A complete processing methodology for 3D monitoring using GNSS receivers

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Two Measurement campaigns (I, II)
Baselines’ Measurement

Relative static positioning method

Only non-trivial baselines are used for the adjustment in order to ensure different conditions and to avoid bias at the calculations
Methodology

The differences of the coordinates $\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij}$ between successive campaigns are calculated.

The full VCV matrix $V_{ij}$ is calculated according to the law of propagation errors by using the full VCV matrix $V_{\delta X, Y, Z}$ of the displacements.

The error ellipses or ellipsoids of the control points are calculated for the detection of the point’s displacement.

For each campaign

- The differences of the coordinates $\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij}$ are calculated.
- The full VCV matrix $V_{ij}$ is calculated.
- The error ellipses or ellipsoids of the control points are calculated.

Methodology

1. Solution of each baseline
2. Determination of the components $\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij}$ in the Cartesian geocentric reference system between the occupation points $i$ and $j$
3. Reliable estimation of the weights of $\Delta X, \Delta Y$ and $\Delta Z$
4. Linear equations are formed and the unknown geocentric coordinates $X, Y, Z$ are calculated by a least squares’ adjustment
5. The full VCV matrix is provided for each campaign
Weights estimation

An objective estimation of the errors $e_x$, $e_y$, $e_z$ of each unknown component of the network is calculated by the root of the variance for each one in VCV matrices.

• indirect observations, equal weight adjustment

$$\Delta x_{ij} = x_j - x_i \quad \Delta y_{ij} = y_j - y_i \quad \Delta z_{ij} = z_j - z_i$$

• The number of equations of each system is equal to the measured baselines.

The condition equations are formed by network’s loops closure by using the measured $\Delta x_{ij}$, $\Delta y_{ij}$, $\Delta z_{ij}$ as follows

$$\Delta x_{ij} + \Delta x_{ik} + \Delta x_{kl} = 0 \quad \Delta y_{ij} + \Delta y_{ik} + \Delta y_{kl} = 0 \quad \Delta z_{ij} + \Delta z_{ik} + \Delta z_{kl} = 0$$

• The number of equations in every system is equal to the number of the unary loops

The objective errors of the unknown components $e_x$, $e_y$, $e_z$ for each point $i$ of the network is presented by the root of the variance for each one in VCV matrices.

The misclosure of the unary loops of the network ($mc$) is the error which the loop contains for three participating baselines. So a decent estimation of this error for each component $\Delta x, \Delta y, \Delta z$ is given by the following equation

$$e_{\Delta x} = \frac{mc_{\text{loop}}}{\sqrt{3}} \quad e_{\Delta y} = \frac{mc_{\text{loop}}}{\sqrt{3}} \quad e_{\Delta z} = \frac{mc_{\text{loop}}}{\sqrt{3}}$$

• Then the mean errors of the baselines’ components determination are calculated as follows.

$L$ is the number of the loops

$$e_{\Delta x} = \frac{\sum |\Delta x_{ij}|}{L} \quad e_{\Delta y} = \frac{\sum |\Delta y_{ij}|}{L} \quad e_{\Delta z} = \frac{\sum |\Delta z_{ij}|}{L}$$

• Considering that for each baseline $i, j$ the following equations are valid

$$e_{\Delta x} = e_{\Delta y} = e_{\Delta z} = \pm \sqrt{e_x^2 + e_y^2 + e_z^2}$$

• Assuming that $e_x = e_x$, $e_y = e_y$, $e_z = e_z$

then

$$e_x = \pm \frac{e_{\Delta x}}{\sqrt{3}} \quad e_y = \pm \frac{e_{\Delta y}}{\sqrt{3}} \quad e_z = \pm \frac{e_{\Delta z}}{\sqrt{3}}$$
Absolute displacements calculation

According to the indirect observations method the following system of regular equations is formed:

\[
\begin{align*}
\Delta X_i &= X_i - X_j, \\
\Delta Y_i &= Y_i - Y_j, \\
\Delta Z_i &= Z_i - Z_j,
\end{align*}
\]

\[
\hat{x} = (A^T \cdot P \cdot A)^{-1} \cdot A^T \cdot P \cdot \hat{a} = N^{-1} \cdot \hat{A}^0 \cdot P \cdot \hat{a}
\]

The absolute position changes of network’s points \( n \) between two sequential measurement campaigns (I and II) are calculated:

\[
\begin{bmatrix}
\Delta X_i^I \\
\Delta Y_i^I \\
\Delta Z_i^I \\
\delta_{Ei}^I \\
\delta_{Ni}^I \\
\delta_{Up_i}^I
\end{bmatrix}
\]

The variances and covariances of these changes:

\[
V_{\Delta X,Y,Z} = V_{X,Y,Z}^I + V_{X,Y,Z}^{II}
\]

The changes \( \Delta X_i^I, \Delta Y_i^I, \Delta Z_i^I \) of each point \( i \) must be converted in an oriented local plane projection, \( \delta_{Ei}, \delta_{Ni}, \delta_{Up_i} \) in order to be more perceptible and to define their directions and their trends in relation to the earth’s surface.

Absolute displacements calculation

- the total rotation matrix \( S_{ALL} \) for the (n-1) unknown points of the network

\[
S_{ALL} = \begin{bmatrix}
-\sin \theta_1 & \cos \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
-\sin \theta_2 & \cos \theta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & -\sin \theta_n & \cos \theta_n & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 0 & \cos \theta_n & \sin \theta_n & 0 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots
\end{bmatrix}
\]

- the position changes of each point \( i \) (\( \Delta X_i^I, \Delta Y_i^I, \Delta Z_i^I \)) in a local projection plan

- The CV matrix for the components \( \delta_{Ei}, \delta_{Ni}, \delta_{Up_i} \) are calculated according to the law of propagation errors by using the appropriate \( J \) matrix as

\[
V_{\delta E,N,Up} = J \cdot V_{\delta X,Y,Z} \cdot J^T
\]

- \( J \) matrix is formed by the partial derivation of the previous equation

\[
J = S_{ALL}
\]
Absolute displacements calculation

- Continuously the change vector $\bar{a}_{i}^{\text{II}}$ and its bearing $b_{i}^{\text{II}}$, in regard to north,
  \[ \bar{a}_{i}^{\text{II}} = \sqrt{(\delta E_{i}^{\text{II}})^{2} + (\delta N_{i}^{\text{II}})^{2}} \]
  \[ b_{i}^{\text{II}} = \arctan \frac{\delta E_{i}^{\text{II}}}{\delta N_{i}^{\text{II}}} \]

- The horizontal displacements could be checked by applying

  **at a glance** general one-dimensional check
  - if $\bar{a}_{i}^{\text{II}} < \delta_{\text{act}} \cdot z$ and $\bar{a}_{i}^{\text{II}} < \delta_{\text{act}} \cdot z$ then there is no horizontal displacement
  - if $\bar{a}_{i}^{\text{II}} > \delta_{\text{act}} \cdot \lambda$ and $\bar{a}_{i}^{\text{II}} > \delta_{\text{act}} \cdot \lambda$ then there is horizontal displacement

- the full check procedure the absolute error ellipse is drawn for each point $i$ for a specific confidence level and the displacement vector of each point is over designed

- the vertical displacements detection
  - if $\delta U_{i}^{\text{II}} < \sigma_{\text{act}} \cdot z$ then is no vertical displacement of the point $i$, otherwise point $i$ has a vertical displacement for the selected confidence level.

A total approach of the absolute displacement’s check could be done by the calculation of the error ellipsoid’s axes for each point.

Relative displacements calculation

- In order to calculate the relative displacements between two points $i$ and $j$ of the network the change’s vectors $\Delta \bar{a}_{E_{ij}}^{\text{II}}, \Delta \bar{a}_{N_{ij}}^{\text{II}}, \Delta \bar{a}_{U_{ij}}^{\text{II}}$ between two sequential measurement campaigns I and II are calculated by using the equations

  \[ \Delta \bar{a}_{E_{ij}}^{\text{II}} = (E_{j} - E_{i})^{\text{II}} - (E_{j} - E_{i})^{\text{I}} = \Delta \bar{a}_{E_{ij}}^{\text{II}} - \bar{a}_{E_{ij}}^{\text{I}} \]

  \[ \Delta \bar{a}_{N_{ij}}^{\text{II}} = (N_{j} - N_{i})^{\text{II}} - (N_{j} - N_{i})^{\text{I}} = \Delta \bar{a}_{N_{ij}}^{\text{II}} - \bar{a}_{N_{ij}}^{\text{I}} \]

  \[ \Delta \bar{a}_{U_{ij}}^{\text{II}} = (U_{j} - U_{i})^{\text{II}} - (U_{j} - U_{i})^{\text{I}} = \Delta \bar{a}_{U_{ij}}^{\text{II}} - \bar{a}_{U_{ij}}^{\text{I}} \]

- Same checks
Discussion

- The influence of the full CV matrix use is very important for the displacements determination as it makes difference not only to the magnitude of the error ellipses’ axes but also to the orientation of its main axis.
- The miscalculation of the ellipse leads to wrong conclusions about the displacements of the control point, as the displacement vector may lie accidentally outside or inside the ellipse.

<table>
<thead>
<tr>
<th>Variances</th>
<th>Full VCV matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{e}$</td>
<td>$\pm$10mm</td>
</tr>
<tr>
<td>$\sigma_{n}$</td>
<td>$\pm$13mm</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0°</td>
</tr>
</tbody>
</table>

Conclusions

- The lack of the full CV matrix as output
- The overestimated standard errors of the baselines solution as well as
- The ”black box” followed procedure, are the main disadvantages of the majority commercial GNSS softwares when used in the 3D monitoring.

In the advantages of the proposed processing methodology are registered

- The linear equations, which are formed, release the procedure from approximations. The entire procedure can be carried out in an easy Excel or Matlab environment as simple equations systems are solved thus no special software development is required.
Conclusions

- The **weight definition proposed technique** avoids the unrealistically optimistic standard errors calculation due to the GNSS ability to collect plethora of data.

- Thereby, it ensures the reliability of the adjustment as it **illustrates the objective achieved standard errors** in the original captured data.

- The use of **specific rotation matrix for each point** in order to calculate either the absolute and relative displacements according to the law of propagation’s error ensure the correctness of the results.

- The **full CV matrix formation** allows the accurate error ellipse or error ellipsoid calculation, the **right evaluation** of the displacements.

Conclusions

- The comparison of the size and the rotation of the error ellipses which are formed by using the full CV matrix or the deficient one prove that there is a strong possibility to extract different conclusions for a point’s displacement as **mainly the ellipse’s orientation is completely different**.

- The proposed processing methodology allows
  - the total surveillance of the adjustment’s steps,
  - the objective weights definition and
  - the full CV matrix formation.

**It is evaluated as efficient and reliable** for such a trustable and serious activity as the 3D monitoring by using GNSS receivers.
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thank you for your attention