2008). *An overview manual for the GRAVSOFT : Geodetic Gravity Field Modelling Programs* (p. 75).

Heiskanen, W. A., & Moritz, H. (1980). *Physical Geodesy* (p. 384). San Fransisco: W.H Freeman and Co.

Hirt, C., Filmer, M., & Featherstone, W. (2010). Comparison and validation of the recent freely available ASTER-GDEM ver1, SRTM ver4. 1 and GEODATA DEM-9S ver3 digital elevation models over Australia. *Australian Journal of Earth …*, 121–136. doi:10.1007/s00190-006-0094-0

Hofmann-Wellenhof, B., & Moritz, H. (2005). *Physical Geodesy* (p. 404). SpringerWein New York.

Hunegnaw, A. (2001). *Geoid determination over Ethiopia with emphasis on downward continuation of gravity anomailes*. Institutionen för geodesi och fotogrammetri, KTH, Geodesy and Photogrammetry.

Jamil, H. (2011). GNSS Heighting and Its Potential Use in Malaysia. In *GNSS Processing And Analysis*. Marrakech, Morocco: FIG Working Week 2011.

Jekeli, C., & Rapp, R. (1980). *Accuracy of the determination of mean anomalies and mean geoid undulations from a satellite gravity field mapping mission*.

Kiamehr, R. (2006). *Precise Gravimetric Geoid Model for Iran Based on GRACE and SRTM*

Data and the Least-Squares Modification of Stokes ' Formula with Some Geodynamic Interpretations. *Royal Institute of Technology (KTH)*. Royal Institute of Technology (KTH).

Kiamehr, R., & Sjöberg, L. E. (2005). Effect of the SRTM global DEM on the determination of a high-resolution geoid model: a case study in Iran. *Journal of Geodesy*, (79), 540–551. doi:10.1007/s00190-005-0006-8

Kiamehr, R., & Sjöberg, L. E. (2012). Scientific Software for Precise Geoid determination Based on the Least-Squares Modification of Stoke's Formula. *Technical Manual, Royal Institute of technology (KTH), Division Geodesy*.

Nahavandchi, H. (1998). *Precise GPS-Gravimetric Geoid determination with Improved Topographic Corrections over Sweden*. *PhD Thesis*. Royal Institute of technology (KTH).

Nsombo, P. (1996). *Geoid Determination Over Zambia*. *PhD Thesis*. Royal Institute of Technology (KTH).

Rapp, R. H., & Pavlis, N. K. (1990). The development and analysis of geopotential coefficient models to spherical harmonic degree 360. *Journal of Geophysical Research: Solid Earth*, *95*(B13), 21885–21911. doi:10.1029/JB095iB13p21885

Rummel, R. (1997). Spherical spectral properties of the earth's gravitational potential and its first and second derivatives. In F. Sansó & R. Rummel (Eds.), *Geodetic Boundary Value Problems in View of the One Centimeter Geoid SE - 11* (Vol. 65, pp. 359–404). Springer Berlin Heidelberg. doi:10.1007/BFb0011710

Sjöberg, L. E. (1984). *Least Squares Modification of Stokes' and Vening Meinesz' Formulas by Accounting for Errors of Truncation, Potential Coefficients and Gravity Data*.

Sjöberg, L. E. (1986). Comparison of some methods of modifying Stokes' formula. *Bollettino Di Geodesia E Sciencze Affini*, *3*(45), 229–248.

Sjöberg, L. E. (1991). Refined least squares modification of Stokes' formula. *Manuscr Geod*, *16*.

Sjöberg, L. E. (2003). A general model for modifying Stokes' formula and its least-squares solution. *Journal of Geodesy*, *77*(7-8), 459–464. doi:10.1007/s00190-003-0346-1

Sjöberg, L. E. (2011). *The KTH Approach to Modelling the Geoid -Extended Lecture Notes* (p. 58). Stockholm.

Sulaiman, S. A. H., Talib, K. H., Wazir, M. A. M., & Yusof, O. M. (2011). Comparison of gravity anomalies from terrestrial gravity and recent geopotential models over West-Malaysia. *System Engineering and Technology (ICSET), 2011 IEEE International Conference on*. doi:10.1109/ICSEngT.2011.5993446

Sulaiman, S. A. H., Talib, K. H., Wazir, M. A. M., & Yusof, O. M. (2013). Evaluation of geoid height derived by geopotential model and existing regional geoid model. In *Signal Processing and its Applications (CSPA), 2013 IEEE 9th International Colloquium on* (pp. 106–110). doi:10.1109/CSPA.2013.6530024

Sulaiman, S. A. H., Talib, K. H., Md Wazir, M. A., & Yusof, O. M. (2013). Cross validation approach in qualification of observed gravity data. Control and System Graduate

Research Colloquium (ICSGRC), 2013 IEEE 4th. doi:10.1109/ICSGRC.2013.6653297

Tscherning, C., & Rapp, R. (1974). *Closed covariance expressions for gravity anomalies, geoid undulations, and deflections of the vertical implied by anomaly degree variance models.* Scientific Interim Report Ohio State. The Ohio State University

Ulotu, P. E. (2009). *Geoid Model of Tanzania from Sparse and Varying Gravity Data Density by the KTH Method*. *PhD Thesis*. Royal Institute of Technology (KTH).