Mathematical Modeling of the Integral Errors in the Image Coordinates of the Points from Photogrammetrical Photos

Assist Dr. Eng. Neli Dimitrova ZDRAVCHEVA, Bulgaria

"*Work is the need of life, wisdom - its aim, satisfaction – its reward.* "
Friedrich Schiller

Keywords: Photogrammetry, Remote Sensing Methods, Calibration of Images, Mathematical Modeling, FEM

ABSTRACT

In the analytical and digital photogrammetry photos are viewed as mathematical models - mostly as central projections. On real images, which are physical models, under the influence of multiple factors there is a deviation in comparison to the theoretical position of the points, i.e. figuratively said, "distortions' relief" appear. In laboratories one can identify individual mistakes, in the real world; however, integrated errors caused by the impact of all factors differ from the sum of the individual components. For mathematical modeling of the field of the total errors in the image coordinates of the points of the photographs the author offers a fundamentally new approach: since the surface of the real deviations of the individual picture in comparison to the theoretical picture is too complex and cannot be precisely defined mathematically by analytical continuous function, valid for the entire image area, then for accounting for local deformations and achieving highly precise results it is logical, according to the author, the photograph to be divided into sub-areas, and for each of them to performed by elements mathematical modeling of the variation in the position of the points. In the report it is done through the finite element method (FEM). There has been established a methodology for calibration of the photos by the construction of an extended mathematical model of collinear dependence (adjustment of bundles of projecting rays) through FEM. There have been derived formulas to calculate the coefficients in the equations of the corrections and so on. .

РЕЗЮМЕ

В аналитичната и дигитална фотограметрия снимките се разглеждат като математически модели - най-често като централни проекции. Върху реалните снимки, които са физически модели, под въздействието на множество фактори се получават отклонения спрямо теоретичното положение на точките, т. е. образно казано възниква "релеф на деформациите". В лабораториите могат да се определят отделни грешки, в реални условия обаче, интегралните грешки, предизвикани от въздействието на всички фактори, се отличават от сумата на отделните съставящи. За математическото моделиране на полето на сумарните грешки в образните координати на точките от фотоснимките авторът предлага един принципно нов подход: Тъй като повърхнината на отклоненията на реалната снимка спрямо теоретичната е твърде сложна и не може да бъде математически точно определена чрез аналитична непрекъсната функция, валидна за цялото образно поле, то за отчитане на локалните деформации и постигането на високоточни резултати е логично, според автора, фотоснимката да се раздели на подобласти и за всяка една от тях да се извърши поелементно математическо моделиране на отклоненията в положението на точките. В доклада то е направено по метода на крайните елементи (МКЕ). Създадена е и методика за калибриране на снимките чрез построяването на един разширен математически модел на колинеарната зависимост (изравнение на снопове от проектиращи лъчи) по МКЕ. Изведени са формули за изчисляване на коефициентите в уравненията на поправките и т. н..

.

1. INTRODUCTION

To deceive ourselves that we know everything in science means blocking the way for its further development and improvement. A basic rule in scientific research is to remember at each step that we could be wrong. No man is perfect and sinless. All scientists, including the greatest geniuses, make mistakes because our human knowledge and imagination are insignificant in comparison to the universal variety and reality. We must rely on the knowledge, achievements and experience of our ancestors and learn from them, our children will learn from us and in this way step by step humankind will continue to see in the veiled secrets and mysteries of reality and nature. And so, by drawing from the wisdom of the ages, we can face ever more confidently the challenges of our world dynamic contemporary world......

Photogrammetry and Remote Sensing (GMF) are very interesting, highly technological, economically profitable, dynamically developing and versatile and are a great example of a modern scientific challenge. The role of the image in GMF is fundamental – regardless of the specific task, the main source of information is Her Majesty the picture. But the fact that is it is so important and decisive does not mean that it is error-free. On the one hand the image is a main source of information, but on the other it is also a main source of errors and therefore of problems those hamper their photogrammetrists and create headaches.

One of the biggest advantages of photogrammetry is that it is a remote method. While geodetic measurements are performed directly in the object space, in photogrammetry the output in resolving all problems in analytical and digital photogrammetry are the image coordinates of the points, which are recorded on the pictures representing physical models.

This is why the direct use of the measured image coordinates in solving various problems leads to approximate, and in some cases (with significant deformations), to incorrect results as well.

Photogrammetry has been developing mainly in two directions [1]: 1. Creation of innovative technical means to expand the opportunities of the photogrammetric systems, to facilitate the work of the operators and to increase the measurement precision; 2. Elaboration of the theoretical foundations of photogrammetry, i. e. Of the mathematical aparatus. At present the accuracy of the final results in processing of the photogrammetric images is limited not so much by the instrumental errors as by errors representing deviations in the position of the points of the picture in terms of the ideal central projection. The reasons for the occurrence of these errors are numerous - distortion of the lenses, refraction, deformation on the photographic materials, the non-adhesion of the film and so on.. (These factors are more than 25). Despite the continuous improvement of the quality of the photo equipment, of the photographic materials and software, an important reserve to achieve higher accuracy of the analytical processing of images is the residual errors in the individual points of image. In the photogrammetric theory photographs are regarded as mathematical models - mostly as central projections of the captured object (in the entire report the frame photos are born in mind) . The parameters of the photogrammetric cameras determine the geometrical characteristics of these projections. This is why a necessary condition for solving various problems in analytical and digital photogrammetry is knowing the interior orientation of the cameras with sufficient accuracy. Furthermore, on real images under the effect of multiple factors deviations are obtained from the theoretical position of the points, i. e. there appears, so to speak, "a relief of deformations".

2. MATHEMATICAL MODELING OF THE ERRORS THROUGH FEM

In creating mathematical models a controversial requirement is made - on the one hand they should be comprehensive enough to account for the influence of all the factors affecting the state of the system, on the other - they should be as simple as possible, so that one can clearly establish mathematically the dependencies among the separate parameters and obtain effective solutions. The functional part of the mathematical model in analytical photogrammetry is best described by the known equations for collinearity [3] .In it the frame photogrametrical picture is seen as an ideal central projection of the captured subject with certain parameters (elements of the interior orientation) . The photography, processing, storage and measurement of the picture are real physical processes with varying parameters. Therefore this mathematical model can be directly applied to solving different problems of analytical and digital photogrammetry only with lowered requirements concerning the accuracy of the final results or when the pictures are taken under ideal conditions. In general, this model is too idealized and loaded with errors. The key to its improvement, in order to obtain high precision results, is in modeling the various errors in order to eliminate them or reduce them to minimum. In other words, the functional part of the mathematical model of collinearity is an approximation of the image geometry. It causes errors in the model – differences are obtained between the measured image coordinates of the points and the corresponding coordinates obtained using the model with true values of its parameters. The approximations, imposed by the functional part of the model should be negligible so that the errors to be random and independent of each other [2]. For this purpose there should be applied more developed and advanced functional models. The model of collinearity has 9 parameters (we consider the case in which the equations of collinearity are made for certain reference points with known coordinates X, Y, Z) - 6 of the external and 3 of the interior orientation of the image. If any corrections for the various systematic errors are made, i.e. if additional parameters are included for their reading an extended mathematical model is obtained:

Reading of errors in the image coordinates can be done in two ways - by estimating each type of error and summing up the individual components or by modeling of the total errors δx, δy. In [2] there have been analyzed the advantages of the second approach and its improvement is discussed further on. According to the author any real photograph represents a two-dimensional field of the theoretical picture. Under the cumulative effect of the different factors in the process of the overall processing of the image (photography, photochemical processing, storage in case of temperature change, atmospheric pressure and humidity, measuring of the stereocomparator and so on.) the ideal, theoretical picture corresponding to the mathematical model based on the condition of collinearity, deforms considerably. I.e. the measured values of the image coordinates of the points of the real image x`, y` differ from their theoretical values x, y, obtained from the corresponding mathematical model of the collinear relationship. In other words, there are variations in the coordinates of the points from the photogrammetric images:

$$U = x - x' \qquad\qquad V = y - y' \qquad\qquad (1)$$

The mathematical modeling of the variations δ (U, V) of the image coordinates of the points from the real photograph in terms of the theoretical central projection is done by the author through FEM [2]. The main requirements for applying it for this purpose are available because the photo is a continuous environment and functions U and V are defined and continuous on its entire area. The studied functions $\delta x = U(x, y)$, $\delta y = V(x, y)$ represent the deviations in the position of the points from the real in comparison to the theoretical image (central projection), i.e. they read the integral errors, caused by the effect of all factors in the process of the overall processing of the image. These functions can be approximated by a discrete model, built from many partly continuous functions, given on a finite number of subareas from the photo. They form a two-dimensional vector field of the deviations of the real photo in comparison to the theoretical one. The functions $U(x, y)$ and $V(x, y)$ can be graphically represented in the form of smooth surfaces over the plane x, y of the image (Figure 1):

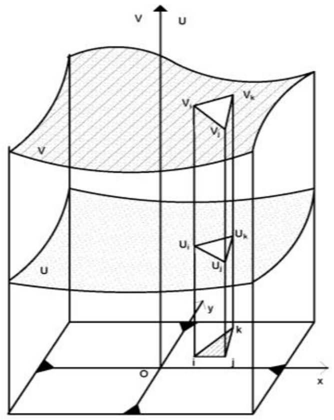

Figure 1

The studied image is divided into triangular finite elements (FE), defined by N_{vz} number of node points. For now we assume that the deviations $\delta_n (U_n, V_n)$, $n = 1, N_{vz}$ in the nodes are known and by them we will represent the deviations $\delta(U, V)$ in the other points form the image. The wanted functions $U(x, y)$ and V (x, y) [3] can be approximated element by element with linear polynomials from this type:

$$U(x,y) = k_1 + k_2 x + k_3 y \qquad V(x,y) = k_4 + k_5 x + k_6 y \quad (2)$$

The six unknowns coefficients $(l = 1,6)$ for each triangular FE are defined fully by the six components of the deviations at its nodal points:

$$\{\delta\}^e = \begin{vmatrix} \delta_i(U_i,V_i) \\ \delta_j(U_i,V_i) \\ \delta_k(U_i,V_i) \end{vmatrix} \qquad (3)$$

For each of the two functions $U(x, y)$ and $V(x, y)$ a system of three equations is drawn, as in formulae (2) we replace the coordinates and the components of the deviations for the three nodal points i, j, k of the specific finite element.

$$\begin{aligned} U_i &= k_1 + k_2 x_i + k_3 y_i & V_i &= k_4 + k_5 x_i + k_6 y_i \\ U_j &= k_1 + k_2 x_j + k_3 y_j & V_j &= k_4 + k_5 x_j + k_6 y_j \\ U_k &= k_1 + k_2 x_k + k_3 y_k & V_k &= k_4 + k_5 x_k + k_3 y_k \end{aligned} \qquad (4)$$

After solving the above two systems these coefficients are obtained $\kappa_l (l = 1,6)$ for the respective finite element $e(i, j, k)$:

$$\begin{aligned} k_1 &= \frac{1}{\Delta}\Big[\left(x_j y_k - x_k y_j\right)U_i + \left(x_k y_i - x_i y_k\right)U_j + \left(x_i y_j - x_j y_i\right)U_k\Big] \\ k_2 &= \frac{1}{\Delta}\Big[\left(y_i - y_k\right)U_i + \left(y_k - y_i\right)U_j + \left(y_i - y_j\right)U_k\Big] \\ k_3 &= \frac{1}{\Delta}\Big[\left(x_k - x_j\right)U_i + \left(x_i - x_k\right)U_j + \left(x_j - x_i\right)U_k\Big] \qquad (5) \\ k_4 &= \frac{1}{\Delta}\Big[\left(x_j y_k - x_k y_j\right)V_i + \left(x_k y_i - x_i y_k\right)V_j + \left(x_i y_j - x_j y_i\right)V_k\Big] \end{aligned}$$

$$k_5 = \frac{1}{\Delta}\left[\left(y_i - y_k\right)V_i + \left(y_k - y_i\right)V_j + \left(y_i - y_j\right)V_k\right]$$

$$k_6 = \frac{1}{\Delta}\left[\left(x_k - x_j\right)V_i + \left(x_i - x_k\right)V_j + \left(x_j - x_i\right)V_k\right]$$

In the above formulas Δ denotes the double area of the respective finite element *(i, j, k)*. The area S is calculated by the famous mathematical formula:

$$S = \frac{1}{2}\left[\left(x_1 - x_2\right)\left(y_1 + y_2\right) + \left(x_2 + x_3\right)\left(y_2 + y_3\right) + \left(x_3 - x_1\right)\left(y_3 + y_1\right)\right] \tag{6}$$

By substituting the obtained values for the coefficients κ_l and making the corresponding transformations formulas (2) acquire this form:

$$\begin{aligned} \delta x &= U\ (x,y) = N_i\ U_i + N_j\ U_j + N_k\ U_k \\ \delta y &= V\ (x,y) = N_i\ V_i + N_j\ V_j + N_k\ V_k \end{aligned} \tag{7}$$

In the above formulas N_i , N_j , N_k indicate the so-called functions of the form [6], which are calculated by the expressions:

$$N_i = \frac{\left(a_i + b_i\ x + c_i\ \mathrm{y}\right)}{\Delta} \qquad N_j = \frac{\left(a_j + b_j\ x + c_j\ \mathrm{y}\right)}{\Delta} \qquad N_k = \frac{\left(a_k + b_k\ x + c_k\ \mathrm{y}\right)}{\Delta}$$

$$\begin{aligned} a_i &= x_j y_k - x_k y_j \\ b_i &= y_j - y_k \\ c_i &= x_k - x_j \\ a_j &= x_k y_i - x_i y_k \\ b_j &= y_k - y_i \\ c_j &= x_k - x_j \\ a_k &= x_i y_j - x_j y_i \\ b_k &= y_i - y_j \\ c_k &= x_j - x_i \end{aligned} \tag{8}$$

where: x, y are the image coordinates of any point on the inside of the discussed FE; x_i, y_i, x_i, y_i, x_k, y_k are the image coordinates of the three nodal points for this e *(i, j, k*). The functions of the form *Ni, Nj, Nk* [3] are presented graphically on Figure 2.

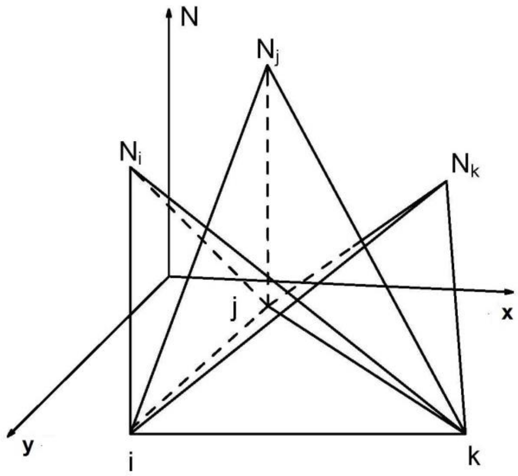

Fig. 2.

At any point of any eFE the sum of the three functions of the form is:

$$N_i + N_j + N_k = 1 \qquad (9)$$

The function of the form N_p of the particular node *p* is equal to a unit in the node *p* and to zero in all other nodal points of the discussed finite element *(p = i, j, k)*. Although the concept of function of the form is widely used, according to the author it would more correctly to call it a function of the influence of the node. Since it reflects the contribution of the nodal deviations on the deviations of any point on the inside of FE in accordance with the approximated function. The functions of the form are also coordinates of the area [6], having interesting geometric interpretation [2]. If the inside of a given FE*e (i, j, k)* there is a fixed arbitrary point *p*, it divides the area of the triangle in three parts - respectively Ai, Aj, A_K that clearly identify the position of the point *p* by the coordinates of the area. The lines, consisting of points with a permanent coordinate of the area *Lp (p = i, j, k)* are straight lines parallel to the side lying to the opposite of the node *p and Lp = 0* is the side itself. It can be concluded that all lines for which *Lp = 0* (their number is equal to the number of FE where the node *p* participates) enclose the sub-area of influence of the node value δ_p. I.e. the errors in the geodetic and the image coordinates of the individual reference point are localized and will not cause a deformation of the entire model.

In the above proposed methodology for modeling integral errors of the images by FEM all arguments and conclusions are made on the assumption that the nodal values of the deviations *δ (U, V)* are known quantities. In reality, however, such information is missing and its obtaining is subject of further examination.

3. ANALYTICAL CALIBRATION OF IMAGES THROUGH FEM

For the analytical calibration through FEM by using test object it is necessary before photography to create a network of marked reference points whose coordinates *X, Y, Z* are determined. The number and the location are of considerable importance [2, 5], as well as the preliminary marking of the reference points and the accuracy with which their coordinates are defined. It is most appropriate in the capacity of calibration values to determine the corrections, $\delta f, \delta x_o, \delta y_o$ to the laboratory defined values of the elements of the internal orientation of the image and the corrections $\delta x_p, \delta y_p$ to the measured image coordinates of the point *p* from the picture for the cumulative effect of all sources of error, regardless of their origin and without their separation in components. I.e. besides the general correction δf to the laboratory determined value of the focal distance for each point *p* of image these corrections are also sought for:

$$\delta x'_P = \delta x_0 + \delta x_P \qquad \delta y'_P = \delta y_0 + \delta y_P \tag{10}$$

Below there are briefly marked basic stages of the established by the author methods [2] for calibration of photographs by FEM using a test object. The first stage consists of creating a network of FE on the image area of the picture - the number, the size and the shape of the individual FE are determined, while keeping the controversial requirement FE to be sufficiently small to ensure a higher accuracy, and on the other hand - with a smaller number of FE with larger size to reduce the workload. Since the photography has a square or rectangular shape, the easiest way to build a network of non-intercepting triangular FE is by joining fixed points from the opposite sides of the image and the resulting quadrangulars are divided into triangles by drawing their diagonals. The size and shape of FE depend on the number and location of the reference points. The calibration network can be also built directly by setting the image coordinates of the nodal points. For each of them two unknowns are introduced: $\delta x_{vz} = U_{vz}(x_{vz}, y_{vz})$ и $\delta y_{vz} = V_{vz}(x_{vz}, y_{vz})$, whose values should be determined as a result of the analytical calibration. The next stage consists of modeling the studied functions $\delta (U, V)$ element by element for each sub-region of the image. Let's fix an arbitrary reference point *p* from the calibrated image. By knowing its measured image coordinates we can define in which finite element *e (i, j, k)* of the already built calibration network this point lies in and the deviation $\delta_p(U_p, V_p)$ can be expressed by the nodal values of the six variations U_i, U_j, U_k, V_i, V_j, V_k in the three peaks *i, j, k* of the particular triangular FE according to formulae (7):

$$\begin{aligned} \delta x'_P &= U_P(x_P, y_P) = N_i\, U_i + N_j\, U_j + N_k U_k \\ \delta y'_P &= \mathrm{V}_P(x_P, y_P) = N_i\, \mathrm{V}_i + N_j\, \mathrm{V}_j + N_k V_k \end{aligned} \tag{11}$$

To make the connection between neighboring FE it is necessary for the nodal values of the studied functions to introduce the same unknown $\delta_{vz}(U_{vz}, V_{vz})$ when performing neighboring local approximations. Thus the continuity of the approximated function $\delta (U, V)$ in its passage through the inter-element limits of the built network of triangular FE is guaranteed by the continuity at the tops of triangles. In the next stage we should find the nodal values of the deviations. Let us imagine that in one hand we hold a real photogrammetric image obtained under specific conditions, and in the other we have its theoretical equivalent, built according to the condition of collinearity (Fig. 3). If the mathematical model of collinearity reflects accurately and adequately the physical nature of the image, when putting the two pictures one over the other, the homonymous points *p and p`* should fully coincide. The presence of mismatches between them shows exactly the opposite, i. e. that the used mathematical model does not reflect and does not fully read the physical nature of the real picture. It is formed in the real physical environment under the influence of multiple deforming factors (Figure 3).

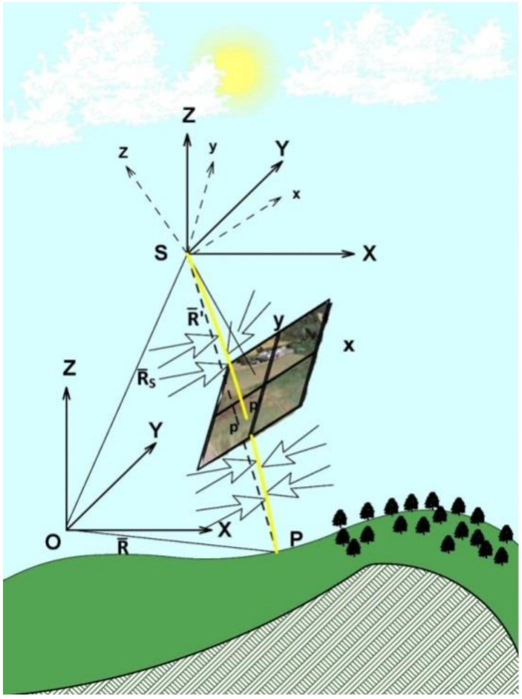

Fig. 3. ⇗ shows the deforming factors

When modeling the integral errors in the pictures by FEM the deviations *δp(Up, Vp*) of the measured image coordinates of the single reference point *p* is expressed by the nodal values of the sex deviations *Ui, Uj, Uk, Vi, Vj, Vk* of the real picture in comparison to the theoretical one in the three peaks *i, j, k*, depending on which FE of the already built calibration network the point falls in. As the full record of the equations is very voluminous, here a variant with using the functions of the form of each FE is given. Taking into account the above considerations and formulas (11) the collinearity equations look this way:

$$X = X_S + (Z - Z_S)\frac{a_1(x + N_iU_i + N_jU_j + N_kU_K) + a_2(y + N_iV_i + N_jV_j + N_kV_K) - a_3 f}{c_1(x + N_iU_i + N_jU_j + N_kU_K) + c_2(y + N_iV_i + N_jV_j + N_kV_K) - c_3 f} \qquad (12)$$

$$Y = Y_S + (Z - Z_S)\frac{b_1(x + N_iU_i + N_jU_j + N_kU_K) + b_2(y + N_iV_i + N_jV_j + N_kV_K) - b_3 f}{c_1(x + N_iU_i + N_jU_j + N_kU_K) + c_2(y + N_iV_i + N_jV_j + N_kV_K) - c_3 f}$$

For each reference point of the picture *p*, whose coordinates *X, Y, Z* are known with the necessary accuracy, two equations of the above type are composed. In this way the adjustment directly include the corrections $\delta_{vz}(U_{vz},V_{vz})$ for the peaks of the calibration network. The result is an extended mathematical model with modeling of the errors through FEM. It is expressed through a system of 2m number of equations (m designates the number of reference points). In this extended mathematical model for analytical calibration of photos by FEM the elements of the exterior orientation of the image X_S, Y_S, Z_S, α, ω, χ and the sought calibration parameters f, U_{vz}, V_{vz} are unknown. The subscript *vz* acquires values from 1 to N_{vz}. Since the next mathematical operations lead to too voluminous and complicated formulas it is advisable to introduce the following auxiliary's names:

$$\begin{aligned} X' &= a_1(x + N_iU_i + N_jU_j + N_kU_k) + a_2(y + N_iV_i + N_jV_j + N_kV_k) - a_3 f \\ Y' &= b_1(x + N_iU_i + N_jU_j + N_kU_k) + b_2(y + N_iV_i + N_jV_j + N_kV_k) - b_3 f \\ Z' &= c_1(x + N_iU_i + N_jU_j + N_kU_k) + c_2(y + N_iV_i + N_jV_j + N_kV_k) - c_3 f \end{aligned} \quad (13)$$

Given these indications, formulas (12) acquire far more compact form:

$$X = X_S + (Z - Z_S)\frac{X'}{Z'} \qquad Y = Y_S + (Z - Z_S)\frac{Y'}{Z'} \quad (14)$$

Equations (12) are in non-linear form regarding the sought unknown and are brought in this form by development in Taylor series. For this purpose we firstly introduce initial values for the unknown:. $X_{s_0}, Y_{s_0}, Z_{s_0}, \alpha_0, \omega_0, \chi_0, f_0, U_{vz_0}, V_{vz_0}$ In this way the corrections to the initial values $\delta X_s, \delta Y_s, \delta Z_s, \delta\alpha, \delta\omega, \delta\chi, \delta f, \delta U_{vz}, \delta V_{vz}$ play the role of unknown values. Therefore for each reference point *p* two equations of the corrections in linear form are obtained:

$$\begin{aligned} &d_1\delta X_s + d_2\delta Y_s + d_3\delta Z_s + d_4\delta\alpha + d_5\delta\omega + d_6\delta\chi + d_7\delta f + d_{7+2ni+1}\delta U_i + \\ &+ d_{7+2nj+1}\delta U_j + d_{7+2nk+1}\delta U_k + d_{7+2ni+2}\delta V_i + d_{7+2nj+2}\delta V_j + d_{7+2nk+2}\delta V_k + l = v \end{aligned} \quad (15)$$

$$\begin{aligned} &d'_1\delta X_s + d'_2\delta Y_s + d'_3\delta Z_s + d'_4\delta\alpha + d'_5\delta\omega + d'_6\delta\chi + d'_7\delta f + d'_{7+2ni+1}\delta U_i + \\ &+ d'_{7+2nj+1}\delta U_j + d'_{7+2nk+1}\delta U_k + d'_{7+2ni+2}\delta V_i + d'_{7+2nj+2}\delta V_j + d'_{7+2nk+2}\delta V_k + l' = v' \end{aligned}$$

If the coordinates of the reference points are defined with equal accuracy, then each of the above equations participates in the adjustment with equal weight. The formulas for the coefficients of the unknown in the above equations of the corrections are derived by the author as a result of the differentiation of (12) and the relevant mathematical operations:

$$\begin{aligned} &d_1 = \frac{\partial X}{\partial X_S} = 1 \qquad d_2 = \frac{\partial X}{\partial Y_S} = 0 \qquad d_3 = \frac{\partial X}{\partial Z_S} = -\frac{X'}{Z'} \qquad d_4 = \frac{\partial X}{\partial \alpha} = (Z - Z_s)\frac{X'^2 + Z'^2}{Z'^2} \\ &d_5 = \frac{\partial X}{\partial \omega} = -(Z - Z_s)\frac{Y'Z'\sin\alpha + X'Y'\cos\alpha}{Z'^2} \\ &d_6 = \frac{\partial X}{\partial \chi} = (Z - Z_s)\frac{Z'(a_2(x + N_iU_i + N_jU_j + N_kU_k) - a_1(y + N_iV_i + N_jV_j + N_kV_k))}{Z'^2} - \\ &\qquad -(Z - Z_s)\frac{X'(c_2(x + N_iU_i + N_jU_j + N_kU_k) - c_1(y + N_iV_i + N_jV_j + N_kV_k))}{Z'^2} \end{aligned} \quad (16)$$

$$d_7 = \frac{\partial X}{\partial f} = -(Z - Z_s)\frac{a_3 Z' - c_3 X'}{Z'^2} \qquad d_{7+2ni+1} = \frac{\partial X}{\partial U_i} = N_i (Z - Z_s)\frac{a_1 Z' - c_1 X'}{Z'^2}$$

$$d_{7+2nj+1} = \frac{\partial X}{\partial U_j} = N_j (Z - Z_s)\frac{a_1 Z' - c_1 X'}{Z'^2} \qquad d_{7+2nk+1} = \frac{\partial X}{\partial U_k} = N_k (Z - Z_s)\frac{a_1 Z' - c_1 X'}{Z'^2}$$

$$d_{7+2ni+2} = \frac{\partial X}{\partial V_i} = N_i (Z - Z_s)\frac{a_2 Z' + c_2 X'}{Z'^2} \qquad d_{7+2nj+2} = \frac{\partial X}{\partial V_j} = N_j (Z - Z_s)\frac{a_2 Z' + c_2 X'}{Z'^2}$$

$$d_{7+2nk+2} = \frac{\partial X}{\partial V_k} = N_k (Z - Z_s)\frac{a_2 Z' + c_2 X'}{Z'^2}$$

$$d'_1 = \frac{\partial Y}{\partial X_S} = 0 \quad d'_2 = \frac{\partial Y}{\partial Y_S} = 1 \quad d'_3 = \frac{\partial Y}{\partial Z_S} = -\frac{Y'}{Z'} \quad d'_4 = \frac{\partial Y}{\partial \alpha} = -(Z - Z_s)\frac{X'Y'}{Z'^2} \qquad (17)$$

$$d'_5 = \frac{\partial Y}{\partial \omega} = (Z - Z_s)\frac{Z'\{[(x + N_i U_i + N_j U_j + N_k U_k)\sin\chi + (y + N_i V_i + N_j V_j + N_k V_k)\cos\chi]b_3 + f\cos\omega\}}{Z'^2} - $$

$$-(Z - Z_s)\frac{Y'^2\cos\alpha}{Z'^2}$$

$$d'_6 = \frac{\partial X}{\partial \chi} = (Z - Z_s)\frac{Z'(b_2(x + N_i U_i + N_j U_j + N_k U_K) - b_1(y + N_i V_i + N_j V_j + N_k V_k))}{Z'^2} - $$

$$-(Z - Z_s)\frac{Y'(c_2(x + N_i U_i + N_j U_j + N_k U_k) - c_1(y + N_i V_i + N_j V_j + N_k V_k))}{Z'^2}$$

$$d'_7 = \frac{\partial Y}{\partial f} = -(Z - Z_s)\frac{b_2 Z' - c_2 Y'}{Z'^2} \qquad d'_{7+2ni+1} = \frac{\partial Y}{\partial U_i} = N_i (Z - Z_s)\frac{b_1 Z' + c_1 X'}{Z'^2}$$

$$d'_{7+2nj+1} = \frac{\partial Y}{\partial U_j} = N_j (Z - Z_s)\frac{b_1 Z' + c_1 X'}{Z'^2} \qquad d'_{7+2nk+1} = \frac{\partial Y}{\partial U_k} = N_k (Z - Z_s)\frac{b_1 Z' + c_1 X']}{Z'^2}$$

In the above formulas for the coefficients in front of the unknown nodal values of the six deviations *Ui, Uj, Uk, Vi, Vj, Vk* of the particular FE the indices *ni, nj, nk* designate the global numbers of the specific three nodal points *i, j. k*, forming the triangle *e*, where the discussed point lies in. The free members l and l 'in the equations of the corrections (11) are calculated by the formulas:

$$l = (X) - X \qquad l' = (Y) - Y \qquad (18)$$

In the above expressions (X) and (Y) indicate the calculated values of the coordinates of the reference point, obtained by the approximate values of the unknown by formulas (12). The next step is composing and solving the normal system of equations by the method of functional iteration, as a result of which we obtain the corrections to the initial values of the unknown in the first approximation. With the corrected values of the unknowns by formulas (16) and (17) we again calculate the coefficients in the equations of the corrections, and by formulas (18) the values of the free members are again calculated. With these values a new system of equations is composed and solved. So the iterations *n* continue until the corrections to the unknown become smaller than the preliminary defined assumptions ε. The final values of the unknown are obtained as a sum of the introduced initial values and the calculated in each iteration corrections to them. The evaluation of the accuracy is carried out by the known LSM formulas using the results of the last iteration - *n*. The number of equations *T* in the system of equations of corrections and the number of the unknowns *K* are determined by the formulas:

$$T = \sum_{q=1}^{c} 2m_q \qquad K = 6.c + d + 2N_v \tag{19}$$

Where: *m* is the number of reference points in the separate images; *c* is the number of photos, which are used for solving the problem; *d* is the number of the defined elements of the interior orientation and can take the following values: d = 3 when it is necessary to determine the coordinates x_0, y_0 of the main point; d = 1 when setting them has no significance and the corrections are directly found $\delta x'_i = \delta x_0 + \delta x_i$ и $\delta y'_i = \delta y_0 + \delta y_i$;*d* = 0; *d = 0* when the calibrated value of the focal distance *f* is known. The question arises what is the minimum number of photos and reference points by which to make calibration, i. e. to compose a pre-defined system of equations. The proposed methodology allows the problem to be solved by a single photo. For the minimum number of reference points at *c = 1* after the alignment of the right sides of formulas (19) we obtain the following formula for the required minimum number of reference points:

$$m_{min} = (6 + d + 2\, N_{vz}) / 2 \tag{20}$$

To enable the adjustment by the LSM it is necessary to use a greater number of reference points, which to be located evenly on the whole photo, if possible. The high precision solution of the problem is obtained by observing certain geometric requirements regarding the parameters of the photo and the nature of the captured area [1,2].

4. CONCLUSION

The proposed methodology for modeling errors by FEM makes it possible to obtain new information about the actual photograph and it may have both theoretical and practical application. In the first case modeling can be used for theoretical study of individual errors of images [2] and of the total errors and for studying the deformations of photogrammetric images. With the found nodal values of the deviations for each FE we can determine the components of the symmetric tensor of the deformations, the major axes of pure deformation and the other parameters of deformation [2,6]. Thus we obtain a complete picture of the "deformed " state of the actual picture in comparison to the theoretical central projection. I. e. we can get a visual picture of the distribution of the studied errors in the different parts of the photograph, and on its basis to select the most appropriate way of reading them - whether through a mathematical model covering the entire picture or by their modeling element by element through FEM .

The practical application of the proposed methodology helps to improve the accuracy in solving various photogrammetric problems (e.g. phototriangulation, photogrammetric mapping and so on) and is expressed in the following: from the analysis of the errors of photographs it can be concluded that systematic errors in all photos taken with the same camera and under the same conditions are practically the same [1,2]. I.e. All points with the same image coordinates from the various images have equal deviations δ *(U, V)*. From these considerations it follows that it is enough only for one photo, let's call it standard, to make calibration by the proposed methodology. On this picture it is necessary to capture a specially prepared test polygon with a certain number of reference points. In the measured image coordinates of all points from the other photographs of the site corrections are introduced, calculated by formulas (7), depending on which FE from the standard image the specific point falls in.

The advantages of the proposed methodology for calibration by FEM in comparison to the methods used so far are in several aspects:

We obtain modeling of integral errors of the image coordinates for each FE separately, and not totally for the whole picture. Furthermore we account for the asymmetry in the distribution of errors over the image regarding its center and localization of the impact of errors in the input data is achieved (the spatial and image coordinates of the reference points), and these are the main drawbacks of the method of polynomials [1, 2, 4]. Calibration by the zone method [1] requires too many areas and therefore reference points, located in strictly defined areas on the image, which in practice is difficult to achieve. In addition, the corrections δx and δy in the individual areas are not linked to any conditions and restrictions and change at a leap at the borders of the zones, and the problem cannot be solved with a single photo. With the calibration through FEM those shortcomings are eliminated. With the proposed methodology the corrections δx ,δy are obtained directly for the peaks of the calibration network, not for the reference points whose images on the photo does not generally coincide with the nodal points. I.e. there is no need for additional transition, which is associated with loss of accuracy.

Through the proposed methodology for mathematical modeling and calibration through FEM of the integral deformational impact of the various interfering factors the aim is to achieve maximum adaptation of the abstract mathematical model of collinearity to the actual physical nature of the image.

LITERATURE:

1. Dubinovskiy, C. B. Калибровка снимков,, M., "Nedra", 1982.
2. Zdravcheva, N. D. Calibration of photos through FEM, dissertation to award the scientific and educational degree "PhD", Sofia, 2013.
3. Zenkevich, O., I. Chang, Метод конечных елементов в теории сооружений и в механикесплошных сред, M., Nedra, 1974.
4. Lobanov, A. N., Фотограмметрия,, M., Nedra, 1984.
5. Lobanov, A. N., C. B. Dubinovskiy, FF Lisenko, Аналитические модели месности и снимков (макетные снимки), M., Nedra, 1984.
6. Streng, G., D. Fix, Theory of the method of the finite elements,M., Mir, 1977.

BIOGRAPHICAL NOTES

Eng. Dr.. Nelly Zdravcheva graduated with honors as a leader with the subject "Geodesy, Photogrammetry and Cartography" at the Higher Institute of Architecture and Construction (now UACEG). She also graduated with honors "Methodology of teaching technical subjects" in the HMI. There she specialized in Applied Mathematics. She is an experienced lecturer and assistant professor in the "Photogrammetry and Cartography" department. She is a PhD of Photogrammetry and Remote Sensing (PRS) and is an author of more than 25 scientific papers and reports, read mainly at international conferences. She is a member of the STU and is a head of the subsection in the "Photogrammetry" section. She reads lectures, seminars and student practices in PRS, architectural photogrammetry, and so on with with the students from the Faculty of Geodesy of UACEG. Her scientific interests are in the field of PRS, architectural photogrammetry, preservation of cultural and historical heritage, pedagogy and education, philosophy, ecology, literature, poetry and design.

CONTACTS

Assist. Dr. Eng. Neli Dimitrova ZDRAVCHEVA, Bulgaria
"University of Architecture, Civil Engineering and Geodesy"
"Hristo Smirnenski" 1 blvd.
Sofia
Bulgaria
Email: neli_z @ abv.bg