Determination of Best Fitting Geoid for Enugu State – Gravimetric Approach

Francis OKEKE, Victor NNAM, Nigeria

Key words: Positioning, Geoid, Gravimetric, Enugu, Gravity, Geodesy.

SUMMARY

In Geodetic and Engineering activities, such geodetic surfaces as the Geoid play a key role in height systems due to the direct relationship that exists between the geoid and the direction of water flow; a major reason why the best fitting local Geoid of every country needs to be accurately determined. An attempt has herein been made to determine a best fitting Geoid for Enugu State, Nigeria using the gravimetric method. A total of two hundred and ninety five (295) stations within the study area were occupied for gravimetric observations. A Hi-Target Differential GNSS system was used on a static mode for the determination of the positions and ellipsoidal heights of the study points and ten (10) Government established control points whose orthometric heights are known were used as reference. A Lacoste and Romberg (G-512) gravimeter was used to measure the gravity values of all the study points. The sling psychrometer was used to measure the Air temperature while the relative humidity, used in correcting the barometric readings was determined from the psychrometric chart. The common corrections needed in a gravity survey such as latitude, tidal, altitude, free air, Bouguer, terrain and drift corrections were all applied. The geoidal quantity N, at a specific control point, P was determined using the classical Stokes formula. The Geoid model achieved was checked at the ten control points established by OSGOF and were found to be consistent to within a RMSE of 2mm (-0.0245m)

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1.0 INTRODUCTION

In recent times, we all have witnessed series of Natural disasters nationally and internationally; these adverse natural occurrences have ravaged physical developments in many parts of the globe and have cost the international community millions of dollars. However, these disasters have raised concerns internationally about sustainable development. Disaster recovery projects require sustainable and reliable geodetic infrastructure which should be based on accurately determined geodetic surfaces; these are the frameworks that enhance the reliability of geographic data for the spatial information management. Orthometric Height, the height of a point above the Geoid is one of the main variables of any spatial information system. This is why it is required for many physical and practical purposes. Orthometric Heights are obtained through the process of Geodetic Levelling; a very tedious, rigorous and expensive method. Global Navigation Satellite System (GNSS) on the other hand, provide an alternative means of determining this quantity (Onwuzuligbo C. U. 2011) although what it directly measures is the ellipsoidal height. With the knowledge of the ellipsoidal height of a point from GNSS observations, the Orthometric height of the same point can be determined if the geoidal undulation of the point is known (Figure 1).

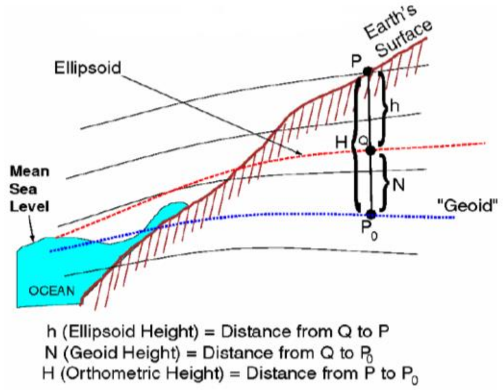

Figure 1: Relationship between the Geoid height, N, the ellipsoidal height, h and the Orthometric height, H. $N = h - H$. *(Source: Ono, M. N. (2009))*

This method of obtaining Orthometric height has been adopted by many nations around the world. This is because the approach is fast, less tedious in difficult terrain and relatively cheap when compared to the geodetic levelling technique.

Depending on data availability and accuracy requirements, there are many principle approaches for determining Geoid models; some of the approaches are gravimetric method, geometric method and the astrogeodetic method (Ono, M. N. (2009).

The gravimetric method is achieved by making gravity measurements and applying the remove-restore (RR) procedure in the determination of a local and regional gravimetric Geoid.

Stoke's classical approach to the determination of gravity field parameters from gravity magnitude observation is based on the solution of the eternal boundary value problem for the disturbing potential "T" generally known as the Brun's formula (Equation 1)

$$N = T/\gamma \qquad (1)$$

Given the stokes formulation (equation 2)

$$N = \frac{R}{4\pi\gamma_m}\int_0^{2\pi}\int_0^{\pi}(\Delta g_0)f(\psi)\sin\psi\delta\psi\delta\alpha \qquad (2)$$

Where ψ = function of the spherical distance between the computation point P and the dummy point Q.

α is the azimuth Q at P

γ_m is a mean value normal gravity and R is a mean Radius

Δg_0 is the mean value of g over the area bounded by $\phi', \phi'+\delta\phi', \lambda'$ and $\lambda'+\delta\lambda'$,

The spectral representation of $\boldsymbol{f(\psi)}$ is as given in equation 3

$$f(\psi) = \sum_2^{\infty}\frac{2n+1}{n-1}Pn(\cos\psi) \qquad (3)$$

Where $\boldsymbol{P_n = }$ **fully – normalized legendre function**

Or

$$\boldsymbol{f(\psi) = \frac{1}{\sin\left(\frac{\psi}{2}\right)} - 6\sin\left(\frac{\psi}{2}\right) + 1 - 5\cos(\psi) - 3\cos(\psi)\ In(\sin\left(\frac{\psi}{2}\right) + sin^2(\frac{\psi}{2}))} \qquad (4)$$

N being small, ψ and α can be treated as if the earth was a sphere, the potential at P of any mass at Q depends only on the distance PQ. Equation 2 therefore changes to (5)

$$N = \frac{R}{2\gamma_m}\int_0^{\pi}(\Delta g_0)f(\psi)\sin\psi\delta\psi \qquad (5)$$

Where:

$$\cos\psi = \sin\varphi\sin\phi' + \cos\phi\cos\phi'\cos(\lambda'-\lambda) \qquad (6)$$

Treating the earth as a sphere rather than a spheroid let Δg_0 and N be expressed as a series of zonal harmonics with P as pole and the earth's centre of mass as the point at which ψ is measured.

2.0 OVERVIEW OF STUDY AREA

Located in South Eastern Nigeria geo-political zone, Enugu state covers an approximate area of about 7161 sq Km and has a population of about 3,267,837 people and density of about 460/km^2 (2006 Census). The centre coordinates of the capital being 6°30′N 7°30′E.

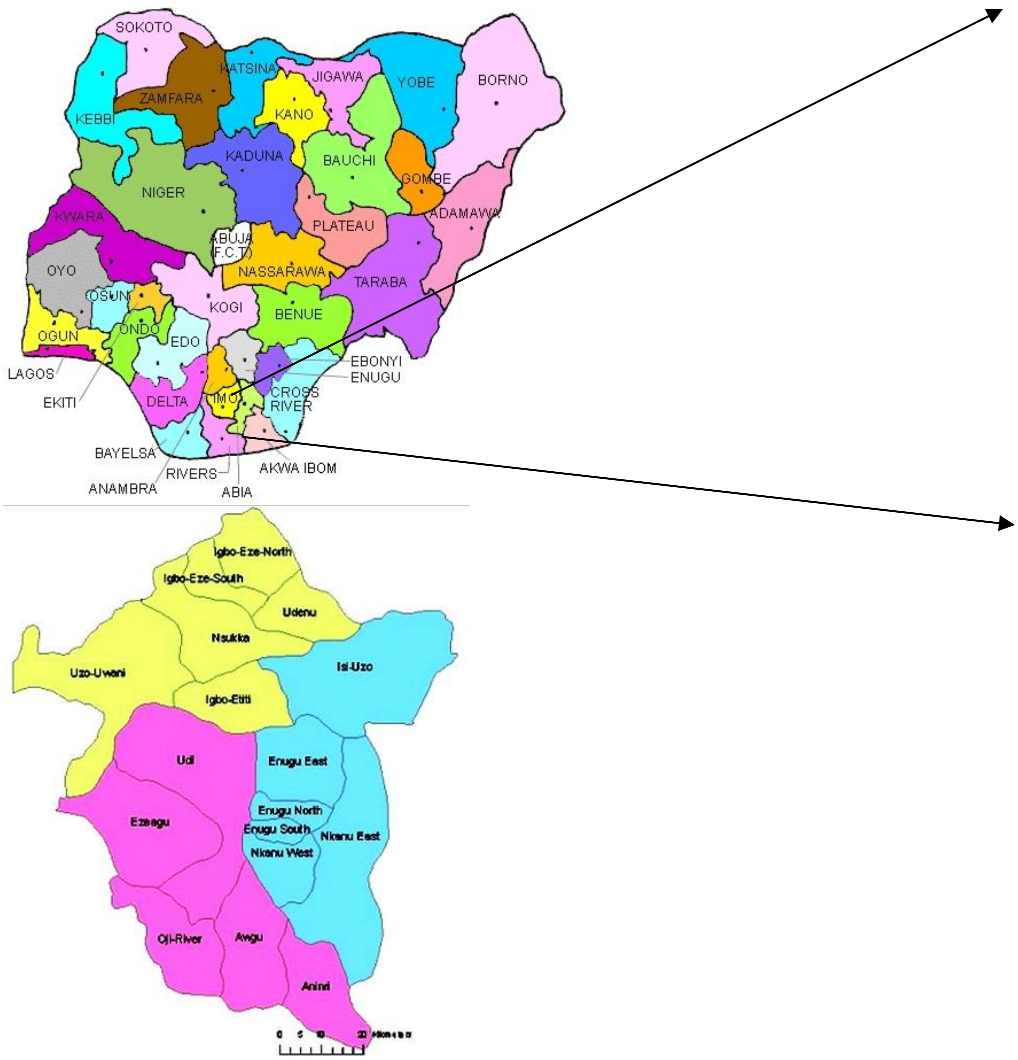

Figure 2: Maps showing Nigeria and an extraction of the study area, Enugu State.

3.0 METHODOLOGY

The gravimetric method of Geoid determination was applied. A total of two hundred and ninety five (295) stations within the study area were occupied for gravimetric observation and most of them are all along accessible roads and tracks. The Gravity measurements were carried out by field party and experts from the National Geological Survey Agency (NGSA), Nigeria. A gravity base station at Nkalagu was used as well as the base station at the General Post-Office Enugu as control. Thirty-six base stations were established during the course of this survey. A Lacoste and Romberg (G-512) gravimeter was used to measure the gravity values of all the two hundred and ninety five (295). A Hi-Target Differential GNSS system was used on a static mode for the determination of the positions and ellipsoidal heights of the points. 10 control points whose orthometric heights are known were used as reference; these points were established by the Office of the Surveyor General of the Federation (OSGOF) by highly accurate spirit levelling before the study. The sling psychrometer was used to measure the Air temperature while the relative humidity, used in correcting the barometric readings was determined from the psychrometric chart. The common corrections needed in a gravity survey such as the corrections for latitude, tide, altitude, free air, Bouguer, terrain and drift corrections were all applied accordingly. Normal gravity on the surface of the reference ellipsoid (WGS84) was computed using the closed form formula given in equation 7 (Meyer et al, 2005) in-order to enhance the computation of gravity anormaly required as major input for evaluating the stokes integral.

$$\gamma = 978032.67714 \left(\frac{1+0.00193185138639 sin^2 \emptyset}{1-0.00669437999013 sin^2 \emptyset}\right) mgals \qquad (7)$$

The geoidal quantity N, at a specific control point, P was determined using the classical Stokes formulae, the Gravity anomaly data obtained from the gravimetric observations was used in the evaluation using the stokes formula in a mat lab program developed for this study. Thus the orthometric heights of the two hundred and ninety five (295) stations were determined.

4.0 DATA USED

Some of the required data for this study include:

1. Gravity data observed by the National Geological Survey Agency (NGSA), Nigeria. These data include absolute gravity and gravity anomaly of each station.
2. The geodetic coordinates (ϕ, λ, h) of existing Government Control points in the area.
3. The 1:5000 topographic map of the study area
4. Parameters of the reference ellipsoid: Angular Unit: Degree (0.017453292); Prime Meridian: Greenwich (0.00000)Φ0; Datum: D_WGS_1984(Minna Datum); Spheroid: WGS_1984; Semimajor Axis: 6378137.0000 b; Semiminor Axis: 6356752.3142451 a; Inverse Flattening: 298.2572235 1/f

5.0 RESULTS

This study was able to achieve results which are shown in this paper. It achieved database of the two hundred and ninety five (295) stations within the study area, their rectangular coordinates, longitude, latitudes, Normal Gravity, Gravity anomaly, Orthometric Heights and Ellipsoidal Heights (Figures 3 – 10).
As a result from a part of an on-going research to determining the hybrid of the study area, the resultant information shall be integrated into the process for completion of the research. The results shall also be submitted to the state and federal authorities.

6.0 ANALYSIS OF RESULTS

The orthometric heights of the known control points and that which was realised from the gravimetric Geoid model were compared. The Standard Deviation is -0.2457m whiles the Root mean square Error (RMSE) is -0.02457m. The results of the statistical analyses show consistency in the results of the study see table 1 below.

Table 1 Comparison of orthometric heights of the known control points and the values realised from the gravimetric Geoid. This is an extract from the main database.

CONTROL ID	Model determined Orthometric Heights (m)	Geometric Orthometric Height(m)	Diff(m)
XSV650	**338.941**	**338.8818**	**-0.0582**
XSV652	**138.269**	**137.5813**	**-0.6877**
XSV649	**152.770**	**152.5444**	**-0.2256**
XSV653	**236.973**	**237.1333**	**0.1603**
XSV655	**439.886**	**439.7824**	**-0.1036**
XSV796	**145.213**	**145.2967**	**0.0837**
XSV653AZ	**239.495**	**239.6504**	**0.1554**
XSV795AZ	**131.398**	**131.4955**	**0.0975**
XSV 909	**91.672**	**91.6841**	**0.0975**
XSV999	**344.685**	**344.9231**	**0.235**
Standard Deviation (SD)			**-0.2457**
Root mean square Error (RMSE)			**-0.02457**

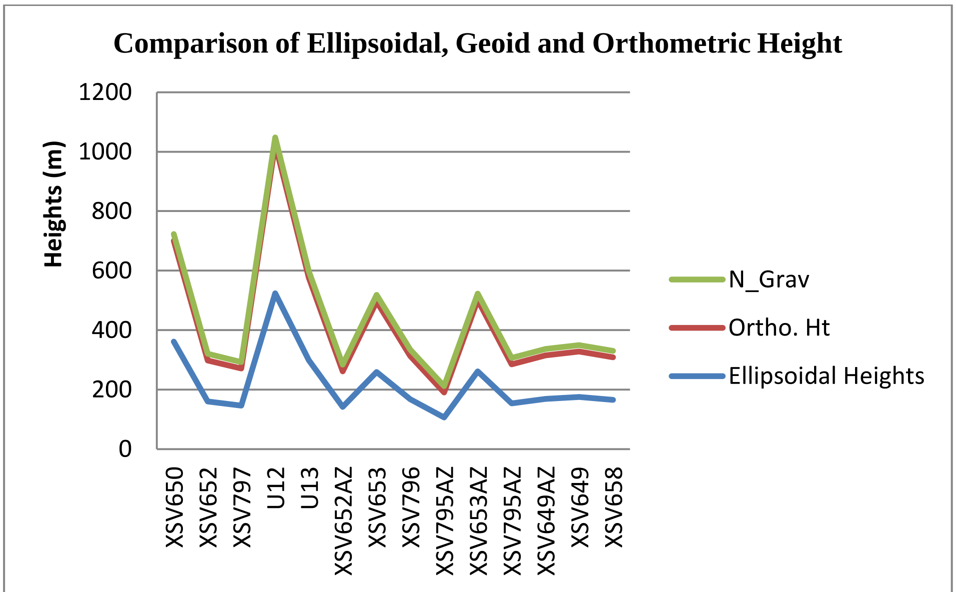

Figure 3: A graphical comparison of the Ellipsoidal, Geoid and Orthometric heights.

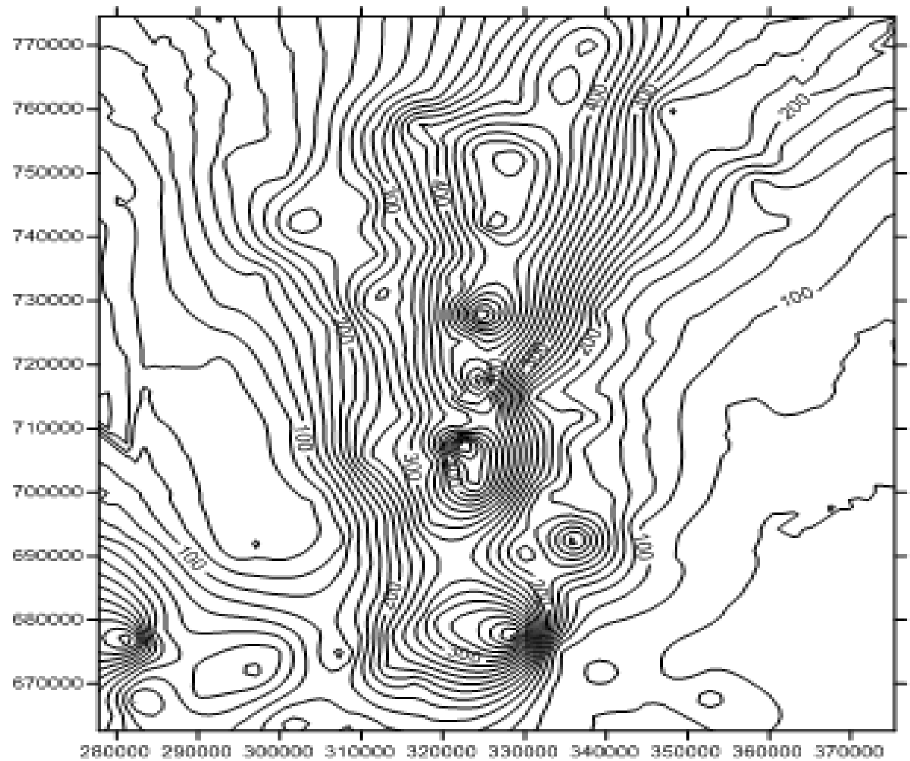

Figure 4: Contour Plot of Ellipsoidal Heights obtained via GNSS post processed static observations within the study area

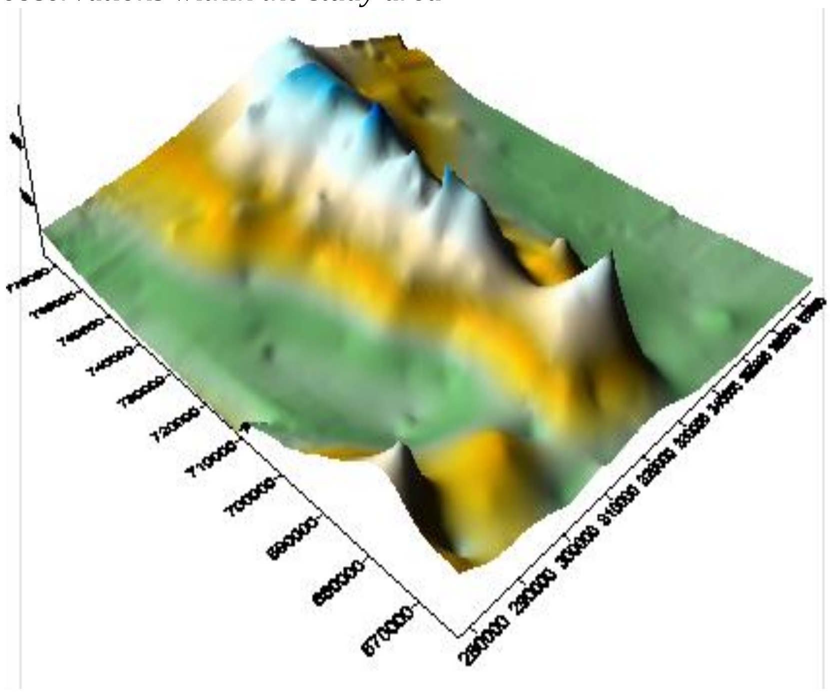

Figure 5: Surface Plot of Ellipsoidal Heights obtained via GNSS.

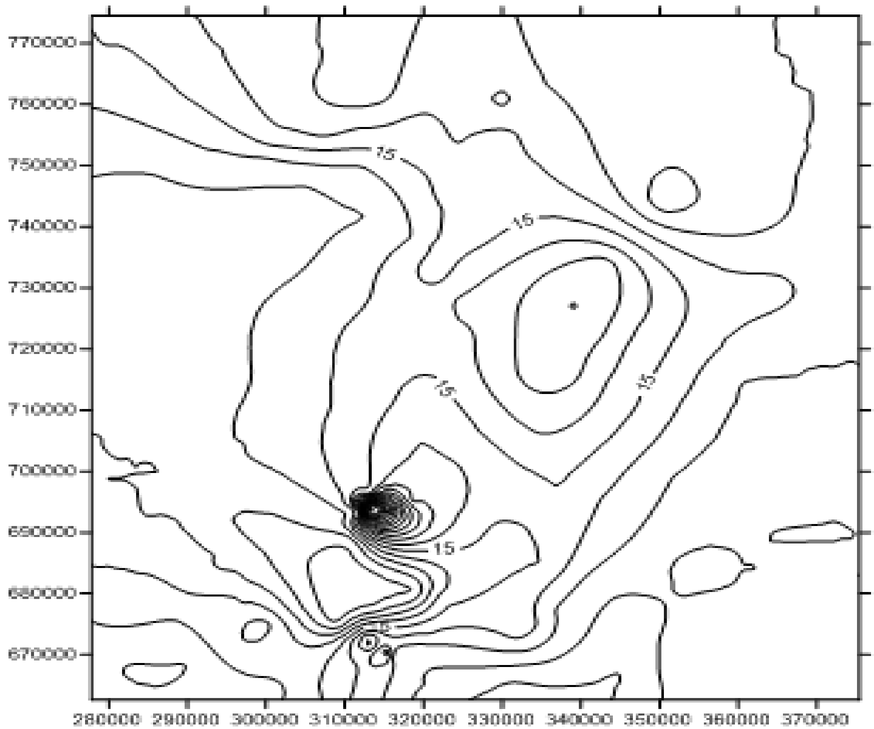

Figure 6: Contour Plot of Gravity Anomaly within the study area.

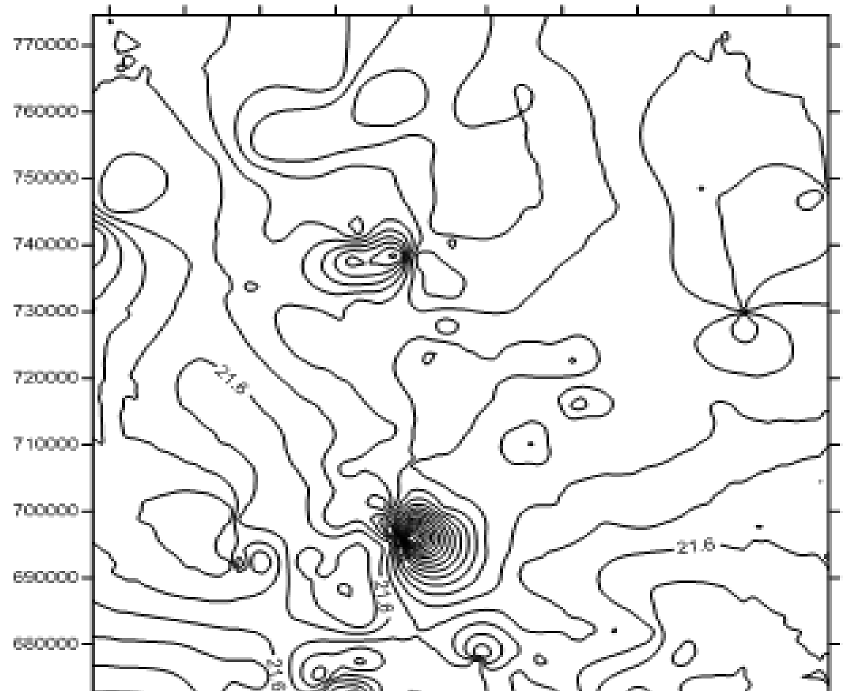

Figure 7: Contour Plot of Gravimetric Geoidal Undulation within the study area

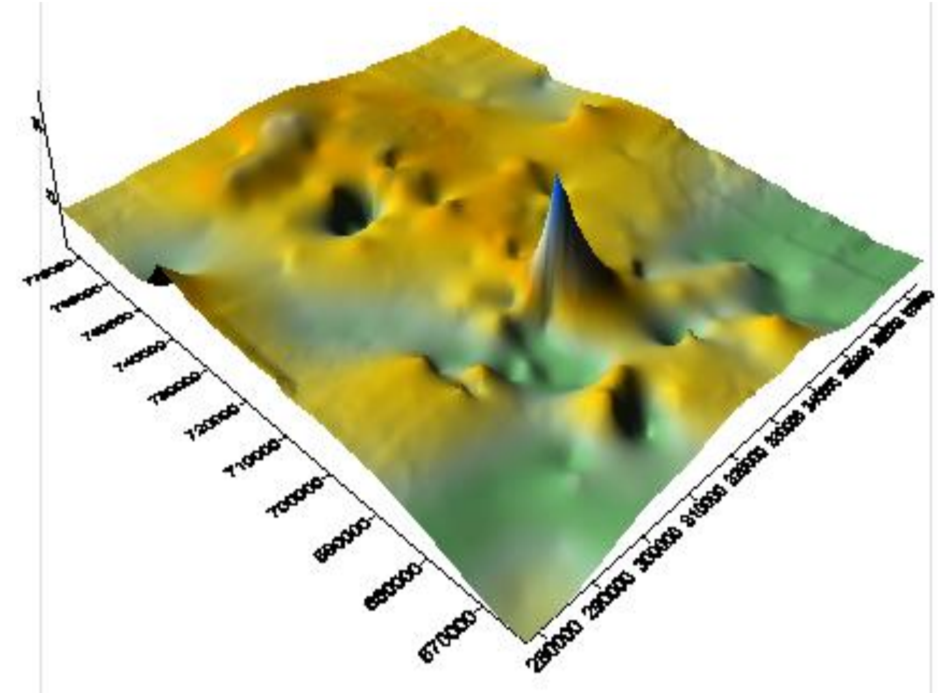

Figure 8: DTM of Gravimetric Geoidal Undulation within the study area

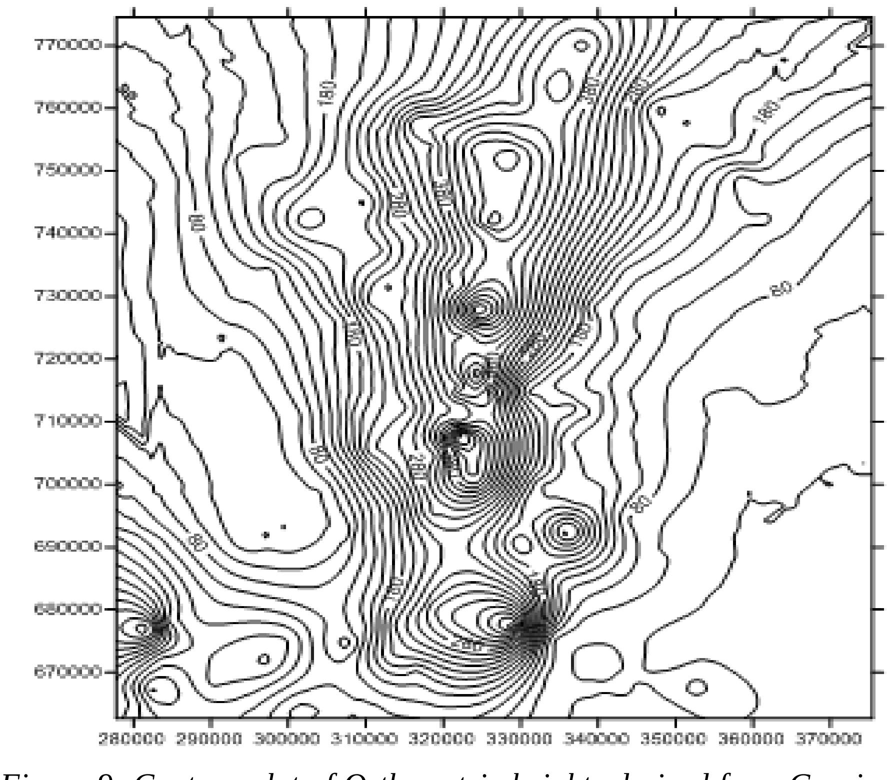

Figure 9: Contour plot of Orthometric heights derived from Gravimetric Geoid model of the study area.

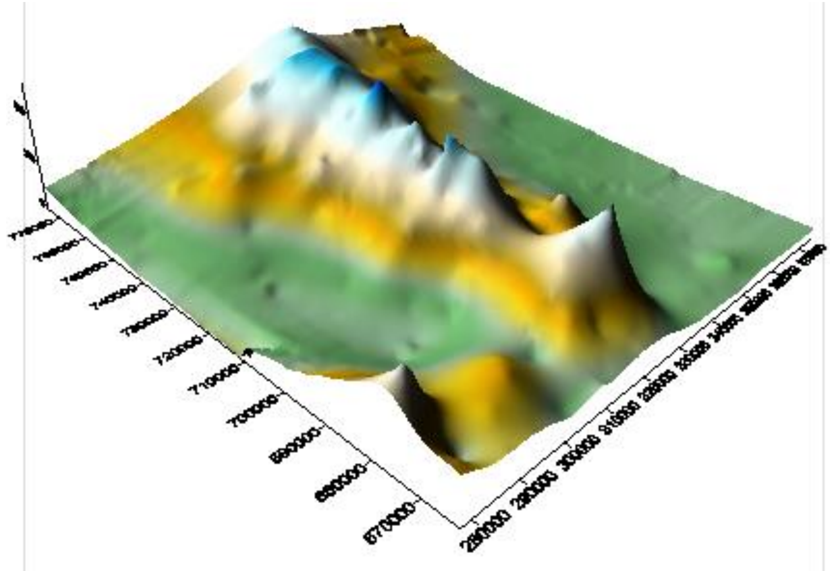

Figure 10: DTM of Orthometric Height of study area.

Figure 3 above shows a graphical comparison of the Ellipsoidal, Geoidal and Orthometric heights In figure 6 above (Gravity anomaly), we observe the inverse relationship between altitudes and normal gravity; that is why areas of higher altitudes like the Udi hills/Ridge have lower normal gravity while areas of lower altitudes have higher normal gravity; in the 3D model, it shows mountains as depressions and valleys as elevations. Figure 4 and 5 above show the contour map and terrain model of the ellipsoidal heights, the result shows consistency with the physical terrain form of the study area. The difference between the orthometric heights (shown in figures 9 and 10) and the ellipsoidal heights (shown in Figures 5 and 6) is the Geoid undulation (shown in figures 7 and 8 above). Figure 8 has a contour interval of 0.1m. The contour interval in figure 4 and 9 are 20m. In figure 10, we can observe that high land/ridge which runs through the centre of map from north to south is the Udi hills which runs from part of Nsukka through Udi and Nkanu and ended in Awgu. These terrain models shown above explains to us the reasons why the Geoid has no direct relationship with the land form and therefore does not get affected by changes on the earth surface.

7.0 CONCLUSION

In order to determine the gravimetric Geoid model of the study area, gravimetric observations were made and the classical stokes formula has been applied to obtain the Geoid undulations which were added to the ellipsoidal heights to get orthometric Heights. Results obtained were compared and a standard deviation of 2.5cm obtained.

The gravimetric method has proven to be a veritable tool for regional geoid modelling even in highly undulating terrains (as is the case within the study area).

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