

## ANALYSIS OF DYNAMIC DEFORMATION PROCESSES WITH ADAPTIVE KALMAN-FILTERING

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**Abstract:** The analysis of dynamic deformation processes is an important task in civil and in mechanical engineering. One main emphasis is set on the derivation of the objects transmission function, represented by systems of ordinary or partial differential equations (“white box”-models). The parametric identification of the initial and boundary conditions and relevant material parameters is one of the new challenges in the application of least-square algorithms. Embedding the investigation of the deformation process in a combination of a theoretical and an experimental system analysis suitable methods are provided with adaptive KALMAN-filtering as central identification tool. In this paper the approach of a full system analysis is shown quantifying a dynamic “white-box”-model for the calculation of thermal deformations of bar-shaped machine elements. The task was motivated from mechanical engineering searching new methods for the precise prediction and computational compensation of thermal influences in the heating and cooling phases of machine tools (i.e. robot arms, etc.). The quantification of thermal deformations under variable dynamic loads requires the modelling of the non-stationary spatial temperature distribution inside the object. Based upon FOURIERS law of heat flow the high-grade non-linear temperature gradient is represented by a system of partial differential equations within the framework of a dynamic Finite Element topology. It is shown that adaptive KALMAN-filtering is suitable to quantify relevant disturbance influences and to identify thermal parameters (i.e. thermal diffusivity) with a deviation of only 0,2%. As result an identified (and verified) parametric model for the realistic prediction respectively simulation of dynamic temperature processes is presented. Classifying the thermal bend as the main deformation quantity of bar-shaped machine tools, the temperature model is extended to a temperature deformation model. In lab tests with an aluminium column control measurements show that the identified model can be used to predict the columns bend with a mean deviation smaller than 10 mgon. Consequently the deformation model is a precise predictor and suitable for realistic simulations of thermal deformations under variable dynamic loads. In future our activities will be primarily focussed on applications in industrial manufacturing.

### 1. Parametric identification of dynamic deformation processes

The creation of a realistic parametric dynamic deformation model is directly associated with the task to combine systems of differential equations (theoretical physical model) with empirical measurements. Consequently basic principles of system theory can be adapted to the geodetic deformation analysis. The parametric identification of a dynamic “white box”-model

[1] requires its integration into a full system analysis which can be splitted into a theoretical and into an experimental part (see Figure 1, [2]).

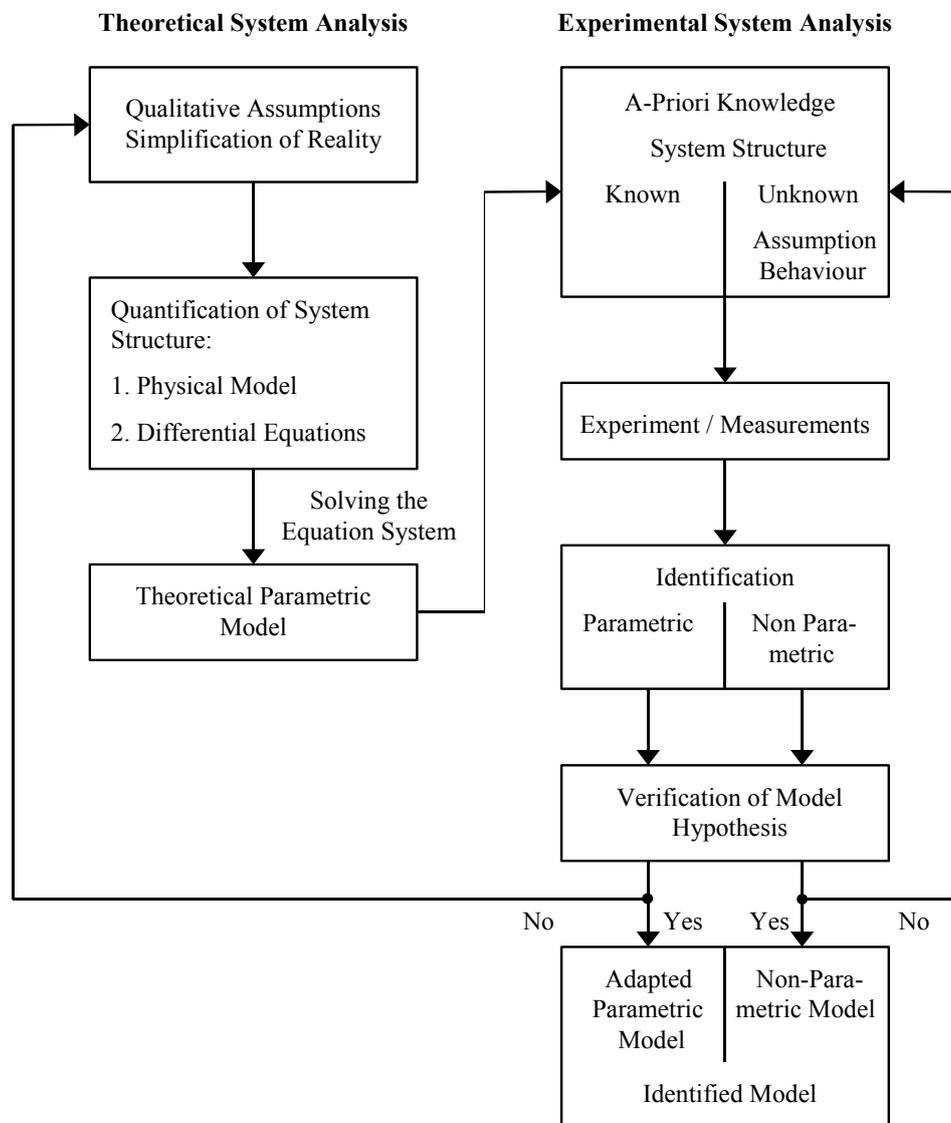


Figure 1: Theoretical and experimental system analysis (ref. to [2])

The theoretical part contains the quantification of the parametric model based on physical relationships (impulse and/or balance equations). The result is a system of ordinary and/or partial differential equations which approximates the real dynamic deformation process.

The adaptation of the deformation model to reality is realized in the experimental part. The parametric identification contains the adaptation of the solved differential equations to reality. It is preliminary focussed on

- the determination of the required initial and boundary conditions
- and the determination of relevant physical parameters (i.e. material properties like temperature coefficients, YOUNG-modulus etc.)

A suitable tool for the experimental system analysis is provided by adaptive KALMAN-filtering (i.e. [3]) which enables the optimal estimation of all desired quantities. The theoretical part of the system analysis is represented by the system equations and the experimental part by the measuring equations (see [4]).

The evaluation of the identification results is realized by the verification process. The first step is executed by the KALMAN-filter and the related innovation test. In case of a parametric identification the verification can be extended to the direct evaluation of the estimated parameters (plausibility check of the numerical quantities by experts) and the longterm comparison of pure deformation predictions with independent measurements. In case of conformity the identified parametric model is suitable for the calculation of realistic simulations with the computer model for further investigations of the deformation process.

## 2. Case study: non-stationary thermal deformation processes in bar-shaped machine tools

To reduce thermal influences on robots and machine tools the development of constructive and compensatory methods is the subject of numerous work in the last years ([5], [6], [7]). Especially in the case of computational compensation methods (see Figure 2) the precise calculation of the expected deformations of single components or kinematic chains represents a central element. It requires the quantification of the relationship between temperature measurements (system input quantities) and the resulting deformations (system output quantities). Within this context modelling of the spatial non-stationary temperature distribution is an essential task.

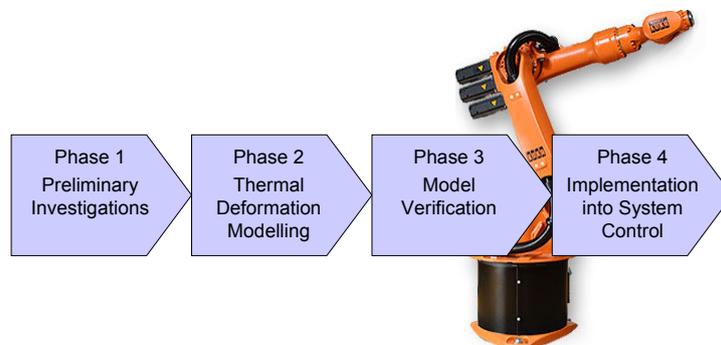


Figure 2: Process chain for computational compensation of thermal influences

Together with the “Institute of Machine Tools” (IFW, University of Stuttgart) the frequently occurring class of bar-shaped bodies was selected as study object. In this case the thermal bend as reaction to one-sided thermal loads can be considered as one main deformation quantity. For the experimental investigations a thin-walled aluminum column with a height of 1,5 m, a diameter of 0,15 m and a fixed base is used (see Figure 3). A one-sided heat source applies heat energy on a longitudinal profile and creates a bend in the  $(x,z)$  vertical plane.

The investigation of an aluminum column is motivated by its non-stationary temperature distribution which can be assumed as representative for bar-shaped machine elements under dynamic thermal loads and its well known material properties which can be used to evaluate the results of parametric identification (verification step). The main objectives of the modelling process are:

- Development of a parametric model for the non-stationary heat flow
- Parametric identification of relevant material parameters with adaptive KALMAN-filtering
- Modular extension to a parametric deformation model for the precise prediction of the thermal bend (“user function”, see [4]) as precondition for a computational compensation strategy

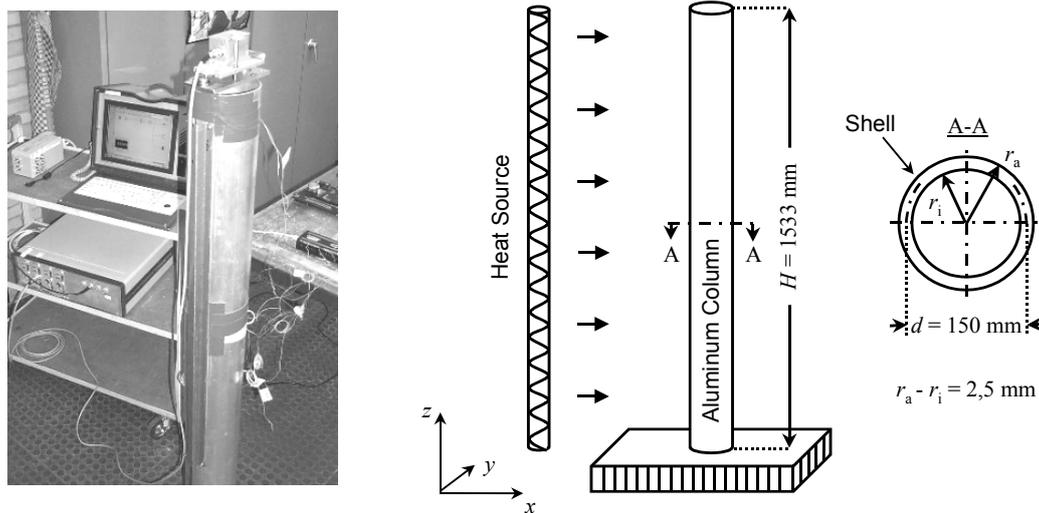


Figure 3: One-sided thermal loading of an aluminium column

### 3. Parametric identification of the non-stationary heat flow model

#### 3.1. Theoretical system analysis

The creation of a parametric dynamic model for the non-stationary heat flow (= “temperature model”) requires the division of the column into vertically arranged finite circular shell segments (see Figure 4a). Inside a single segment the heat flow is assumed

- to be homogenous in its cross section  $A$
- to have a one-dimensional dispersion (see Figure 4b)

Deviations from these assumptions are considered as stochastic model disturbances.

Using FOURIER’s law of heat flow (i.e. [8]) the non-stationary temperature distribution of each segment can be quantified by a homogenous linear partial differential equation (PDE) of 2<sup>nd</sup> order.

$$\begin{aligned}
 \text{(a)} \quad & \frac{\partial T_i}{\partial t} - \alpha \frac{\partial^2 T_i}{\partial \xi^2} = 0 \quad \text{with } \alpha = \frac{\lambda}{\rho c} \quad ; \quad i = 1 \dots s \text{ segments} \\
 \text{(b)} \quad & T_i(\xi, t = t_0) = T_{0,i}(\xi) \quad 0 < \xi < L \quad \text{initial values} \\
 & T_i(\xi = 0, t) = T_i(\xi = L, t) = T_{u,i}(t) \quad t \geq t_0 \quad \text{boundary values}
 \end{aligned} \tag{1}$$

In (1a)  $T_i(\xi, t)$  describes the local and temporal progress of the temperature distribution. The material parameter  $\alpha$  is the thermal diffusivity consisting of thermal conductivity  $\lambda$ , density  $\rho$  and specific heat capacity  $c$ . The specification of the initial and boundary values (1b) with  $T_{0,i}(\xi)$  as “initial temperature distribution” and  $T_{u,i}(t)$  as “boundary temperature” enables the solution of (1a).

The boundary temperatures  $T_{u,i}(t_k)$  are injected by the external heat source and can be assumed as system input quantities  $u_i(t_k)$  (or “correcting variables” in terms of KALMAN-filtering). The continuous temperature distribution  $T_i(\xi, t)$  can be sampled in discrete points  $\xi_j$  and times  $t_k$ . In terms of KALMAN-filtering it represents the temperature state  $\mathbf{x}_{\text{temp},i} = T_i(\xi_j, t_k)$  of a segment.

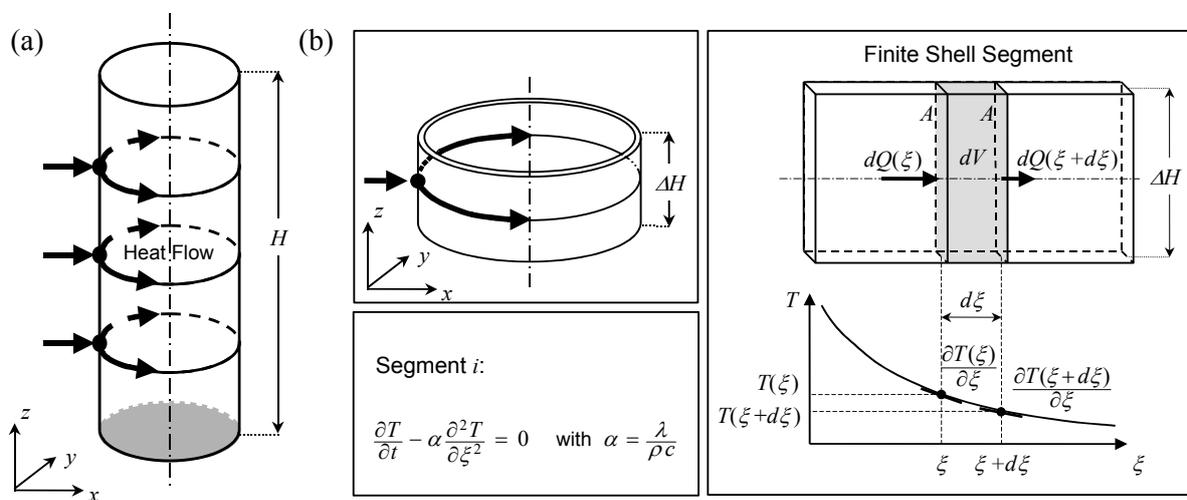


Figure 4: Assumed heat flow of a finite shell segment

### 3.2. Experimental system analysis

The adaptation of the temperature model (1) to reality contains the identification of the thermal diffusivity  $\alpha$  and the temperature state quantities  $\mathbf{x}_{\text{temp},i}$ . It is realized by adaptive KALMAN-filtering.

The structure of the disturbed system equations of the adaptive KALMAN-filter is shown in (3). On the right side the first hypermatrix represents the “extended transition matrix” for  $\mathbf{x}_{\text{temp}}$ , the second matrix the “extended coefficient matrix of correcting variable” for  $u$  (= injected boundary temperature  $T_u$ ) and the third matrix the “extended coefficient matrix of disturbing variables”  $\mathbf{w}$  and  $w_p$ . The stochastic influence of the model disturbances  $\mathbf{w}$  (i.e. convective heat exchange caused by air flow) was evaluated in preliminary investigations and added by “disturbance temperatures”

$$\mathbf{w}_k = (T_{w,i})_k \quad i = 1, 2, \dots, q \quad (2)$$

The quantity  $w_p$  is the stochastic part of a “random walk”-process (i.e. [9]) disturbing  $\alpha$ .

$$\begin{aligned}
 \begin{pmatrix} \frac{\tilde{\mathbf{x}}_{\text{Temp},k+1} - \bar{\mathbf{x}}_{\text{Temp},k+1}}{n,1} \\ \frac{\tilde{\mathbf{x}}_{\text{p},k+1} - \bar{\mathbf{x}}_{\text{p},k+1}}{1,1} \end{pmatrix} &= \begin{pmatrix} \frac{\mathbf{T}_{k+1,k}}{n,n} & \frac{\mathbf{T}_{\text{p},k+1,k}}{n,1} \\ \mathbf{O} & \mathbf{E} \end{pmatrix} \begin{pmatrix} \frac{\tilde{\mathbf{x}}_{\text{Temp},k} - \hat{\mathbf{x}}_{\text{Temp},k}}{n,1} \\ \frac{\tilde{\mathbf{x}}_{\text{p},k} - \hat{\mathbf{x}}_{\text{p},k}}{1,1} \end{pmatrix} \\
 &+ \begin{pmatrix} \frac{\mathbf{B}_{k+1,k}}{n,1} \\ \mathbf{O} \\ \frac{\mathbf{O}}{1,1} \end{pmatrix} \begin{pmatrix} \tilde{u}_k - u_k \\ 1,1 \end{pmatrix} \\
 &+ \begin{pmatrix} \frac{\mathbf{S}_{k+1,k}}{n,q} & \frac{\mathbf{T}_{\text{p},k+1,k}}{n,1} \\ \mathbf{O} & \mathbf{E} \end{pmatrix} \begin{pmatrix} \mathbf{w}_k \\ q,1 \\ \mathbf{w}_{\text{p},k} \\ 1,1 \end{pmatrix}
 \end{aligned} \tag{3}$$

The identification of  $\alpha$  is carried out by a thermal load step and additional temperature measurements (measuring equations of the KALMAN-filter). The measurements are realized by NiCr-Ni-thermocouples. Because of the thin-walled column the radial heat flow can be neglected and surface measurements are suitable for this task. The identification results are shown in Figure 5.

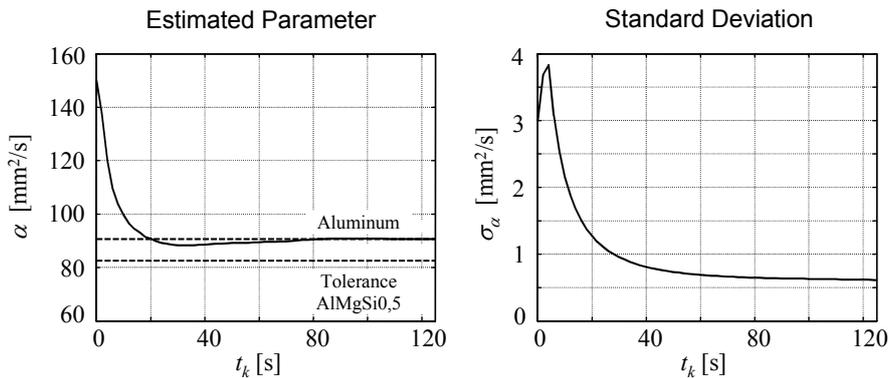


Figure 5: Identification results for thermal diffusivity  $\alpha$

After the identification phase the mean deviation of the estimated parameter is about 0,2 % to the theoretical value of pure aluminum (see [10]). The standard deviation is compatible with this result. Using a spectroscopic material analysis for the first verification step the material is classified as aluminum alloy AlMgSi0,5 [10] with an unknown portion of magnesium and silicon. These uncertainties result in a narrow tolerance band for the possible value of  $\alpha$ . The Filter restricts the band to its upper boundary (see Figure 5).

In the second verification step the capability of the temperature model to predict a realistic spatial and temporal progress of the temperature distribution must be checked. To do this the aluminum column is exposed to thermal loads which are significantly different from the identification phase. Temperature measurements are executed in different sections of the column and compared with the predicted results [11]. The mean deviations (*r.m.s.*-values) are within a range of 0,8 K to 1,4 K and within  $3\sigma$  of the thermocouples. Consequently the temperature model can be assumed to be verified.

#### 4. Modelling of the thermal bend as user function

The extension to a dynamic thermal deformation model is realized by combination of the verified temperature model with a user function. Within this concept the thermal bend is considered to be the system output (see Figure 6).

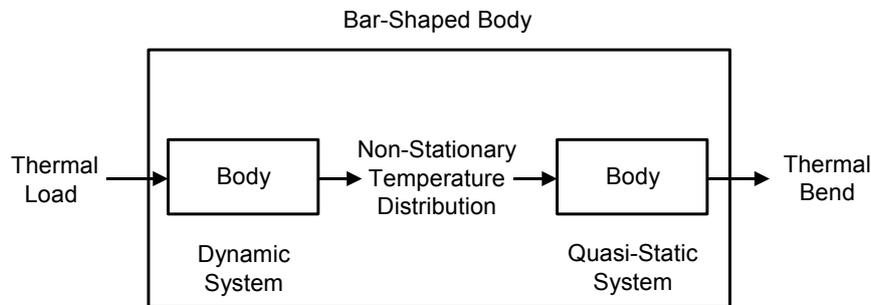


Figure 6: Thermal bend as user function

Neglecting thermal stresses (i.e. [12]) and assuming a circular arc as idealised bending line the inclination  $\varphi$  of the column is a representative geometrical quantity for the thermal bend (see Figure 7b). According to [13] the very simple functional model is given with

$$\varphi(t_k) = \frac{H \alpha_T \Delta T(t_k)}{d} = \frac{H \alpha_T (T_u(t_k) - T(\xi_5, t_k))}{d} \quad (4)$$

In equation (4)  $\alpha_T$  is the temperature extension of aluminum. In this special case the coefficient is assumed to be a priori known. Measuring the progress of the external thermal load  $T_u$  the identified temperature model (1) can be used to predict the spatial and temporal progress of the temperature distribution. Extracting the diametric temperature  $T_5$  (see Figure 7b) the user function (4) can be used to predict the inclination  $\varphi$  as system output.

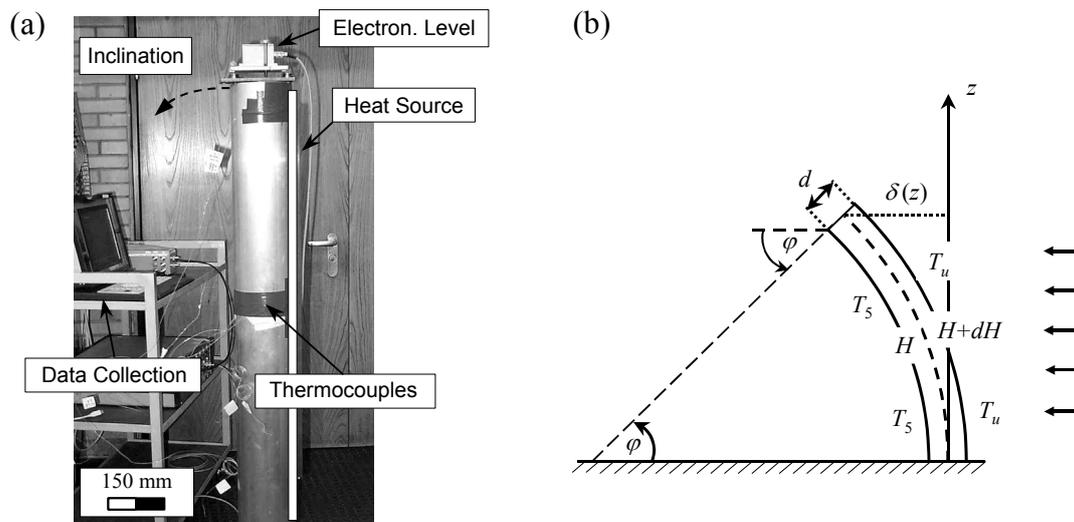


Figure 7: User function: thermal bend of the aluminum column

Figure 7a shows the experimental configuration for the verification of the deformation model. The metrological acquisition of the inclination is realized by a precise electronic level Q-flex 1400 (from HONEYWELL). In Figure 8 the comparison of the predicted (—) and the measured (---) inclination  $\varphi$  is shown for a one-sided thermal load step with a subsequent heating phase of 200 s. The mean deviation can be quantified by a *r.m.s.*-value of about 10 mgon. This result corresponds with a relative error of 2,5 % related to the maximum inclination of the column. Considering computational compensation methods in industrial manufacturing acceptable error tolerances are within a range of about 1–2 %. Consequently these first results can be evaluated as very promising.

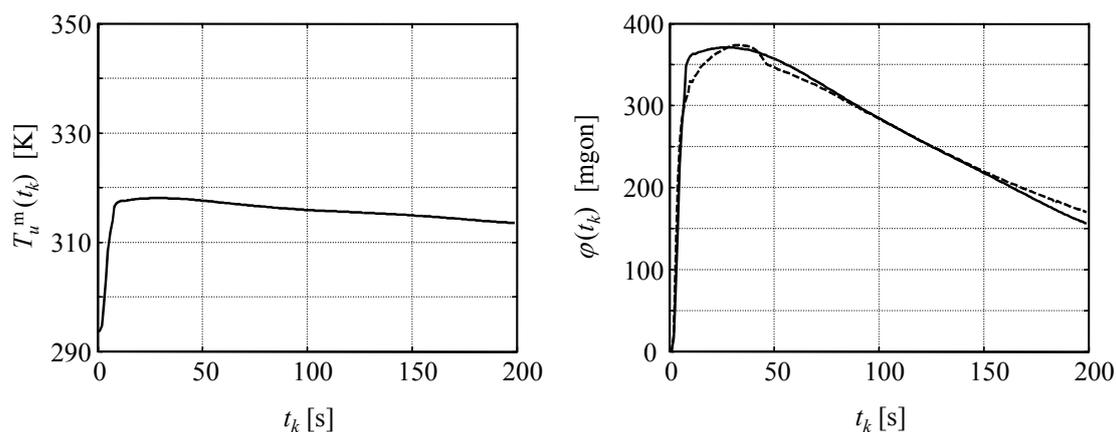


Figure 8: Verification by comparison of predicted and measured inclination  $\varphi$

Figure 9 exemplarily shows the capability of the identified “white box” deformation model to simulate a large variety of different thermal deformation processes (of course only within the range of validity of the physical model).

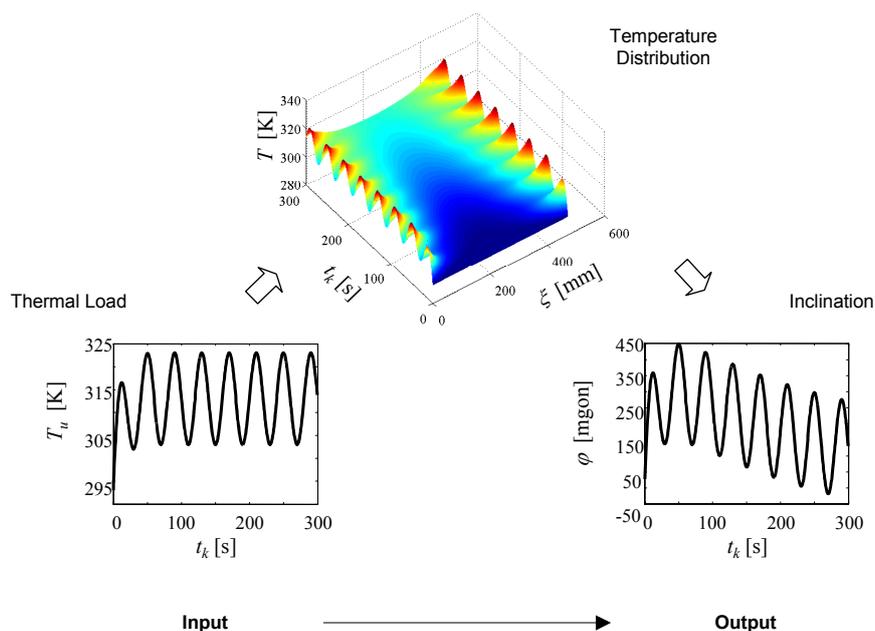


Figure 9: Simulation of the inclination progress under a periodical thermal load

In this case the aluminum column is loaded with a thermal step combined with a periodic progress of the induced heat energy. The calculation of the inclination shows that the column also performs a periodical movement which is superposed to the general equalization process caused by heat adjustment.

## 5. Conclusions

Comparing the predicted and measured thermal induced inclination the efficiency of a dynamic parametric deformation model is demonstrated. One main advantage is the independency of varying thermal loads – of course only within the range of the physical assumptions.

The calibrated deformation model is suitable for realistic simulations. Specifying only the progress of the injected boundary temperatures  $T_{u,i}$  the system output (inclination  $\varphi$ ) can be calculated by the computer model. This allows the investigation of a wide spectrum of thermal influences without additional experiments.

The achieved accuracy in calibration and prediction provides a very promising base for future computational compensation methods in industrial manufacturing. Investigating the influence of the convective heat flow and the distribution of the thermal stresses there's still room for improvement. The adaptive KALMAN-filter is an efficient tool to consider these quantities by deterministic and stochastic methods.

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