Polynomial Transformation Of Cassini Coordinates To UTM Coordinates On The Excel Spreadsheet

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Key Words: Transformation, Polynomial, Cassini, UTM, Scale

ABSTRACT

A procedure for transforming Cassini grid coordinates (planimetric only) to UTM coordinates on the Excel spreadsheet is outlined. A general second degree polynomial is used to compute twelve parameters which includes two scale parameters, one for the eastings, and the other for the northings and ten other unknowns. Three interconnected worksheets are used to compute the transformation. The first worksheet is used for data entry. The second worksheet computes the twelve transformation parameters necessary for the transformation by use of entries in the first worksheet, while the third worksheet computes the transformed coordinates in UTM by use of the entries in the first worksheet and the computed parameters in the second worksheet. A general third degree polynomial can also be used for the transformation by computing twenty unknowns. Ellipsoidal Cassini co-ordinates for the common points can be also computed on the excel spreadsheet, and where corresponding ellipsoidal coordinates in UTM are available, a seven parameter transformation can be obtained.

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1. INTRODUCTION

In the past all surveys in Kenya were based on Cassini-Solder projection, but the current trend is to change over to the universal transverse mercator (UTM). The Cassini Soldner Projection was used in the earliest surveys in Kenya and the grid units are in feet. The universal transverse mercator (UTM) is often preferred to Cassini because of difficulty in measuring scale and direction on Cassini, moreover survey computations are simpler when bearings are correct. A need therefore arises from time to time for conversion of coordinates from cassini to UTM and vice versa. This may be necessary when working in an area where some of the points are in cassini projection while others are in UTM projection.

The conversion of Cassini coordinates to UTM Coordinates can be performed by use of existing softwares, indeed GIS softwares will support the transformation of data between different projection systems. However where such programs are not available, the transformation can be accomplished by use of the Excel spreadsheet. The laptop computer can be used in the field for quick coordinate transformation from cassini to UTM. Presented here are procedures and steps necessary for the transformation from cassini to UTM coordinates and vice versa on the Excel spreadsheet. The steps and procedures are simple due to the fact that no prior knowledge of any programming language is necessary because the Excel commands are inbuilt.

2. COORDINATE TRANSFORMATION

A coordinate transformation is performed where the target coordinate reference system is based on a different datum to the source reference system. In our case the target coordinate system is UTM while the source is cassini. This transformation from cassini to UTM and vice versa can be accomplished by two methods: transformation by use of polynomials of degree 2, 3 or higher. Alternatively the transformation may be achieved via computing the geographical coordinates from the grid coordinates in both systems for the common points to compute a seven parameter transformation of a scale, three rotation elements, and three translation elements. This latter method was demonstrated in **[2]** on the excel spreadsheet and hence the former method will be demonstrated here.

3. TWELVE PARAMETER TRANSFORMATION

We start off with grid coordinates (x_i, y_i) on the cassini and (X_i, Y_i) on UTM projection for a number of points. The objective is to obtain the coordinate transformation parameters from cassini to UTM. The steps in the transformation as follows: The UTM projection is conformal whereas the cassini projection is not. To obtain conformal coordinates on the cassini projection a correction is added to the cassini eastings as shown in appendix 2. The cassini coordinates are

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then transformed to UTM coordinates by the computation of scale, rotation and two translation elements. The formulae necessary for this transformation are shown in the appendix1. However, since we are transforming between two different types of projections, the transformation is not necessarily linear as explained in **[1]**, a second degree polynomial is therefore used to compute a twelve parameter solution by use of six common points in both systems. This includes the two parameters of scale, two translation elements and eight other parameters. The formulae necessary for the computation of these twelve parameters is also shown in appendix1. Our solution will therefore consist of solving for twelve transformation parameters.

Whereas, a minimum of six points is necessary to solve for the twelve unknowns, an extra one point is added in this case to allow for redundancy.

4. THE SOLUTION

The solution is by use of three different sheets on the same worksheet, however all three are interlinked making it possible for many transformations by simply changing the coordinates entry. Each sheet has a unique function as follows:

Sheet 1: Coordinate entry

The coordinates of the points to be transformed from cassini to UTM are entered in this sheet. The cassini coordinates are entered first with the corresponding coordinates in UTM being entered on the same row as the cassini coordinates as shown in Table 1. Seven pairs of coordinates in both systems for the seven points to be used to compute the twelve parameters are entered in three rows A23:F29. The stations with cassini coordinates only follow, A30:F31 in this example. The cassini coordinates for these two points will be transformed to UTM coordinates.

The cassini coordinates are not conformal and therefore a correction term is added to the cassini eastings to obtain conformal cassini coordinates. The following formula is entered in D23 as shown in appendix 2.

D23: = C23+C23^3/(6*$C$7*$C$8)+C23^5/(24*$C$7^2*$C$8^2)

The copy command is used to copy this formula to D24:D31. C7 and C8 are entered as absolute cell references.

Sheet 2: Transformation parameters

The computations for the transformation parameters are illustrated in Table 2. The necessary steps for the formation of the observation equations for the least squares solution are shown in Table 2.

The entries F23:F29 in Table 1 are copied to A5:A11 in Table 2 as follows: -

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A5: ='sheet 1'!F23

TABLE 1:   COORDINATES ENTRY

	A	B	C	D	E	F
1						
2			**Transformation of Cassini Coordinates to UTM by use of a second Degree Polynomial**			
3			CASSINI	UTM		
4	parameters					
5	flattening, f		0.003	0.003407561		
6	ecentricity, e2		0.007	0.006803511		
7	semi major axis,a		20926348.000	6378249.145		
8	semi minor axis,b		20855232.837	6356514.87		
9	Spheroid		Clarke 1858	Clarke 1880		
10	projection			TM		
11	Unit of measure		feet	metre		
12	Meridian of origin		37 E	39E		
13	latitude of origin		Equator	Equator		
14	SF. origin			0.9996		
15	false origin			500000W		
16				10000000S		
17	Datum		New(1960) arc	New(1960) arc		
18						
19						
20				COORDINATES		
21			CASSINI (feet)		UTM (metres)	
22	STATION	NORTHINGS	EASTINGS	CONFORMAL EASTINGS	NORTHINGS	EASTINGS
23	1	-54102.10	-182848.40	-182850.735	9861717.50	221784.70
24	2	-54093.10	-128079.90	-128080.702	9861732.90	238487.30
25	3	-54089.00	-91567.50	-91567.793	9861742.70	249621.40
26	4	-144786.50	-182828.20	-182830.534	9834060.90	221813.70
27	5	-144775.60	-128066.60	-128067.402	9834079.40	238514.60
28	6	-144770.80	-91559.20	-91559.493	9834091.10	249647.40
29	7	-90366.30	-128075.00	-128075.802	9850671.50	238497.60
30	8	-90369.00	-146330.20	-146331.397		
31	9	-126639.30	-128069.60	-128070.402		

Sheet 1

These are the UTM eastings for the common points. The copy command is then used to copy this formula to A6:A11.

Similarly entries E23:E29 in Table 1 are copied to A12:A18 in Table 2 as follows:

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A12: ='sheet 1'!E23

These are the UTM northings for the common points. The copy command is used to copy this formula to A13:A18. Overall, the combined entries for the UTM eastings and northings which are in A5:A18 for the seven points, form the *l* matrix (**14x1**) component of the observation equation as shown in Table 2.

Each station gives rise to two equations: The following formulae are entered for the first equation for the eastings component.

C5: ='sheet 1'!D23

D5: ='sheet 1'!B23

E5: ='sheet 1'!D23^2

F5: ='sheet 1'!D23*'sheet 1'!B23

G5: ='sheet 1'!B23^2

H5: =0

I5: =0

J5: =0

K5: =0

L5: =0

M5:=1

N5: =0

The copy command is used to copy these formulae to C6:N11 for the other six points.

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TABLE 2: THE SOLUTION OF THE TWELVE PARAMETERS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1							Solution of the twelve parameters							
2														
3	**I**							**A**						
4														
5	221784.70		-182850.73	-54102.10	33434391162.62	9.89260873E+09	2927037224.41	0	0	0	0	0	1	0
6	238487.30		-128080.70	-54093.10	16404666325.56	6.92828224E+09	2926063467.61	0	0	0	0	0	1	0
7	249621.40		-91567.79	-54089.00	8384660751.82	4.95281037E+09	2925619921.00	0	0	0	0	0	1	0
8	221813.70		-182830.53	-144786.50	33427004118.07	2.64713931E+10	20963130582.25	0	0	0	0	0	1	0
9	238514.60		-128067.40	-144775.60	16401259491.74	1.85410350E+10	20959974355.36	0	0	0	0	0	1	0
10	249647.40		-91559.49	-144770.80	8383140780.75	1.32551411E+10	20958584532.64	0	0	0	0	0	1	0
11	238497.60		-128075.80	-90366.30	16403411135.09	1.15737364E+10	8166068175.69	0	0	0	0	0	1	0
12	9861717.50		0	0	0	0	0	-182850.73	-54102.10	3.3434391163E+10	9.8926087312E+09	2927037224.41	0	1
13	9861732.90		0	0	0	0	0	-128080.70	-54093.10	1.6404666326E+10	6.9282822426E+09	2926063467.61	0	1
14	9861742.70		0	0	0	0	0	-91567.79	-54089.00	8.3846607518E+09	4.9528103665E+09	2925619921.00	0	1
15	9834060.90		0	0	0	0	0	-182830.53	-144786.50	3.3427004118E+10	2.6471393093E+10	20963130582.25	0	1
16	9834079.40		0	0	0	0	0	-128067.40	-144775.60	1.6401259492E+10	1.8541034986E+10	20959974355.36	0	1
17	9834091.10		0	0	0	0	0	-91559.49	-144770.80	8.3831407807E+09	1.3255141067E+10	20958584532.64	0	1
18	9850671.50		0	0	0	0	0	-128075.80	-90366.30	1.6403411135E+10	1.1573736373E+10	8166068175.69	0	1

	C	D	E	F	G	H	I	J	K	L	M	N	O	P
21							A^T							
22														
23	-182850.7347	-128080.7024	-91567.7932	-182830.5339	-128067.4021	-91559.4931	-128075.8023	0	0	0	0	0	0	0
24	-54102.1000	-54093.1000	-54089.0000	-144786.5000	-144775.6000	-144770.8000	-90366.3000	0	0	0	0	0	0	0
25	33434391162.6187	16404666325.5566	8384660751.8215	33427004118.0743	16401259491.7444	8383140780.7454	16403411135.0942	0	0	0	0	0	0	0
26	9892608731.1927	6928282242.6301	4952810366.4844	26471393093.3111	18541034985.7436	13255141066.8569	11573736373.4903	0	0	0	0	0	0	0
27	2927037224.4100	2926063467.6100	2925619921.0000	20963130582.2500	20959974355.3600	20958584532.6400	8166068175.6900	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	-182850.73	-128080.70	-91567.79	-182830.5	-128067.4	-91559.49	-128075.80
29	0	0	0	0	0	0	0	-54102.10	-54093.10	-54089.00	-144786.5	-144775.6	-144770.80	-90366.30
30	0	0	0	0	0	0	0	33434391162.62	16404666325.56	8384660751.82	3.343E+10	1.64E+10	8383140780.75	16403411135.09
31	0	0	0	0	0	0	0	9892608731.19	6928282242.63	4952810366.48	2.647E+10	1.854E+10	13255141066.86	11573736373.49
32	0	0	0	0	0	0	0	2927037224.41	2926063467.61	2925619921.00	2.096E+10	2.096E+10	20958584532.64	8166068175.69
33	1	1	1	1	1	1	1	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	1	1	1	1	1	1	1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
36	A^TI								A^TA					
37														
38	-2.18460E+11		1.32839E+11	9.161501E+10	-2.00628E+16	-1.306E+16	-1.06597E+16	0	0	0	0	0	-933032.46	0
39	-1.62742E+11		9.161501E+10	7.982648E+10	-1.306E+16	-1.06597E+16	-1.03167E+16	0	0	0	0	0	-686983.4	0
40	3.07520E+16		-2.00628E+16	-1.306E+16	3.18299E+21	1.97586E+21	1.52455E+21	0	0	0	0	0	1.3284E+11	0
41	2.14461E+16		-1.306E+16	-1.06597E+16	1.97586E+21	1.52455E+21	1.37958E+21	0	0	0	0	0	9.1615E+10	0
42	1.89063E+16		-1.06597E+16	-1.03167E+16	1.52455E+21	1.37958E+21	1.41041E+21	0	0	0	0	0	7.9826E+10	0
43	-9.18877E+12		0	0	0	0	0	1.328385E+11	9.161501E+10	-2.00628E+16	-1.306E+16	-1.06597E+16	0	-933032.46
44	-6.76184E+12		0	0	0	0	0	9.161501E+10	7.982648E+10	-1.306E+16	-1.06597E+16	-1.03167E+16	0	-686983.4
45	1.30823E+18		0	0	0	0	0	-2.00628E+16	-1.306E+16	3.18299E+21	1.97586E+21	1.52455E+21	0	1.3284E+11
46	9.01743E+17		0	0	0	0	0	-1.306E+16	-1.06597E+16	1.97586E+21	1.52455E+21	1.37958E+21	0	9.1615E+10
47	7.85398E+17		0	0	0	0	0	-1.06597E+16	-1.03167E+16	1.52455E+21	1.37958E+21	1.41041E+21	0	7.9826E+10
48	1.65837E+06		-933032.4617	-686983.4	1.32839E+11	9.161501E+10	7.982648E+10	0	0	0	0	0	7	0
49	6.89381E+07		0	0	0	0	0	-933032.4617	-686983.4	1.32839E+11	9.161501E+10	7.982648E+10	0	7

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N
51	Solution N⁻¹*AᵀI							inverse of normal matrix (AᵀA)⁻¹						
52														
53	0.304875211	A1	1.53085E-08	-6.114E-09	5.27178E-14	5.75401E-15	-3.46243E-14	0	0	0	0	0	0.00075956	0
54	-0.000216198	A2	-6.114E-09	1.66447E-08	-2.5187E-14	8.01869E-15	7.75055E-14	0	0	0	0	0	0.00030775	0
55	-2.56154E-10	A3	5.27178E-14	-2.5187E-14	1.90088E-19	9.19754E-23	-1.26709E-19	0	0	0	0	0	2.3914E-09	0
56	-7.66805E-11	A4	5.75401E-15	8.01869E-15	9.19754E-23	5.76133E-20	1.4551E-21	0	0	0	0	0	7.8154E-10	0
57	2.50167E-10	A5	-3.46243E-14	7.75055E-14	-1.26709E-19	1.4551E-21	3.86868E-19	0	0	0	0	0	9.6508E-10	0
58	0.000214365	B1	0	0	0	0	0	1.53085E-08	-6.114E-09	5.27178E-14	5.75401E-15	-3.46243E-14	0	0.00075956
59	0.304877265	B2	0	0	0	0	0	-6.114E-09	1.66447E-08	-2.5187E-14	8.01869E-15	7.75055E-14	0	0.00030775
60	3.48808E-11	B3	0	0	0	0	0	5.27178E-14	-2.5187E-14	1.90088E-19	9.19754E-23	-1.26709E-19	0	2.3914E-09
61	-5.08559E-10	B4	0	0	0	0	0	5.75401E-15	8.01869E-15	9.19754E-23	5.76133E-20	1.4551E-21	0	7.8154E-10
62	-3.11581E-11	B5	0	0	0	0	0	-3.46243E-14	7.75055E-14	-1.26709E-19	1.4551E-21	3.86868E-19	0	9.6508E-10
63	277528.2466	Ao	0.000759559	0.000307751	2.39138E-09	7.81537E-10	9.65076E-10	0	0	0	0	0	64.972346	0
64	9878255.154	Bo	0	0	0	0	0	0.000759559	0.000307751	2.39138E-09	7.81537E-10	9.65076E-10	0	64.972346

Sheet 2

The following formulae are then entered for the second equation for the northings component.

C12: =0

D12: =0

E12: =0

F12: =0

G12: =0

H12: ='sheet 1'!D23

I12: ='sheet 1'!B23

J12: ='sheet 1'!D23^2

K12:= 'sheet 1'!D23*'sheet 1'!B23

L12:= 'sheet 1'!B23^2

M12:0

N12:=1

The copy command is used to copy this formula to C12:H18 for the other six points. Finally, the entries from the above for the cassini northings and eastings in C5:N18 for the seven points forms the *A* matrix (**14x12**) component of the observation equation.

The size of the observation equation is dependent on the number of points. Once the observation equation is formed and the fact that the Excel spreadsheet has the facility to manipulate matrices, the computation with least squares could be performed simply through the usual

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matrix manipulations. The steps involved comprised the formation of the normal equations, and the evaluation of the unknown parameters from the normal equations. This is illustrated below:-

The normal equation is formed through the matrix multiplication of the transpose (A^T) of the A matrix with the A matrix. The transpose matrix is obtained by first selecting the cells C23:P34. The following entry is then made in cell C23 as follows

C23:=**TRANSPOSE**(C5:N18)

The *shift* and *ctrl* keys are held down and the enter key pressed. The result is the transpose matrix of A in cells C23:P34.

The normal matrix is then computed through matrix multiplication by first selecting cells C38:N49, and making the following entry in cell C38 as follows

C38:=**MMULT**(C23:P34, C5:N18)

The *shift* and *ctrl* keys are again held down and the enter key pressed. The result is the normal matrix in cell C38:N49.

The inverse of the normal matrix is obtained by first selecting the cells C53:N64 and making the following entry in cell C53 as follows

C53=:**MINVERSE**(C38:N49)

The *shift* and *ctrl* keys are again held down and the enter key pressed. The result is the inverse matrix in cell C53:N64.

Since the l matrix has already been formed, the $A^T l$ is obtained is via use of the **MMULT** function as demonstrated above. The results are entered in cells A38:A49. Finally the unknown parameters are similarly computed via matrix multiplication i.e. $N^{-1}*A^T l$ and the results entered in cells A53:A64. The results of this computation are shown on Table 2, Sheet 2.

Sheet 3: Transformation

The transformation of cassini to UTM coordinates is performed in Sheet 3, as illustrated in table 3. The coordinates' entry in Sheet 1 and the transformation parameters in Sheet 2 are used to compute the UTM coordinates. This is accomplished through the following steps:

The cassini coordinates in Sheet 1 are copied to Sheet 3 using the following commands.

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B6: ='sheet 1'!B23

C6: ='sheet 1'!C23

The formulae are then copied to B7:C14.

TABLE 3:   THE TRANSFORMED COORDINATES

	A	B	C	D	E	F	G	H	I	J
1	TRANSFORMED COORDINATES									
2										
3	STATION	CASSINI (feet)		UTM (metres)						
4		NORTHINGS	EASTINGS	NORTHINGS	EASTINGS					
5										
6	1	-54102.10	-182848.40	9861717.501	221784.696		9861717.500	221784.700	-0.0005	0.0036
7	2	-54093.10	-128079.90	9861732.899	238487.309		9861732.900	238487.300	0.0013	-0.0089
8	3	-54089.00	-91567.50	9861742.701	249621.395		9861742.700	249621.400	-0.0008	0.0054
9	4	-144786.50	-182828.20	9834060.900	221813.704		9834060.900	221813.700	0.0005	-0.0036
10	5	-144775.60	-128066.60	9834079.401	238514.591		9834079.400	238514.600	-0.0013	0.0089
11	6	-144770.80	-91559.20	9834091.099	249647.405		9834091.100	249647.400	0.0008	-0.0054
12	7	-90366.30	-128075.00	9850671.500	238497.600		9850671.500	238497.600	0.0000	0.0000
13	8	-90369.00	-146330.20	9850666.099	232930.513					
14	9	-126639.30	-128069.60	9839610.081	238508.702					

Sheet 3

The transformed UTM coordinates are computed from the following formulae (for the northings and eastings)

D6:=sheet2!$A$58*Sheet1!D23+Sheet2!$A$59*Sheet1!B23+Sheet2!$A$60*(Sheet1!D23^2) +Sheet2!$A$61*Sheet1!D23*Sheet1!B23+Sheet2!$A$62*(Sheet1!B23^2)+Sheet2!$A$64

D7:=Sheet2!$A$53*Sheet1!D23+Sheet2!$A$54*Sheet1!B23+Sheet2!$A$55*(Sheet1!D23^2)+Sheet2!$A$56*Sheet1!D23*Sheet1!B23+Sheet2!$A$57*(Sheet1!B23^2)+Sheet2!$A$63

These formulae are then copied to D7:E14. The two new points are transformed from Cassini to UTM coordinates. This has been presented here as an example, it is important to have more than six points in the transformation to act as a check (for this the residuals are calculated in sheet 3 cells I6:J12) and also to increase the redundancy. The reverse computations follow the same procedure.

5. COMPUTATION OF ELLIPSOIDAL CASSINI COORDINATES

As stated earlier, the cassini projection is not conformal. To obtain conformal coordinates a correction is applied to the cassini eastings. Once conformal cassini coordinates are obtained, the ellipsoidal coordinates in cassini are computed as in UTM projection. This was demonstrated in **[2]** on the Excel spreadsheet. It is then possible to compute a seven parameter

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transformation between cassini and UTM projection for a set of given coordinates as was again shown in **[2].**

CONCLUSION

The results obtained by the use of the Excel spreadsheet are the same as would be obtained by use of a computer program. The computations on Excel are simple since for every operation, the Excel commands are easily called because they are inbuilt. Since the three sheets are interlinked, changing the coordinates entry makes computation of many transformations possible. Once created junior surveyors can use it with ease. One draw-back with the Excel spreadsheet is the fact that it computes up to 15 significant figures, hence it is important to limit the size of the entries by changing the decimal point and applying it later or use of initial values and applying an iteration. One way to avoid this problem of numerical instability the polynomial may be used after reducing the coordinates in both source and target coordinate reference system to manageable numbers.

A third degree polynomial was also used to compute the twenty unknowns, two scale elements, two translation elements and twenty other unknowns.
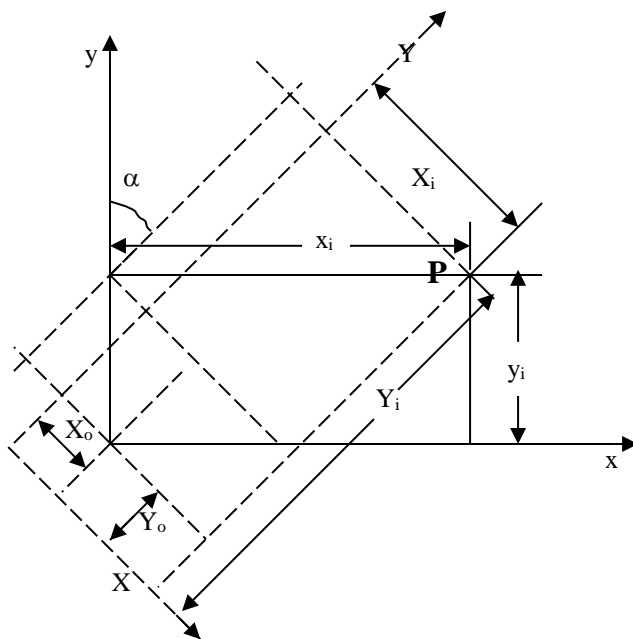
APPENDIX 1

FIGURE 1

Consider a point **P** with coordinates (X_i, Y_i) on one coordinate system and (x_i, y_i) in the other as shown in figure 1, the coordinates (X_i, Y_i) in terms of (x_i, y_i) are given by

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$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = k \begin{pmatrix} x_i \cos\alpha & -y_i \sin\alpha \\ x_i \sin\alpha & y_i \cos\alpha \end{pmatrix} + \begin{pmatrix} X_o \\ Y_o \end{pmatrix} \qquad (1)$$

This can be written as

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} kcx_i & -ksy_i \\ ksx_i & kcy_i \end{pmatrix} + \begin{pmatrix} X_o \\ Y_o \end{pmatrix} \qquad (2)$$

where,

k is the scale factor
c is the cosine of the rotation angle α
s is the sine of the rotation angle α
X_o, Y_o are the translation elements

These four parameters (k, α, X_o, Y_o) must be determined to be able to perform a coordinate transformation. However, in some cases, it is not sufficient to assume a linear relationship between the two sets of coordinates particularly if they are on different types of projections. In such a case a second order transformation is required as explained in **[1].** The general second degree polynomial is given by: -

$$X = A_o + A_1 x + A_2\, y + A_3 x^2 + A_4\, xy + A_5\ y^2 \qquad (3a)$$

$$Y = B_o\ + B_1\, x + B_2\, y + B_3\, x^2 + B_4 xy + B_5\, y^2 \qquad (3b)$$

This general second order degree polynomial can be written in matrix form as follows:

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} A_1 x + A_2 y + A_3 x^2 + A_4 xy + A_5 y^2 \\ B_1 x + B_2 y + B_3 x^2 + B_4 xy + B_5 y^2 \end{pmatrix} + \begin{pmatrix} A_o \\ B_o \end{pmatrix} + \begin{pmatrix} v_{xi} \\ v_{yi} \end{pmatrix} \qquad (4)$$

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Equation (4) can be written in matrix form as follows.

$$
\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} x & y & x^2 & xy & y^2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & x & y & x^2 & xy & y^2 & 0 & 1 \end{pmatrix} + \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ A_o \\ B_o \end{pmatrix} + \begin{pmatrix} v_{xi} \\ v_{yi} \end{pmatrix} \quad (5)
$$

From equation (5), twelve unknowns need to be computed i.e. A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4, B_5, A_0 B_0 to solve the problem with a minimum of 6 common points.

APPENDIX 2

To convert non-conformal cassini coordinates to conformal cassini coordinates a correction is added to the cassini eastings as follows:

$$
x_c = \frac{x^3}{6R^2} + \frac{x^5}{24R^4} + ...
$$

Where R is the radius of the earth and is given by

$$
R = \sqrt{a*b}
$$

a, b are the semi-major and semi-minor axis of the ellipsoid respectively as shown in **[1]**.

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1. Allan A.L., Hollwey J.R., Maynes J.H.B., 1977. Practical Field Surveying and Computations. 689 pages

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