NATIONAL TECHNICAL UNIVERSITY OF ATHENS
SCHOOL OF RURAL AND SURVEYING ENGINEERING
DEPARTMENT OF TOPOGRAPHY
LABORATORY OF GENERAL GEODESY

A complete processing methodology for 3D monitoring using GNSS receivers

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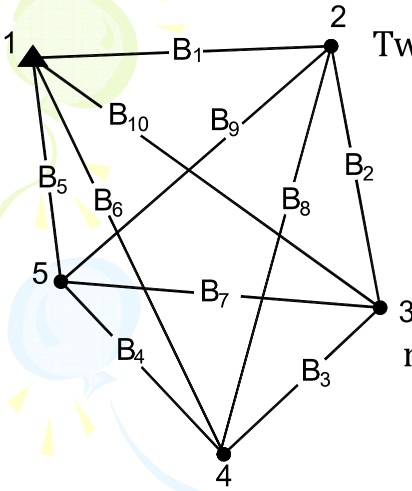
FIG Working Week
17 - 21 May, Bulgaria
From the wisdom of the ages to the challenges of modern world
FIG SOFIA 2015
Sofia, Bulgaria 17- 21 May 2015

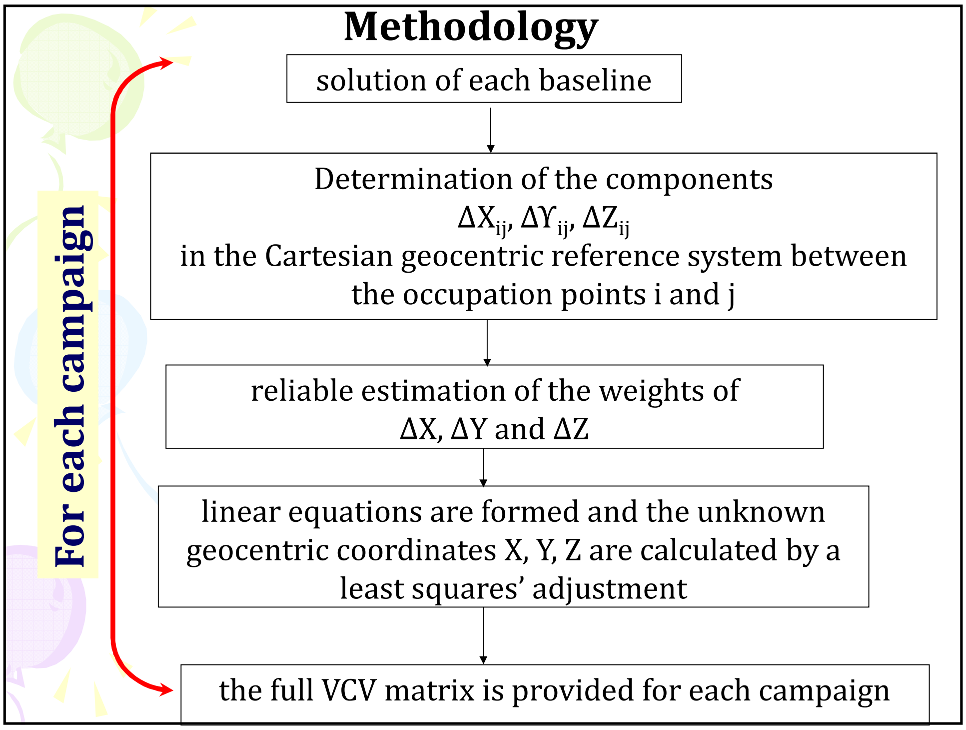


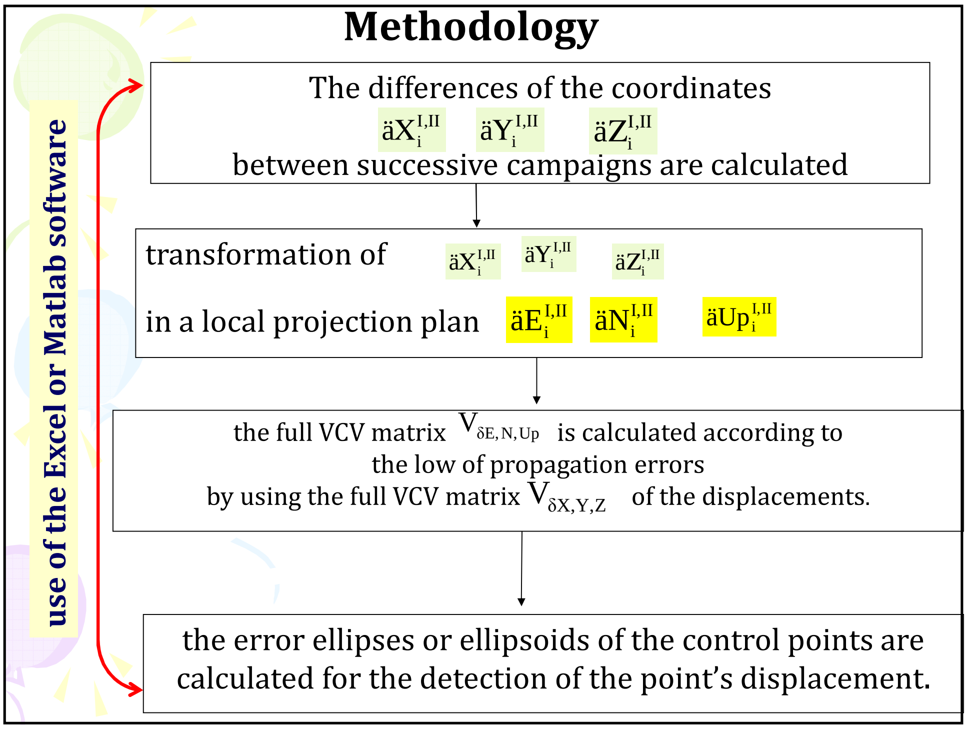
Two Measurement campaigns (I, II)

Baselines' Measurement

relative static positioning method

Only non-trivial baselines are used for the adjustment in order to ensure different conditions and to avoid bias at the calculations


Methodology
For each campaign
solution of each baseline
Determination of the components
ΔXij, ΔYij, ΔZij
in the Cartesian geocentric reference system between the occupation points i and j
reliable estimation of the weights of
ΔX, ΔY and ΔZ
linear equations are formed and the unknown geocentric coordinates X, Y, Z are calculated by a least squares' adjustment
the full VCV matrix is provided for each campaign


Methodology
use of the Excel or Matlab software
The differences of the coordinates
äXi I,II äYi I,II äZi I,II
between successive campaigns are calculated
transformation of äXi I,II äYi I,II äZi I,II
in a local projection plan äEi I,II äNi I,II äUpi I,II
the full VCV matrix VδE,N,Up is calculated according to
the low of propagation errors
by using the full VCV matrix VδX,Y,Z of the displacements.
the error ellipses or ellipsoids of the control points are calculated for the detection of the point's displacement.

Weights estimation

An objective estimation of the errors e_{X_i} e_{Y_i} e_{Z_i}
a preliminary least square equal weight adjustment could be applied according to either the method of indirect observations or the method of condition equations

1st method

- indirect observations ,equal weight adjustment

$\Delta X_{ij} = X_j - X_i$ $\Delta Y_{ij} = Y_j - Y_i$ $\Delta Z_{ij} = Z_j - Z_i$

- The number of equations of each system is equal to the measured baselines.

2nd method

- The condition equations are formed by network's loops closure by using the measured ΔX_{ij}, ΔY_{ij}, ΔZ_{ij} as follows

$\Delta X_{ij} + \Delta X_{jk} + \Delta X_{ki} = 0$ $\Delta Y_{ij} + \Delta Y_{jk} + \Delta Y_{ki} = 0$ $\Delta Z_{ij} + \Delta Z_{jk} + \Delta Z_{ki} = 0$

- The number of equations in every system is equal to the number of the unary loops

The objective errors of the unknown components e_{X_i} e_{Y_i} e_{Z_i} for each point i of the network is presented by the root of the variance for each one in VCV matrices.

Weights estimation

3rd method empirical

- The misclosure of the unary loops of the network (mc) is the error, which the loop contains for three participating baselines. So a decent estimation of this error for each component ΔX,ΔY,ΔZ is given by the following equation

$$e_{\Delta X_{ij}} = \pm \frac{mc_{loop_X}}{\sqrt{3}} \quad e_{\Delta Y_{ij}} = \pm \frac{mc_{loop_Y}}{\sqrt{3}} \quad e_{\Delta Z_{ij}} = \pm \frac{mc_{loop_Z}}{\sqrt{3}}$$

- Then the mean errors of the baselines' components determination are calculated as follows.
- L is the number of the loops

$$e_{\Delta X_m} = \pm \frac{\sum_{t=1}^{L} \left| e_{\Delta X_{ij}} \right|}{L} \quad e_{\Delta Y_m} = \pm \frac{\sum_{t=1}^{L} \left| e_{\Delta Y_{ij}} \right|}{L} \quad e_{\Delta Z_m} = \pm \frac{\sum_{t=1}^{L} \left| e_{\Delta Z_{ij}} \right|}{L}$$

- Considering that for each baseline i, j the following equations are valid

$$e_{\Delta X_{ij}} = e_{\Delta X_m} = \pm\sqrt{e_{X_i}^2 + e_{X_j}^2} \quad e_{\Delta Y_{ij}} = e_{\Delta Y_m} = \pm\sqrt{e_{Y_i}^2 + e_{Y_j}^2} \quad e_{\Delta Z_{ij}} = e_{\Delta Z_m} = \pm\sqrt{e_{Z_i}^2 + e_{Z_j}^2}$$

- Assuming that $e_{X_i} = e_{X_j}$ $e_{Y_i} = e_{Y_j}$ $e_{Z_i} = e_{Z_j}$

then

$$e_{X_i} = \pm \frac{e_{\Delta X_{ij}}}{\sqrt{2}} \quad e_{Y_i} = \pm \frac{e_{\Delta Y_{ij}}}{\sqrt{2}} \quad e_{Z_i} = \pm \frac{e_{\Delta Z_{ij}}}{\sqrt{2}}$$

Absolute displacements calculation

According to the indirect observations method the following system of regular equations is formed $\Delta X_{ij} = X_j - X_i$ $\Delta Y_{ij} = Y_j - Y_i$ $\Delta Z_{ij} = Z_j - Z_i$

$$\hat{x} = (A^T \cdot P \cdot A)^{-1} \cdot Á^{Ô} \cdot P \cdot äl = N^{-1} \cdot Á^{Ô} \cdot P \cdot äl$$

- The absolute position changes of network's points n between two sequential measurement campaigns (I and II) are calculated

$$\delta^{II\text{-}I} = \begin{bmatrix} äX_i^{I,II} \\ äY_i^{I,II} \\ äZ_i^{I,II} \\ äX_{i+1}^{I,II} \\ äY_{i+1}^{I,II} \\ äZ_{i+1}^{I,II} \\ . \\ . \\ . \\ äX_{n-1}^{I,II} \\ äY_{n-1}^{I,II} \\ äZ_{n-1}^{I,II} \end{bmatrix}$$

- The variances and covariances of these changes $V_{\delta X,Y,Z} = V^{I}_{X,Y,Z} + V^{II}_{X,Y,Z}$
- The changes $äX_i^{I,II}$, $äY_i^{I,II}$, $äZ_i^{I,II}$ of each point i must be converted in **an oriented local plane projection** , $\delta East_i$, $\delta North_i$ and δUp_i in order to be more perceptible and to define their directions and their trends in relation to the earth's surface

Absolute displacements calculation

- the total rotation matrix S_{ALL} for the (n-1) unknown points of the network

$$S_{ALL} = \left[\begin{array}{cccccccccccc}
-\sin ë_i & \cos ë_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\sin ö_i \cdot \cos ë_i & -\sin ö_i \cdot \sin ë_i & \cos ö_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\cos ö_i \cdot \cos ë_i & \cos ö_i \cdot \sin ë_i & \sin ö_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sin ë_{i+1} & \cos ë_{i+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sin ö_{i+1} \cdot \cos ë_{i+1} & -\sin ö_{i+1} \cdot \sin ë_{i+1} & \cos ö_{i+1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos ö_{i+1} \cdot \cos ë_{i+1} & \cos ö_{i+1} \cdot \sin ë_{i+1} & \sin ö_{i+1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin ë_{n-1} & \cos ë_{n-1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin ö_{n-1} \cdot \cos ë_{n-1} & -\sin ö_{n-1} \cdot \sin ë_{n-1} & \cos ö_{n-1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos ö_{n-1} \cdot \cos ë_{n-1} & \cos ö_{n-1} \cdot \sin ë_{n-1} & \sin ö_{n-1}
\end{array}\right]$$

- the position changes of each point i ($äE_i^{I,II}$, $äN_i^{I,II}$, $äUp_i^{I,II}$) in a local projection plan
- The CV matrix for the components $äE_i^{I,II}$, $äN_i^{I,II}$, $äUp_i^{I,II}$ are calculated according to the law of propagation errors by using the appropriate J matrix as

$$V_{\delta E,N,Up} = J \cdot V_{\delta X,Y,Z} \cdot J^T$$

- J matrix is formed by the partial derivation of the previous equation
- $J = S_{ALL}$

Absolute displacements calculation

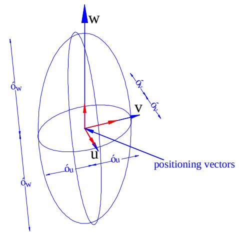
- Continuously the change vector $\text{är}_i^{I,II}$ and its bearing $b_i^{I,II}$, in regard to north,

$$\text{är}_i^{I,II} = \sqrt{(\delta E_i^{I,II})^2 + (\delta N_i^{I,II})^2} \qquad b_i^{I,II} = \arctan\frac{\delta E_i^{I,II}}{\delta N_i^{I,II}}$$

- The **horizontal displacements** could be checked by applying
- **at a glance** general one –dimension check
  if $\text{är}_i^{I,II} < \text{ó}_{\delta E_i^{I,II}} \cdot z$ and $\text{är}_i^{I,II} < \text{ó}_{\delta N_i^{I,II}} \cdot z$ then there is no horizontal displacement
  if $\text{är}_i^{I,II} > \text{ó}_{\delta E_i^{I,II}} \cdot \lambda$ and $\text{är}_i^{I,II} > \text{ó}_{\delta N_i^{I,II}} \cdot \lambda$ then there is horizontal displacement
- **the full check** procedure the absolute error ellipse is drawn for each point i for a specific confidence level and the displacement vector of each point is over designed
- the **vertical displacements detection**
  if $\delta Up_i^{I,II} < \sigma_{\delta Up_i^{I,II}} \cdot z$ then is no vertical displacement of the point i, otherwise

point i has a vertical displacement for the selected confidence level .

a total approach of the absolute displacement’s check could be done by the calculation of the error ellipsoid’s axes for each point



Relative displacements calculation

- In order to calculate the relative displacements between two points i and j of the network the change’s vectors $\Delta\text{äE}_{i,j}^{\text{É,ÉÉ}}$, $\text{ÄäÍ}_{i,j}^{\text{É,ÉÉ}}$ $\text{ÄäUp}_{i,j}^{\text{É,ÉÉ}}$ between two sequential measurement campaigns I and II are calculated by using the equations

$$\Delta\text{äE}_{i,j}^{\text{É,ÉÉ}} = (E_j - E_i)^{II} - (E_j - E_i)^{I} = \text{äE}_j^{\text{É,ÉÉ}} - \text{äE}_i^{\text{É,ÉÉ}}$$

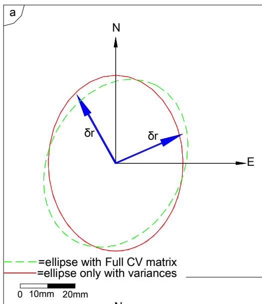
$$\text{ÄäÍ}_{i,j}^{\text{É,ÉÉ}} = (N_j - N_i)^{II} - (N_j - N_i)^{\text{É}} = \text{äN}_j^{\text{É,ÉÉ}} - \text{äN}_i^{\text{É,ÉÉ}}$$

$$\Delta\delta Up_{i,j}^{I,\,II} = (Up_j - Up_i)^{II} - (Up_j - Up_i)^{I} = \delta Up_j^{I,\,II} - \delta Up_i^{I,\,II}$$

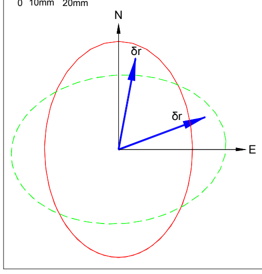
- Same checks

Discussion

- The influence of the full CV matrix use is very important for the displacements determination as it makes difference not only to the magnitude of the error ellipses' axes but also to the orientation of its main axis
- The miscalculation of the ellipse leads to wrong conclusions about the displacements of the control point, as the displacement vector may lie accidentally outside or inside the ellipse.



	$\sigma_{\delta E_i}$ =±10mm $\sigma_{\delta N_i}$ =±13mm $\sigma_{\delta E_i \delta N_i}$ =5.023 $\cdot 10^{-5}$ m^2	
	Only variances	Full VCV matrix
σ_u	±10mm	±12mm
σ_v	±13mm	±11mm
θ	0°	27° 41′



	$\sigma_{\delta E_i}$ =±16mm $\sigma_{\delta N_i}$ =±11mm $\sigma_{\delta E_i \delta N_i}$ =9.093 $\cdot 10^{-6}$ m^2	
	Only variances	Full VCV matrix
σ_u	±16mm	±16mm
σ_v	±11mm	±11mm
θ	180°	86° 10′

Conclusions

- The **lack of the full CV matrix as output**
- the **overestimated standard errors** of the baselines solution as well as
- the **"black box"** followed procedure, are the main disadvantages of the majority commercial GNSS softwares when used in the 3D monitoring.

In the advantages of the proposed processing methodology are registered

❖ the **linear equations**, which are formed, release the procedure from approximations. The entire procedure can be carried out in an **easy Excel or Matlab** environment as simple equations systems are solved thus no special software development is required.

Conclusions

❖ The **weight definition proposed technique** avoids the unrealistically optimistic standard errors calculation due to the GNSS ability to collect plethora of data.

❖ Thereby, it ensures the reliability of the adjustment as it **illustrates the objective achieved standard errors** in the original captured data.

❖ The use of **specific rotation matrix for each point** in order to calculate either the absolute and relative displacements according to the law of propagation's error ensure the correctness of the results

❖ The **full CV matrix formation** allows the accurate error ellipse or error ellipsoid calculation, the **right evaluation** of the displacements .

Conclusions

❖ The comparison of the size and the rotation of the error ellipses which are formed by using the full CV matrix or the deficient one prove that there is a strong possibility to extract different conclusions for a point's displacement as **mainly the ellipse's orientation is completely different**.

❖ The proposed processing methodology allows

✓ the total surveillance of the adjustment's steps,

✓ the objective weights definition and

✓ the full CV matrix formation.

it is evaluated as efficient and reliable for such a trustable and serious activity as the 3D monitoring by using GNSS receivers.

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