THE GRAVIMETRIC QUASIGEOID MODEL OVER UGANDA

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INTRODUCTION

• Need for geoid/quasigeoid model? --- GNSS-HEIGHT DETERMINATION

\[ N = h - N' \]
\[ h = \zeta + \phi \]

- Adverseometric height
- Geoidal height
- Normal height
- 

\[ \zeta \] - height anomaly/quasigeoid height
INTRODUCTION

Quasigeoid determination

• Directly using Stokes formula/modification

\[ \zeta_f = \frac{R}{4 \pi \sigma} \int \int S(r, \varphi) \Delta g \, d\sigma \]

• Indirectly

\[ \zeta_f = N_f + (\zeta - N) \]

\( (\zeta - N) \) = quasigeoid-geoid separation

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INTRODUCTION

Quasigeoid determination

• Based on the Uganda Gravimetric Geoid Model 2014—KTH method

\[ \bar{N}_{L, M} = \frac{R}{4 \pi \sigma} \int \int S(\sigma) \Delta g \, d\sigma + \sum_{k=0}^{M} \left( Q_{g, k}^L + s_k \right) \Delta g_{\text{GGM}}^{\text{GGM}} + \]

\( \Delta N_{\text{comb}} \) + \( \Delta N_{\text{mod}} \) + \( \Delta N_{\text{ref}} \) + \( \Delta N_{\text{m}} \)

\( \sigma \) = spherical cap

\( R \) = mean Earth radius

\( \gamma \) = mean normal gravity on reference ellipsoid

\( S(\sigma) \) = modified Stokes' function

\( M \) = maximum degree of GGM

\( L \) = maximum degree of modification

\( Q_{g, k} \) = Molodensky truncation coefficients
Data used for the determination of the geoid model

- 7,839 terrestrial gravity data from BGI
- World Gravity Map 2012 surface gravity anomalies – BGI
- SRTM3 DEM—CGIAR-CSI
- GOCE-only GGM—GO_CONS_GCF_2_TIM_R5 (maximum degree=280)
- 10 GNSS/levelling points

Internal & external accuracy assessment of UGG2014

- Internal accuracy= 11.5 cm
- External accuracy before & after 4-parameter fitting = 11.6 cm and 7.4 cm
Determination of the quasigeoid-geoid separation

- Approximate formula (Heiskanen & Moritz, 1967)

\[ \zeta - N = \frac{\Delta g_\omega H}{\gamma_n} \]

\( \Delta g_\omega \) = Bouguer gravity anomaly

\( H \) = topographic height (SRTM3)

\( \gamma_n \) = normal gravity at latitude 45°

- Strict formula (Sjöberg, 2006; 2010)

\[ \zeta - N = \frac{T(r_p, \Omega)}{\gamma_0(\phi)} - \frac{V'_{\text{quad}}(r_p, \Omega)}{\gamma_0(\phi)} + \frac{V_{\text{quad}}(r_p, \Omega)}{\gamma(\Omega)} \]

<table>
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<tr>
<th>Formula</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Std.</th>
<th>RMS</th>
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<tr>
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<td>0.16</td>
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</table>

- Topographic bias in the strict formula --harmonic series expansion

\[ V'_{\text{quad}}(r_p, \Omega) = 2\pi G \rho \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left( H_{n0}^m + \frac{2}{3R} H_{n1}^m \right) \gamma_n(\Omega) \]
Topographic bias over Uganda

UGQ2014 & its evaluation

\[ \zeta_p = N_p + (\zeta - N) \]

GNSS/levelling residuals over 10 points before and after the 4-parameter fit (units: cm)

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<tr>
<td>Approximate</td>
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<tr>
<td>Before</td>
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<td>Before</td>
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<tr>
<td>After</td>
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<td>12.91</td>
<td>0.0</td>
<td>6.65</td>
<td>6.31</td>
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</tbody>
</table>
Conclusion

- Standard errors of ellipsoidal heights = 2.2 cm
- Standard errors of normal-orthometric heights = 1.0 cm
- Standard error of UGQ2014 before & after fitting are 15.4 cm and 5.8 cm
- Satisfactory given the poor quality & quantity of terrestrial gravity data
- Comparison of approximate & strict formulas shows that --- introduce errors of 2.6 m in the QGGS --- 35 cm in the final quasigeoid heights
- Future work ---
  - GNSS/levelling observations to create a more homogeneous dataset
  - Government– airborne gravimetry for better quality gravity data
THANK YOU FOR YOUR ATTENTION

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