

# HEIGHTING WITH GPS: POSSIBILITIES AND LIMITATIONS

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## ABSTRACT

Global Positioning System (GPS) surveying is now seen as a true three dimensional tool and GPS heighting can be a viable alternative to other more conventional forms of height measurement. This paper examines the limitations and possibilities of GPS heighting. The first part of the paper details the limitations of GPS heighting, including those factors that affect the GPS height measurement itself and the associated issues of geoid modeling and compatibility with the local vertical datum. The second part of the paper examines the possibilities for GPS heighting, focusing on three application areas currently generating interest for the practicing surveyor; deformation monitoring, real time GPS surveying and machine monitoring and guidance. These applications cover the range of achievable GPS heighting accuracy.

## INTRODUCTION

Global Positioning System (GPS) surveying has been used extensively and with great success for the production and propagation of survey control. During the development of GPS surveying the focus was typically on horizontal control with the ability of GPS to measure height being seen as an added extra. GPS surveying has now matured to the point where it is seen as a true three dimensional tool. However, application of GPS to the measurement of height can be complex and solving the problems involved can account for the majority of the effort in finalising a GPS surveying project.

GPS measures heights related to the ellipsoid. In some cases ellipsoidal heights alone are sufficient for the type of survey being undertaken. However, many applications require heights that are related to a physically meaningful surface such as the geoid, or at least some attempt at realizing the geoid such as a surface based on locally observed mean sea level. Such physically meaningful heights take the form of orthometric or normal heights. For this discussion the difference between orthometric and normal heights is not that significant and the term orthometric height will be used for convenience throughout this paper.

Before examining the possibilities for GPS heighting it is necessary to understand the limitations.

## LIMITATIONS TO GPS HEIGHTING

In practice, GPS heighting typically involves measuring ellipsoidal heights with GPS, applying some form of geoid model and making any adjustment to fit the resulting orthometric heights to the existing vertical datum. Therefore, in examining the limitations of GPS heighting it is necessary to consider three broad areas:

- limitations of the GPS measurement
- limitations due to the available geoid model
- limitations due to vertical datum issues

Some or all of these issues vary in importance depending on the overall extent of the GPS survey in question. GPS surveys over national and continental scale are typically associated

with datum level geodetic operations and need to consider many more issues than day to day surveys which extend over a few kilometres or less. In considering limitations to GPS heighting, this paper will attempt to highlight when the scale of the project is relevant.

### Limitations of the GPS measurement

Obviously, the first limitation in GPS heighting is the quality of the GPS solutions used to obtain a height. Three broad categories of GPS observation types are possible:

- Point Positioning which is the stand alone navigation mode for which GPS was designed;
- Differential GPS (DGPS) which uses a differential correction approach but which is primarily based on pseudo range measurements and
- GPS Surveying using a differential approach but primarily based on measurement of the phase of the GPS signals.

While DGPS and even Point Positioning may be useful for producing heights in certain applications, the term GPS Heighting is typically taken to refer to the use of phase measurement techniques that can be grouped under the broad heading of GPS Surveying. This paper concentrates on heighting using these higher precision GPS Surveying techniques.

Within GPS Surveying, an overall consideration is whether the phase ambiguities have been resolved to integer values. Ambiguity resolution affects all three dimensions, not only height. For the measurement techniques known as *Rapid Static* and *Real Time Kinematic (RTK)*, which are used for shorter baselines, ambiguity resolution is a prerequisite and should be achieved for most day to day GPS surveying applications. It is important to realise that RTK uses the smallest possible amount of data and even the best algorithms sometimes resolve the ambiguities incorrectly. To avoid such errors, which can reach the metre level, it is important to build redundancy into a survey by, for example, occupying stations more than once.

Two aspects that can affect the overall quality of the baseline solution are errors in the ephemeris or in the starting coordinates used in the processing. The effect of these can reach several parts per million and apply to all three dimensions. Assuming that the broadcast ephemeris quality remains as high as in recent times, its effect will be minimal for most applications over short baselines. However, it should be noted that obtaining a WGS84 three-dimensional starting position of a reasonable quality (say +/- 10m or better) could be more problematic in some areas of the world.

Another error source to consider is multi-path. Reflective surfaces can mean that some of the signal reaching the antenna does not travel on a direct path from the satellite. The effects of multi-path can reach the decimetre level in three dimensions. Observation over time to allow the satellite geometry to change sufficiently enables the effect of multi-path to be reduced through averaging. However, with the short observation times typical of the Rapid Static and RTK techniques in common use, it is necessary to pay attention to this issue. While modern hardware and software designs include various ways of reducing the effects of multi-path, even over short observation times, it is important to choose stations so as to reduce the likelihood of multi-path and build redundancy into the survey to enable detection of any remaining effect.

Atmospheric delay is another issue to be considered. For a short baseline one can reasonably assume that the radio signals measured by both receivers pass through the same part of the atmosphere. However, as baseline length increases that assumption begins to break down and atmospheric effects need more consideration. Two components of the atmosphere are relevant, the ionosphere and the troposphere.

Problems due to the upper atmospheric layer known as the ionosphere can affect all three dimensions and become significant on lines longer than, say 20km. For such longer baselines, processing software is able to take advantage of the fact that the ionospheric effect is related to the frequency of the signal and dual frequency measurements can be used to remove most of the effect. The effect of the ionosphere is greater near the poles and the geomagnetic equator and varies over time in association with solar disturbance cycles. Therefore, it should be noted that for certain areas and certain times the ionospheric effect can be significant, even over short baselines.

The effect of the troposphere is particularly significant for height measurement. Unlike the ionosphere, tropospheric delay cannot be mitigated using dual frequency measurement. Furthermore, the GPS signal can be delayed due to both the dry and wet components of the troposphere. Most GPS processing includes models to account for the dry component of the troposphere. However, it is difficult to model the wet component given its greater variability and estimating the wet delay as part of the overall baseline estimation process is the best approach.

Over longer baselines (say 100km or more) it is typical for many hours of GPS data to be observed and the high level of redundant data allows for the tropospheric delay to be estimated at regular intervals through the data set (e.g. one delay each hour). For such long baselines, ignoring the tropospheric delay can cause a height error of several centimetres (see, for example, Dodson et al, 1996).

For many day to day surveys, the baselines are typically quite short and the effect of the troposphere is less significant. Also the data observation times are short meaning that less data is available to estimate the delay even if it were significant. Generally speaking, such surveys can simply use the software model for the dry component and the remaining effect will not be significant. However, it should also be noted that the tropospheric effect could be significant when there is a significant change in height between the ends of the baseline. For steep baselines, even when relatively short, there may be situations where longer observation time and estimating the delay may be warranted.

Tidal phenomena may be significant for GPS heighting in certain circumstances. These include the earth tide and the ocean tide's variable loading of the crust in and adjacent to the coastal zone. While not usually significant for day to day GPS surveying over short baselines, there can be significant differential effects for baselines of 100km and longer; amounting to centimetre level errors in height. Some software packages enable modeling of these tidal effects for those situations when they are significant.

The other major source of error for GPS heighting involves the antenna. The first and most obvious problem is that the height of the antenna above the survey mark must be correctly measured. Many RTK systems use a pole for the roving antenna to decrease occupation time compared to tripod usage. An advantage of this for heighting is that the fixed height of the

pole minimises the possibility of incorrect antenna heights. When variable height tripods are used it is important to have a field routine for checking the height measurement at each station. Use of a slant height (to the outside of the ground plane) and comparison to the vertically measured height is a technique in common usage. Measurement in both metric and imperial units is another approach.

A less obvious antenna issue arises when various antenna types are mixed in the same survey. The problem with antenna mixing is that different antenna may have their effective antenna phase centre (also called the electrical centre) in different positions. This effect can be most significant in the height component and can reach values of several centimetres. The International GPS for Geodynamics Service (IGS) has had to address this issue to account for the many types of antennae used at its various permanent tracking stations. Antenna models are available and they can be applied in some software packages to mitigate the effects of antenna mixing (see Mader and MacKay, 1996).

Most GPS surveying applications use receivers and antennae that are all from the same manufacturer and this problem is minimised. However, there may be situations where a survey mixes antennae that have significantly different characteristics and surveyors need to be aware of this issue. One situation where this could arise is when using data from a base station run by another organisation. At present such a possibility is limited mainly to post processed applications but the increasing popularity of RTK along with adoption of standard RTK data formats is likely to lead to mixing of receiver and antenna types from different manufacturers, even in RTK surveying. For such situations antenna modeling will need to be addressed to ensure reliable height measurement.

#### Accuracy of GPS measurement for Height

Despite all of the issues outlined above, the *bottom line* for the practicing GPS surveyor is what can be achieved using typical Rapid Static and RTK approaches? A pragmatic approach to answering that question is to look at the accuracy claims of manufacturers. A quick scan of product brochures or information on web sites of a number of manufacturers led to the following:

- Leica state baseline rms values for their new SR530 for real time static of 5mm + 2ppm and real time stop & go and kinematic of 10mm + 2ppm. No differentiation is made between horizontal and vertical accuracies.
- Ashtech produce a RTK system that uses both GPS and the Russian GLONASS systems and measures single frequency data from both systems. The stated vertical accuracy for the GG-RTK system is 1cm + 1ppm at 1 sigma.
- Javad Positioning Systems make a general statement of 1mm + 1ppm for dual frequency and 2mm + 2ppm for single frequency.
- Trimble gives a quite comprehensive outline in the data sheet for their 4800 GPS Total Station product (summarised in the following Table). While not specifically stated, it would appear that these are 1 sigma values. Note that the term *fast static* used by Trimble and the term *rapid static* used in this paper are the same. Also note that for RTK the stated accuracy varies according to the update rate used (1Hz is a rate of 1 update per second and 5Hz equals 5 per second).

Mode	Accuracy	Latency
Static and Fast Static	5mm + 1ppm Horizontal	

	10mm + 1ppm Vertical	
Post processed kinematic	10mm + 2ppm Horizontal <10km	
	20mm + 1ppm Horizontal >10km	
	20mm + 1ppm Vertical	
RTK at 1 Hz	10mm + 2ppm Horizontal	0.4 sec
	20mm + 2ppm Vertical	
RTK at 5 Hz	30mm + 2ppm Horizontal	0.1 sec
	50mm + 2ppm Vertical	

If one accepts these figures then the vertical accuracy possible for the types of GPS surveying modes used in most day to day surveys are shown in the Tables below. The table shows values in millimetres for baseline lengths of 1, 5 and 10km. As well as 1 sigma, the table also shows 3 sigma values to give an indication of the worst results that may be expected.

Mode	mm	+ ppm	Error in mm (1 sigma)			Error in mm (3 sigma)		
			1km	5km	10km	1km	5km	10km
Fast Static	10	1	11	15	20	33	45	60
Kinematic	20	1	21	25	30	63	75	90
RTK 1 Hz	20	2	22	30	40	66	90	120
RTK 5 Hz	50	2	52	60	70	156	180	210

It must be remembered that the issues and accuracy values outlined in this section are only for the GPS measurements. For day to day surveys over project areas less than 10km in extent, the GPS measurement is often the least significant part of the GPS Heighting problem.

#### Limitations due to Geoid Model

GPS surveying measures differences in ellipsoidal heights ( $h$  in Figure 1) and to produce physically meaningful heights such as orthometric heights ( $H$ ) there is a need for a sufficiently precise model of the separation between the geoid and the ellipsoid; the geoid height ( $N$ ). Also, GPS surveying can measure that ellipsoidal height difference over large distances very efficiently. These two points can highlight problems in the existing geoid model or vertical datum, or both.

In some areas of the world the only available geoid model is a global geopotential model (GGM). A GGM is typically computed as a series of spherical harmonic expansions to a maximum degree and order. Many recent GGM use an expansion to degree and order 360. That means they are able to resolve features in the geoid with a wavelength down to half a degree (nominally 55km). With such resolution, even state of the art models such as the Earth Geopotential Model 1996 (EGM96 - Lemoine et al, 1996) are limited to absolute accuracy at the metre level and relative accuracy at the several decimetre level.

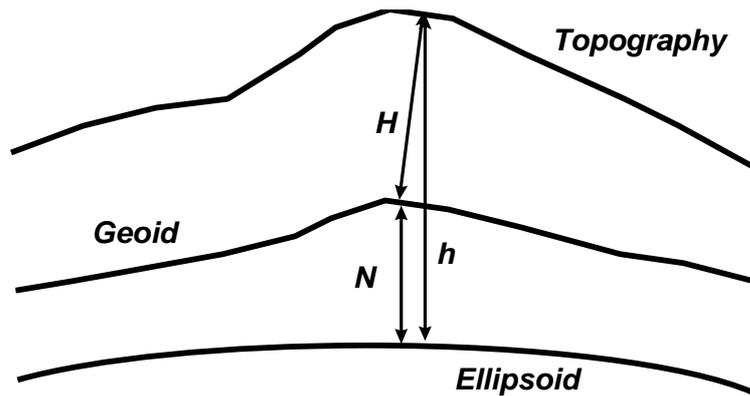


Figure 1 Geoid Height

In many regions of the world, it is desirable to improve upon the accuracy possible using only the GGM by computing a local geoid model. The model typically takes the form of a grid of geoid heights to be interpolated by users as required. Local geoid models are derived in two parts. The long wavelength component comes from the best available GGM while the short wavelength component is computed using locally observed gravity data. The short wavelength component of the geoid can be computed using several techniques, including doing the integration directly (with quadratures or rings), least squares collocation or fast fourier transform. Sunkel (1996) gives a summary of recent geoid developments in various countries. Obviously, geoid height accuracy is strongly influenced by how well the gravity data used in the computation represents the actual gravity field and recent improvements include:

- use of satellite altimetry data for increasing gravity data density offshore,
- use of digital elevation model data (DEM) to account for terrain effects on the gravity field,
- use of DEM data to reflect high frequency gravity field variations and improve the gridding or interpolation of the raw gravity data.

The limiting factors for the inherent accuracy of a geoid model are the amount of variability in the gravity field and in the terrain. A geoid model will typically be least accurate in areas with rugged terrain and highly variable underlying geology.

However in applying a geoid model, its inherent accuracy is not the only limitation. How well the geoid model can be used in conjunction with the existing vertical datum also requires consideration.

#### Limitations due to Vertical Datum

The definition of the vertical datum in many areas of the world has often been localized with realization through published orthometric or normal heights from local adjustment of networks of spirit level, barometric and trigonometric heighting observations. Sometimes multiple vertical datum developed with each propagating from a single point such as a tide gauge. Some regional or national vertical datum have been developed by constraining the adjustment of the leveling and heighting observations to the height of mean sea level at one or more tide gauges.

In the realisation of the Australia Height Datum (AHD) for example, the adjustment of 97,320km of leveling was constrained to mean sea level at 30 tide gauges. Oceanographic influences along with possible errors in the leveling mean that the surface formed by the base of AHD could be distorted significantly from the purely geopotential surface of the geoid (see for example Featherstone, 1998). In such cases a possible solution is to address the problem at the level of the vertical datum by developing an appropriate model of the distortion and adding it to the geoid model.

Whether or not such a distortion process has been incorporated into the geoid model, it is prudent for the GPS survey to verify the agreement between the geoid model and the vertical datum in a given project area. Occupying at least three existing stations with vertical datum height values as part of the GPS survey can do this. If there is some residual local distortion it can be removed as part of the process of adjusting the survey.

Another issue for modern vertical datum is the need to refine our information management in relation to heights. Prior to GPS, most geodetic databases typically stored only an orthometric (or normal) height for a station because that was the typical form for height observations. With GPS, vertical datum will increasingly be made up of stations at which heights are observed directly as orthometric heights and others at which ellipsoidal heights are observed. There is also a need to treat a geoid height like any other observation type in that it is of a particular quality at a particular time. Without careful management of these different data types any problems can blur into one another and make maintenance and improvement of the vertical datum difficult.

#### POSSIBILITIES FOR GPS HEIGHTING

The rest of this paper will examine the possibilities for GPS heighting, focusing on three application areas currently generating interest for the practicing surveyor and covering the full range of achievable accuracy.

##### Possibilities for Deformation Monitoring

While many applications for GPS surveying need to produce orthometric or normal heights there are some applications where ellipsoidal heights alone are useful. One such application is vertical deformation monitoring where the most important issue is to quantify a change in height over time and whether any change is relative to the geoid or ellipsoid is not particularly relevant.

Using GPS for deformation monitoring brings the normal advantages of GPS surveying such as no requirement for inter-visibility between stations and the ability to span large distances with high precision. Also, deformation applications require many repeated observations over time and GPS is well suited to automated survey processes that can significantly reduce cost.

Given the limitations outlined earlier in this paper, issues to consider in designing a GPS deformation survey include ambiguity resolution, quality of ephemeris and starting coordinates, multi-path, troposphere, tidal phenomena, antenna modeling and antenna height measurement.

It is possible to account for those issues that are significant over short baselines using commercially available equipment and software and obtain centimetre accuracy heights. However, short baselines are not always practical in deformation surveys where it is

necessary to measure to stable fixed stations well outside the deformation zone. Therefore, the accuracy possible from Rapid Static and RTK will not satisfy many deformation monitoring requirements and longer observation periods with data processing in static mode may be required. Centimetre accuracy is possible even over baselines longer than 100km with occupation times of several days and using specialised data processing.

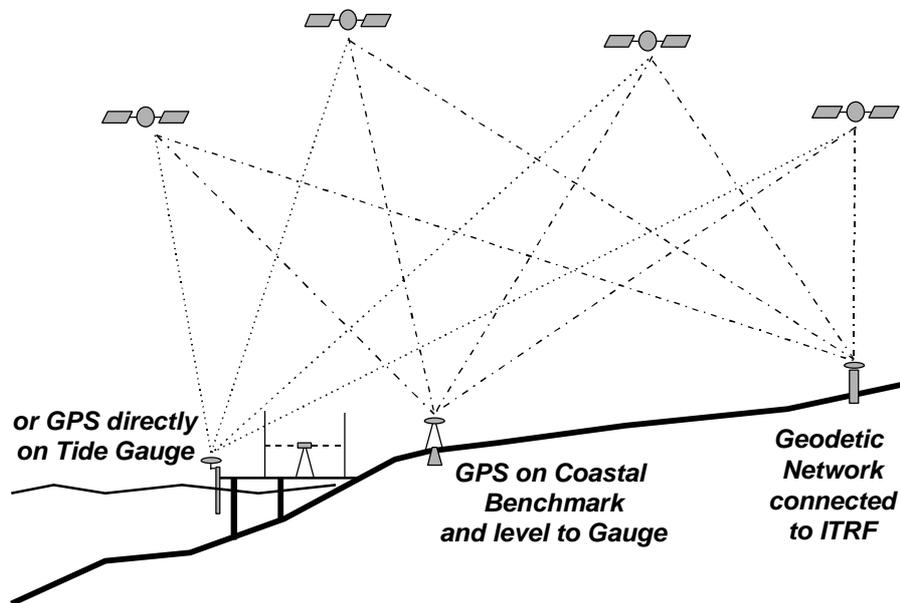


Figure 2 Monitoring Vertical Deformation of a Tide Gauge using GPS

One of the most demanding current applications for GPS heighting is in the centimetre or better accuracy required for monitoring the vertical deformation of high precision tide gauges being used to measure possible sea level rise due to global warming. Early thinking on this problem saw the need for coastal and inland arrays of marks tied to each other and to the global fiducial network by GPS and with leveling from the coastal array to the tide gauge. GPS observations took several days repeated at regular epochs. However, there were problems with isolating the deformation between the arrays and the gauge as well as accuracy limitations with episodic GPS. The state of the art approach recommends use of a permanent tracker mounted directly on the gauge as the best means for obtaining the required accuracy. Both approaches are illustrated in Figure 2. For a comprehensive description of the state of the art in this demanding application see Neilan et al (1997).

#### Application in Real Time GPS Surveying

The high productivity of RTK with its requirement for only a few epochs of data recording, means that many surveyors would like to apply the technique to the height measurements required in engineering applications. However, some caution is required and it is necessary to consider how RTK errors increase with baseline length using a particular equipment configuration. For initial route finding or contour and detail surveys requiring accuracy at the several centimetre level, RTK may well be suited. For many engineering surveys however, the heighting accuracy required is at the one centimetre level and RTK may be suitable but should be restricted to baselines shorter than a kilometre. For projects extending more than a kilometre, several RTK base stations may be required. One way of improving the accuracy of RTK over longer baselines is to observe for longer periods at a point. This approach can be thought of as *real time rapid static*, and brings improved accuracy while maintaining the logistical advantages of real time results.

Whether range is extended by multiple base stations or by longer observation time, for projects extending over many kilometres the issues of geoid and local vertical datum distortion will need to be considered. Many systems allow incorporation of geoid models into the real time data processing. However, that assumes any local distortions are incorporated in the geoid model to an accuracy sufficient for the project. Some manufacturers provide an additional feature referred to as field calibration whereby any remaining three dimensional distortion can be determined in the field using real time procedures to occupy existing control stations. Obviously, field calibration is subject to any error in the GPS observations over the baseline lengths required to connect to control.

In practice then, when contemplating RTK for centimetre level GPS heighting, it is necessary to develop field procedures that address the many possible error sources. Such procedures also need to be flexible and assessed on a case by case basis.

### Application in Machine Monitoring and Guidance

Many GPS manufacturers are promoting the use of RTK techniques to automate machine monitoring and guidance for application in agricultural, earth moving and construction equipment. It must be realised that such procedures are subject to the same error sources as any other RTK surveying technique and, as outlined in the tables earlier, those errors vary according to baseline length and the update rate being used (e.g. 1Hz vs 5Hz).

It is important to realise that a major reason for the high precision of surveys supporting engineering applications is to give a margin for error. Normally, such margins are desirable given the error which can propagate through the several steps and sets of measurements which may be required over the life of a construction process. If, for some applications, the machinery is positioned directly from the local control in a single step that margin of error may not need to be so stringent. However, other applications will require a high accuracy that is at the limits achievable with GPS in real time. In road construction for example, real time machine guidance may be feasible for earth moving but not for pavement laying equipment.

The major concern is a tendency toward black box thinking and someone who is not aware of all the issues could misuse these highly automated systems. These include the GPS measurement, geoid and datum issues discussed in this paper that can affect the final achievable accuracy for a machine guidance system. It is to be hoped that surveyors who are aware of these issues will continue to be involved in connecting and assessing the local control, establishing the base stations required and monitoring overall quality.

### CONCLUSION

This paper examined in detail the limitations of GPS heighting, including GPS measurement, geoid and datum issues. GPS measurement accuracy can be limited by ambiguity resolution, quality of ephemeris and starting coordinates, multi-path, troposphere, tidal phenomena, antenna modeling and antenna height measurement. The effect of availability and quality of geoid models was also addressed. The characteristics of the vertical datum and its suitability for combination with the geoid model and ellipsoidal heights from GPS were shown to also be significant limitations to the achievable height accuracy. It was pointed out throughout the paper that all these limitations vary in significance from one situation to another.

The paper then examined the possibilities for GPS heighting by looking at the three applications of deformation monitoring, real time GPS surveying and machine monitoring and guidance. These applications cover the complete range of achievable accuracy. The significance of the limitations of GPS heighting also varies between these applications. The paper closed with a caution against too automated an approach to the use of real time GPS techniques in machine systems especially where high accuracy heights are required.

## REFERENCES

- Dodson, A.H., P.J. Shardlow, L.C.M. Hubbard, G. Elgered and P.O.J. Jarlemark, 1996, Wet Tropospheric Effects on Relative GPS Height Determination, *Journal of Geodesy*, Vol. 70, pp 188-202.
- Featherstone, W.E., 1998, Do we need a Gravimetric Geoid or a model of the Australina Height Datum to Transform GPS Heights in Australia, *The Australian Surveyor*, Vol. 43, No. 4, December, pp 273-280.
- Lemoine, F.G., D.E., Smith, R. Smith, L. Kunz, N. Pavlis, M. Klosko, D.S. Chinn, M.H. Torrence, R.G. Williamson, C.M. Cox, K.E. Rachlin, Y.M. Wang, E.C. Pavlis, S.C. Kenyon, R. Salman, R. Trimmer, R.H. Rapp and R.S. Nerem, 1996, The Development of the NASA GSFC and DMA Joint Geopotential Model, *Proceedings of the International Symposium on Gravity, Geoid and Marine Geodesy*, Tokyo, Japan, September 30 - October 5.  
(Internet copy available from: <http://cddis.gsfc.nasa.gov/926/egm96/egm96.html>)
- Mader, G.L. and J.R. MacKay, 1996, Calibration of GPS Antennas, *IGS Workshop Proceedings*, Silver Spring, Md. March 19-21, 1996. available from Jet Propulsion Laboratory, Pasadena, California. JPL Publication 96-23 10/96. pp.81-105.
- Neilan, R.E., P.A. Van Scoy and P.L. Woodworth (Editors), 1997, *Workshop on Methods for Monitoring Sea Level*, *Proceedings of the Workshop*, Pasadena, California, March 17 - 18, 1997. available from Jet Propulsion Laboratory, Pasadena, California. JPL Publication 97-17 3/98.
- Sunkel, H., 1996, *International Geoid Commission Activity Report 1991 - 1995*, International Geoid Commission, Commission XII of Section III of the International Association of Geodesy.  
(Internet copy available from: <http://www.ngs.noaa.gov/~dennis/geoid.html>)