Robustness Analysis of the GPS network of Oran city, Algeria (7542)

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1. Problematic and objectives.
4. Program realized: Robana_3DNet.
6. Conclusion and perspectives.
Problematic & objectives

- Conventionally, 3D geodetic networks are established by the union of horizontal and vertical networks.
- From decade, GPS networks have become more and more important (reference networks, topographic surveys, surveillance, auscultation, ..)
- Usual statistical analysis: evaluation of random errors! / Systematic errors or biases <values and effects / network>?
  - Assessment of the reliability of networks approach (Theory of Baarda).
  - Approach to quantifying the deformation potential of networks (Concept of robustness).

Objective: Complete analysis of 3D geodetic networks (GPS) in terms of quality, reliability and robustness. / realization of a program (ROBANA_3DNET).

Analysis of GPS networks

1. Adjustment of GPS network:

   Obs. relations of GPS Baseline
   \[
   \begin{align*}
   v_x &= \Delta X^0 - \Delta X^m + d X_t - d X_s \\
   v_y &= \Delta Y^0 - \Delta Y^m + d Y_t - d Y_s \\
   v_z &= \Delta Z^0 - \Delta Z^m + d Z_t - d Z_s
   \end{align*}
   \]

   Least Squares Solution: \( \Sigma_{i=0}^{m} p_i v_i^2 \) Minimum.
   - Parameters: \( \bar{X} = N^{-1}K = -(D^T.P.D)^{-1}D^T.P.B \)
   - Residuals: \( \bar{v} = D.\bar{X} + B \)
   - A posteriori variance factor:
     \[ \sigma_0^2 = \frac{\bar{v}^T P \bar{v}}{(m-n)} = \frac{\sum_{i=0}^{m} p_i v_i^2}{m-n} \]

   Matrix writing:
   \[
   \begin{align*}
   V &= D.X + B \\
   D: (0 0 1 0 -1 0 0)
   \end{align*}
   \]

   Weight of Obs.
   \[
   P = \begin{pmatrix}
   \sigma_0^2 & 0 & \cdots & 0 \\
   0 & \sigma_0^2 & \cdots & 0 \\
   \vdots & \vdots & \ddots & \vdots \\
   0 & 0 & \cdots & \sigma_0^2/\sigma_m^2
   \end{pmatrix}
   \]
   \( \sigma_0^2 \): A priori variance factor
2. Statistical Analysis:

(a) Analysis of quality of GPS obs.:
- Factor of a posteriori variance: Khi-2 Test
- Gross errors of observations: Student Test

(b) Analysis of quality of estimated parameters:
- Precisions of network parameters
- Absolute Error Ellipsoids
  (2D: error ellipse; 1D: error interval)

3. Robustness Analysis of GPS networks

1. Reliability of GPS networks:
   Network ability to detect and estimate the effects that undetected blunders may have on a solution (adjusted parameters). « Baarda’s theory »

   Reliable network: minimise non detectable errors in obs. ⇒ minimise effects of these errors on adjusted parameters.

   - Redundancy numbers: $R = I - A(A^T A)^{-1} A^T$
   - Inner Reliability: $V = \sigma y_1 \frac{\delta}{\sqrt{\delta}}$
   - External Reliability: $V = (A^T P A)^{-1} A^T P V A$

   $\alpha$: Type I Error (prob. of rejecting a good obs)
   $\beta$: Type II Error (prob. of accepting a blunder)
   $\delta$: Non-centrality parameter
2. Robustness of GPS networks:
Combinaison of reliability and deformation of network [Vanicek et al., 1991].

- Deformation?
  - Displacement field:
  \[
  \Delta X_i = \begin{bmatrix}
  \Delta x_i \\ \Delta y_i \\ \Delta z_i
  \end{bmatrix}
  \quad \text{where}
  \begin{align*}
  a_i &= \frac{\partial u_i}{\partial x}(x_i - x) + \frac{\partial v_i}{\partial y}(y_i - y) + \frac{\partial w_i}{\partial z}(z_i - z) + \eta_i \\
  b_i &= \frac{\partial u_i}{\partial y}(x_i - x) + \frac{\partial v_i}{\partial z}(y_i - y) + \frac{\partial w_i}{\partial x}(z_i - z) + \eta_i \\
  c_i &= \frac{\partial u_i}{\partial z}(x_i - x) + \frac{\partial v_i}{\partial x}(y_i - y) + \frac{\partial w_i}{\partial y}(z_i - z) + \eta_i
  \end{align*}
  \]
  - Strain tensor:
  \[
  E(x, y, z) = \begin{bmatrix}
  \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\
  \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\
  \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}
  \end{bmatrix}
  \]

- Computation of optimal displacements:
  \[
  d_i = \sqrt{u_i^2 + v_i^2 + w_i^2}
  \]

- Displacement thresholds: [GSD/Canada, 1996]
  - Measure of robustness:
    \[
    d_i \leq \delta_i : \text{Robust Point} \quad d_i > \delta_i : \text{Weak Point}
    \]

For probability of 95%:
3. Strain tensor 3D:

Strain tensor:

\[ E(x, y, z) = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \]

\[ E = S + A \]

Symmetric part

\[ S = \frac{1}{2} \begin{bmatrix} \epsilon_{xx} & \frac{1}{2}(\epsilon_{xy} + \epsilon_{yx}) & \frac{1}{2}(\epsilon_{xz} + \epsilon_{zx}) \\ \frac{1}{2}(\epsilon_{xy} + \epsilon_{yx}) & \epsilon_{yy} & \frac{1}{2}(\epsilon_{yz} + \epsilon_{zy}) \\ \frac{1}{2}(\epsilon_{xz} + \epsilon_{zx}) & \frac{1}{2}(\epsilon_{yz} + \epsilon_{zy}) & \epsilon_{zz} \end{bmatrix} \]

Anti-symmetric part

\[ A = \begin{bmatrix} 0 & \frac{1}{2}(\epsilon_{xy} - \epsilon_{yx}) & \frac{1}{2}(\epsilon_{xz} - \epsilon_{zx}) \\ \frac{1}{2}(\epsilon_{yx} - \epsilon_{xy}) & 0 & \frac{1}{2}(\epsilon_{yz} - \epsilon_{zy}) \\ \frac{1}{2}(\epsilon_{zx} - \epsilon_{xz}) & \frac{1}{2}(\epsilon_{zy} - \epsilon_{yz}) & 0 \end{bmatrix} \]

3. Strain tensor 3D:

Primitives of Deformation:

**Dilatation**

- Average:
  \[ \sigma = \frac{1}{3}(\lambda_1 + \lambda_2 + \lambda_3) = \frac{1}{3}(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) \]

- Total:
  \[ \lambda = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \]

**Twist**

- Component:
  \[ \omega_{xy} = \frac{1}{2}(\epsilon_{xy} - \epsilon_{yx}) \]
  \[ \omega_{xz} = \frac{1}{2}(\epsilon_{xz} - \epsilon_{zx}) \]
  \[ \omega_{yz} = \frac{1}{2}(\epsilon_{yz} - \epsilon_{zy}) \]

- Total:
  \[ \Omega = \sqrt{\omega_{xy}^2 + \omega_{xz}^2 + \omega_{yz}^2} \]

**Shear**

**Simple**

- \[ \epsilon_{xy} = -\epsilon_{yx} = \frac{1}{2}(\epsilon_{xy} - \epsilon_{yx}) = \frac{1}{2}(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) \]

- \[ \epsilon_{xz} = -\epsilon_{zx} = \frac{1}{2}(\epsilon_{xz} - \epsilon_{zx}) = \frac{1}{2}(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) \]

- \[ \epsilon_{yz} = -\epsilon_{zy} = \frac{1}{2}(\epsilon_{yz} - \epsilon_{zy}) = \frac{1}{2}(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}) \]

**Pure**

- \[ \nu_{xy} = -\nu_{yx} = \frac{1}{2}(\epsilon_{xy} + \epsilon_{yx}) = \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \]

- \[ \nu_{xz} = -\nu_{zx} = \frac{1}{2}(\epsilon_{xz} + \epsilon_{zx}) = \frac{1}{2}(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) \]

- \[ \nu_{yz} = -\nu_{zy} = \frac{1}{2}(\epsilon_{yz} + \epsilon_{zy}) = \frac{1}{2}(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}) \]

- Total:
  \[ \gamma = \sqrt{\nu_{xy}^2 + \nu_{xz}^2 + \nu_{yz}^2} \]

.. For an easier interpretation of strain tensor
3. Strain tensor 3D:

Invariants of deformation primitives [Vanicek et al., 2001] and [Berber, 2006]:

\[ \sigma = \frac{1}{3}(\lambda_1 + \lambda_2 + \lambda_3) = \frac{1}{3}(\varepsilon_{\mu\nu} + \varepsilon_{\nu\mu} + \varepsilon_{\mu\nu}) \]

- Mean Dilatation
- Max Shear
- Total Differential Rotation

... In this study, 03 components are chosen among different strain tensor primitives, which are deformation invariants in the 3D case.

Here the robustness is expressed in deformation.

4. Algorithm:

- This procedure is implemented at the developed program Robana_3DNet
ROBANA_3DNet (ROBustness ANAlysis of 3D geodetic NETworks) : MATLAB program, Adjustment & Reliability and Robustness Analyses of GPS nets. [DGS /CTS, Arzew].

Modules : Adjustment / Statistical Analysis / Reliability Analysis/ Robustness Analysis
Application: GPS network of Oran City

WinPrism / PROCESS
- 28 sessions (45 points)
- $\sigma$ GPS baselines = $\pm 4.3$ mm ($\pm 3.2$ mm, $\pm 1.5$ mm, $\pm 2.3$ mm : $\Delta X$, $\Delta Y$ and $\Delta Z$, resp.)
### Application: GPS network of Oran City

#### 1. Adjustment:

<table>
<thead>
<tr>
<th>Number of Parameters</th>
<th>Number of Observations</th>
<th>Number of Freedom Degree</th>
<th>Number of Fixed Points</th>
<th>$\sigma_0$ [m]</th>
<th>Probability Thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>132</td>
<td>147 GPS Baselines</td>
<td>349</td>
<td>01 (point 37)</td>
<td>1.</td>
<td>$\alpha = 5%$</td>
</tr>
<tr>
<td></td>
<td>(67 components)</td>
<td></td>
<td></td>
<td></td>
<td>$\beta = 20%$</td>
</tr>
</tbody>
</table>

#### Precisions:

- $\sigma_0$: mean standard deviation.
- Khi-2 Test: positive.
- Student Test: 7 baselines suspected.

<table>
<thead>
<tr>
<th>$\sigma_a$ (m)</th>
<th>$\sigma_b$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.008</td>
<td>0.004</td>
</tr>
<tr>
<td>0.002</td>
<td>0.007</td>
</tr>
<tr>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>0.013</td>
<td>0.007</td>
</tr>
</tbody>
</table>

#### Error Domains:

- Average: 0.001 m
- RMS: 0.007 m
- Minimum: 0.003 m (pts: 33, 40)
- Maximum: 0.007 m (pts: 35)
2. Analyse de fiabilité:

- Redundancy numbers
  
  Measure of absorption of a blunder: indication of reliability of obs.

- Inner Reliability (FI)

<table>
<thead>
<tr>
<th>Observation</th>
<th>$F(x)$</th>
<th>$F(y)$</th>
<th>$F(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.006</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>RMS</td>
<td>0.003</td>
<td>0.001</td>
<td>0.003</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.016</td>
<td>0.010</td>
<td>0.012</td>
</tr>
</tbody>
</table>

- External Reliability (FE)

<table>
<thead>
<tr>
<th>Observation</th>
<th>$Fe(F)$</th>
<th>$Fe(x)$</th>
<th>$Fe(y)$</th>
<th>$Fe(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>-0.004</td>
<td>-0.001</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>RMS</td>
<td>0.004</td>
<td>0.002</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.015</td>
<td>-0.006</td>
<td>-0.008</td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>0.006</td>
<td>0.004</td>
<td>0.007</td>
<td></td>
</tr>
</tbody>
</table>

- External reliability of network is of about few mm → reliable network.
- However, max values ≤ 15 mm have important dimension of error ellipses.
2. Analysis of robustness:

Network is robust?

In terms of displacements:

<table>
<thead>
<tr>
<th></th>
<th>Disp. (δ) (m)</th>
<th>Threshold (δ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.003</td>
<td>0.024</td>
</tr>
<tr>
<td>RMS</td>
<td>0.007</td>
<td>0.060</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.030</td>
<td>0.040</td>
</tr>
</tbody>
</table>

Robustness in orientation (twist)

Robustness in configuration (shear)

Robustness in scale (dilatation)

ppm: part per million
Reliability Analysis of aspects and robustness of 3D geodetic networks (GPS) are treated. 

Valorization of the work: realization of **Robana_3DNet** program (in MATLAB), at (DGS / CTS) → [Adjustment, Statistical Analysis, reliability and robustness analyses: GPS networks].

Validation of the program: test network (45 points) of the GPS network of the city of Oran (2009).

Results: \( \sigma \) GPS baselines ± 4 mm (PROCESS / WinPrism); \( \sigma \) GPS points ± 9 mm in position and height. Error Domains (ellipses and ranges of errors) are more important in points of network edge.

Network reliability: redundancy (15%, 98%); bias (internal reliability) ~ 8 mm; external network reliability 4 mm in horizontally and 1 mm in altimetry → reliable network.

Network Robustness: Robust network throughout except few pts on perimeter where displacements and deformations are significant due to low number of connections and to configuration.

Design of GPS networks: criteria of reliability and robustness.

Outlook:

To enrich ROBANA_3DNET program, it is necessary to:

- Validate the program on other GPS networks, including large networks.
- Develop an automatic diagnosis of the network analysis results.
- Integrate a module of significance of primitive strain tensor, based on Monte Carlo method.
Thank you for your attention