

The Key Technologies for High-Precision and High-Frequency GNSS Marine Positioning Research

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Key words: GNSS marine positioning, Beidou short message, Tropospheric delay, GNSS/INS integrated navigation

SUMMARY

The 21st century is the century of the ocean. Modern marine activities are rapidly advancing toward a new era of automation. Positioning is the prerequisite for the normal operation of unmanned Surface vessels. However, current GNSS marine precise positioning still faces challenges such as the difficulty in generating atmospheric enhancement information, broadcasting the enhancement information, and achieving robust positioning in the high-dynamic marine environment. To address this, a GNSS marine high-frequency, low-cost precise positioning system based on Beidou short message communication was designed. A theoretical model for non-isotropic tropospheric delay with high spatiotemporal resolution in the marine environment was proposed, breaking through the constraints of isotropic and anisotropic frameworks, and improving the accuracy of marine tropospheric delay estimation from the Decimeter level to the Centimeter level. An innovative high-frequency fusion positioning method based on a single Beidou short message communication was developed, overcoming the limitations of small bandwidth and low frequency of Beidou short messages. This increased the marine positioning frequency from 1/60 Hz to 1 Hz with a single Beidou short message. Additionally, an anti-interference adaptive GNSS/INS integrated navigation model suitable for unmanned vessels was proposed, addressing the issue that the general motion constraint model for vehicles is not applicable to marine platforms in GNSS signal denial environments. This improvement prevented the divergence and poor robustness of marine positioning results, enhancing the positioning accuracy from the meter level to the decimeter level. Based on the aforementioned innovative achievements, a fast, cost-effective, and robust Beidou marine precise positioning hardware and software platform has been established. This platform provides precise positioning technical support for marine search and rescue, as well as the intelligent operation and maintenance of marine ranches, yielding significant socio-economic benefits.

摘要

21 世纪是海洋的世纪。现代海洋活动正加速迈向无人化新纪元。定位是无人艇正常作业的前提。然而，目前 GNSS 海洋精密定位仍面临海洋大气增强信息生成难，增强信息播发难、海洋高动态环境稳健定位难的问题。对此，设计了基于北斗短报文的

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GNSS 海洋高频低成本精密定位系统：提出了海洋高时空分辨率对流层延迟非各向同性理论模型，突破了各向异性和各向同性框架束缚，使海洋对流层延迟估计精度从 **dm** 级提高到 **cm** 级；创新了基于单台北斗短报文通信的海洋高频融合定位方法，突破了北斗短报文带宽小，频率低的限制，将单北斗短报文海洋定位频率从 1/60Hz 提高到 1Hz；提出了适用于无人艇的抗差自适应 GNSSINS 组合导航模型，解决了 GNSS 信号拒止环境中，车辆通用运动约束模型不适用于海上载体，导致海洋定位结果易发散、稳健性差的难题，使定位精度从 **m** 级提高到 **dm** 级。基于上述创新成果，搭建了快速、经济、稳健的北斗海洋精密定位软硬件平台，为海上搜救、海洋牧场智慧化运维的业务工作提供了精密定位技术支撑，取得了显著社会效益。

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1. Introduction

Marine positioning technology provides location services for maritime transportation, resource exploration, and other ocean-related activities, serving as the technical foundation for all marine operations. Global Navigation Satellite Systems (GNSS), represented by the BeiDou Satellite Navigation System (BDS) and GPS, are the most widely used positioning methods. With the advancement of intelligent ships and unmanned vessels, the demand for high-precision GNSS positioning has become increasingly urgent. However, current high-precision marine positioning faces three key challenges: First, the inability to deploy GNSS reference stations at sea makes it difficult to generate atmospheric correction data. Second, the lack of mobile communication networks at sea hinders the dissemination of augmentation information. Third, the highly dynamic marine environment poses difficulties in achieving continuous and stable positioning.

In terms of GNSS augmentation information generation, current technologies for generating orbit and clock corrections are already quite mature (Gu et al., 2023; Hadas & Bosy, 2015). Ionospheric delay compensation has also achieved relatively high accuracy through methods such as dual-frequency observation combinations (Wanninger & Thiemann, 2025), meeting the requirements of routine applications. In contrast, tropospheric delay is significantly affected by variations in meteorological conditions. Existing tropospheric delay estimation techniques can satisfy the demands of most land-based applications under normal meteorological conditions (Hopfield, 1969; Saastamoinen, 1972; Böhm et al., 2006, 2015; Niell, 1996; Landskron & Böhm, 2018). However, in the marine environment, particularly under extreme weather conditions, meteorological factors exhibit strong spatiotemporal variations, and the distribution of water vapor is inhomogeneous. As a result, existing models are unable to accurately characterize the non-isotropic nature of marine tropospheric delay (Xu et al., 2024, 2025).

The absence of communication networks at sea restricts the dissemination of GNSS augmentation information. Currently, high-precision marine navigation and positioning primarily rely on commercial Precise Point Positioning (PPP) services, such as OmniSTAR and Furgo's StarFix, which are costly. The unique short message communication function of China's independently developed BDS offers a solution with Chinese characteristics to address the issue of "communication blind zones" at sea (Ji et al., 2019; Nie et al., 2020; Gu et al., 2022). However, BeiDou short message communication has inherent bottlenecks, including limited channel bandwidth and low transmission frequency. With the advancement of Low Earth Orbit (LEO) satellite navigation augmentation technologies, utilizing LEO satellites to relay or broadcast independent navigation augmentation signals and establishing high-precision, high-availability augmentation services can effectively enhance GNSS performance in terms of accuracy, convergence speed, reliability, and integrity (Di Vruno et al., 2023; Yang et al., 2024).

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Therefore, integrating BDS and LEO to construct a heterogeneous maritime network, investigating efficient compression and coding strategies for both BDS short messages and LEO signals, and achieving seamless BDS-LEO augmented positioning represent a feasible approach to addressing the challenges of high-precision marine positioning, warranting further in-depth exploration and research.

In intelligent navigation systems for ships and unmanned surface vessels, precise positioning constitutes the core component for achieving autonomous decision-making and control. However, in highly dynamic marine environments, the intensity of disturbances such as wind, waves, and currents increases significantly. GNSS antennas are susceptible to being submerged by waves or affected by sea spray, leading to signal quality degradation. Furthermore, in complex scenarios such as bridges, drilling platforms, and sea caves, GNSS signals are prone to obstruction or multipath effects, resulting in reduced positioning accuracy or even complete failure. To enhance system robustness, researchers commonly adopt the strategy of GNSS/Inertial Navigation System (INS) integration (Mao et al., 2022; Zhang et al., 2018). However, due to the inherent cumulative errors of Inertial Measurement Units (IMUs), the system's positioning accuracy degrades rapidly. To mitigate this issue, researchers have proposed major solutions including artificial intelligence, leveraging the vehicle's own motion characteristics, and integrating environmental information to compensate for IMU errors (Zhang et al., 2022, 2023; Xu et al., 2023). Although these methods have improved the navigation stability of vessels in GNSS-denied environments, they remain largely confined to data fusion at the sensor level, and have not yet fully exploited the dynamic coupling mechanism between vessel motion and environmental disturbances. Future research could consider environmental dynamic characteristics as a novel information source, providing more physically meaningful support for robust positioning in complex marine environments.

In conclusion, this paper conducts research on key technologies for BDS and LEO satellite-integrated augmentation to enable high-precision marine positioning. A multimodal classification and estimation model for marine tropospheric delay is constructed based on dynamic variational data assimilation. A maritime heterogeneous network communication and augmented positioning method is developed, integrating BDS short messages with LEO satellites. Additionally, a BDS/INS stable positioning and attitude determination technology is investigated, leveraging wind-wave-current coupled prediction and control-motion dynamics. These contributions collectively provide technical support for the precise navigation and cooperative operations of intelligent, unmanned, or minimally crewed equipment, such as unmanned surface vessels.

2. Theoretical Model for the Non-Isotropic Characteristics of Tropospheric Delay with High Spatiotemporal Resolution over Marine Areas

To address the challenge of low accuracy and slow convergence in real-time tropospheric delay estimation over marine areas, which consequently degrades positioning precision and speed, a theoretical method for estimating marine tropospheric delay with high spatiotemporal resolution considering non-isotropic characteristics is proposed. This method encompasses techniques for the non-isotropic modeling and prediction of slant tropospheric delay, as well as high-precision zenith tropospheric delay estimation considering diurnal variations. It achieves

centimeter-level accuracy in marine tropospheric delay estimation, reducing the convergence time for decimeter-level positioning for maritime users from 30 minutes to 10 minutes.

2.1 Non-Isotropic Modeling and Prediction Technology for Slant Tropospheric Delay

To address the issue that frequent variations in marine meteorological factors cause significant changes in slant tropospheric delay with azimuth, leading to large estimation errors that severely impact marine positioning accuracy and initialization speed, we have broken away from the existing theoretical framework of tropospheric anisotropy and isotropy. We have revealed the non-isotropic nature of tropospheric delay, analyzed its spatiotemporal characteristics, and defined its three manifestations: positive anisotropic, isotropy by undermined, and negative anisotropic, as illustrated in Figure 1.

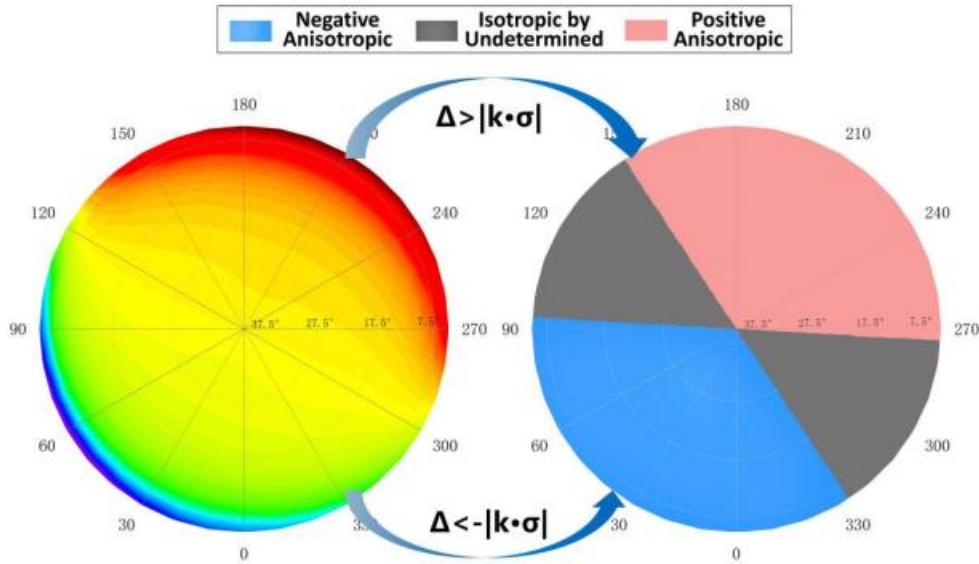


Figure 1. Theoretical model of the non-isotropic tropospheric delay

$$k = \begin{cases} 0.80, |u| \leq e^0 \\ 0.1 \frac{e_0}{|u|} \left(\frac{e_1 - |u|}{e_1 - e_0} \right)^2 + 0.70, e^0 \leq |u| < e^1 \\ 0.70, e^1 \leq |u| \end{cases} \quad (1)$$

Where, $|u| = |\Delta/\sigma|$, $e_0 = 0.5$, $e_1 = 2.0$, This three-segment sliding window function, constructed based on the IGG-III weight function, constitutes a crucial component for determining the thresholds in the non-isotropic classification model. It ensures the continuity of the non-isotropic classification results and provides support for the construction of a slant tropospheric delay estimation and prediction model that accounts for non-isotropic characteristics.

Based on the non-isotropic characteristics of the troposphere, a quadrant-adaptive horizontal gradient model was reconstructed. Utilizing the NARX neural network, a high-resolution ($0.25^\circ \times 0.25^\circ$) non-isotropic slant tropospheric delay model was developed, with a focus on predicting slant tropospheric delays in China's coastal regions under complex marine atmospheric conditions. The model achieves an accuracy of 5 cm, which represents an order of magnitude improvement over the commonly used VMF1 model and the VMF1 model

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augmented with horizontal gradients. This technology can provide technical support for the realization of long-range BDS real-time kinematic positioning. Furthermore, this technology has been integrated into a high-frequency marine fusion positioning method based on single-station BDS short message communication and has been successfully applied to decimeter-level positioning and navigation of vessels, reducing the convergence time for maritime users from 30 minutes to 10 minutes, as illustrated in Figure 2.

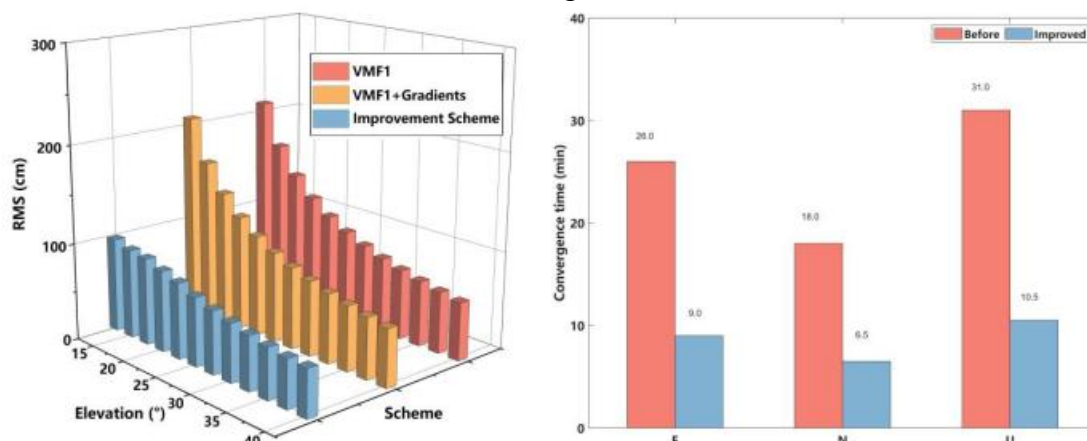


Figure 2. Accuracy of the non-isotropic slant tropospheric delay model and the convergence time after its application in the positioning algorithm

2.2 High-Precision Zenith Tropospheric Delay Estimation Technology Considering Diurnal Variations

To address the limitation that current zenith tropospheric delay models fail to accurately capture the diurnal variations of tropospheric delay in marine areas characterized by abundant and highly variable water vapor content, we utilized high spatiotemporal resolution Numerical Weather Prediction (NWP) ERA5 and ERA5-Land data provided by the European Centre for Medium-Range Weather Forecasts from 2015 to 2018. We proposed a fitting model incorporating annual, semi-annual, and diurnal variations without phase functions to precisely model the tropospheric delay. Consequently, a global meteorological-parameter-free zenith tropospheric delay model considering diurnal variations was developed. The model achieves an estimation accuracy of 3.8 cm. The estimation accuracy for marine zenith tropospheric delay is comparable to that over inland areas and is significantly superior to the commonly used UNB3 (accuracy: 5.1 cm), UNB3m (accuracy: 5.4 cm), and EGNOS models (accuracy: 5.6 cm). It is comparable to the internationally recognized GPT3 model, which has the highest accuracy, while utilizing only one-third of the meteorological data required by the GPT3 model, as illustrated in Figure 3.

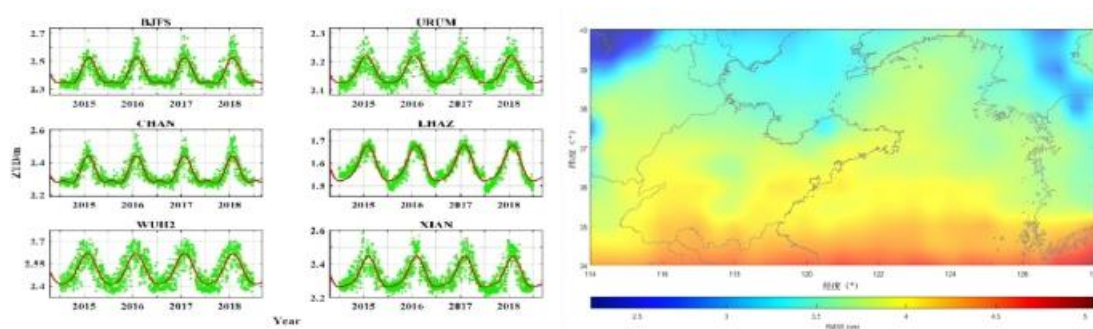


Figure 3. Diurnal variation characteristics and the accuracy of the zenith tropospheric delay estimation model considering diurnal variations

3. High-Frequency Marine Fusion Positioning Method Based on Single-Station BDS Short Message Communication

This paper systematically establishes a high-frequency marine fusion positioning method based on single-station BDS short message communication. The method includes techniques for the simplification, re-encoding, and recovery of multi-mode multi-frequency BDS/GNSS observations and corrections, as well as a fusion positioning approach combining temporal baseline and relative positioning based on NWP and single-station BDS short message equipment. It overcomes the current limitations of low frequency and narrow bandwidth in BDS short messages, which lead to high costs and discontinuous results in marine positioning, achieving continuous marine positioning at the thousand-yuan level based on BDS.

3.1 Simplification, Re-encoding, and Recovery Technology for Multi-Mode Multi-Frequency BDS/GNSS Observations and Corrections

To overcome the limitation of the narrow single-transmission bandwidth of BDS short messages, a simplification method leveraging the temporal correlation of observations from BDS/GNSS reference stations is proposed. This method involves transmitting inter-epoch differential information for pseudorange and phase observations, correction data, Real-Time Service (RTS) data, and orbit and clock data, as illustrated in Figure 4. A simplified compression and encoding scheme was designed to convert International GNSS Service (IGS) real-time ephemeris corrections into equivalent range corrections, significantly reducing the volume of correction data required for multi-mode multi-frequency BDS/GNSS differential positioning. Furthermore, data recovery processing schemes for the user receiver at the first epoch, second epoch, and subsequent epochs were established. This enables the number of satellites for which corrections are broadcast in a single BDS short message to increase from over a dozen to more than a hundred, greatly increasing the number of available satellites for BDS/GNSS marine positioning, enhancing data redundancy, and improving the accuracy of marine positioning results.

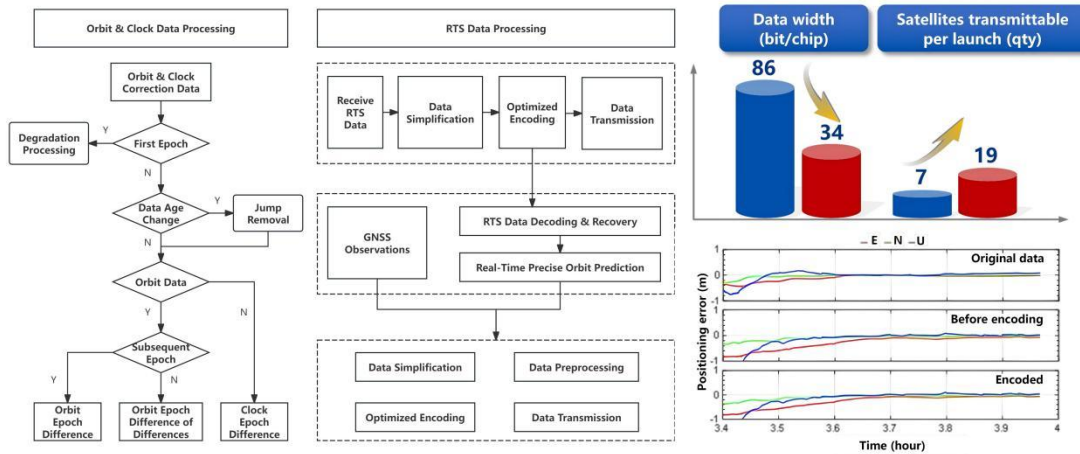


Figure 4. Compression encoding scheme and its effectiveness

3.2 Fusion Positioning Method Combining Temporal Baseline and Relative Positioning Based on NWP and Single-Station BDS Short Message Equipment

To overcome the limitation of the low broadcast frequency of BDS short messages, a fusion method integrating spatial relative positioning and temporal baseline positioning was innovated by leveraging the temporal correlation of base station BDS/GNSS observation data. A relative positioning method for roving stations at integer minutes based on BDS short messages was proposed, enabling high-precision baseline resolution between the base station and roving station at integer-minute intervals. A cumulative temporal baseline positioning method was created to achieve continuous positioning for roving stations at non-integer minute epochs. To address the challenge of rapid accuracy degradation in fusion positioning caused by error accumulation, an atmospheric-augmented temporal baseline precise positioning algorithm based on NWP was developed, incorporating the tropospheric delay prediction model from Innovation Point 1. This algorithm satisfies the requirements of high-dynamic precise positioning applications in open seas, as illustrated in Figure 5.

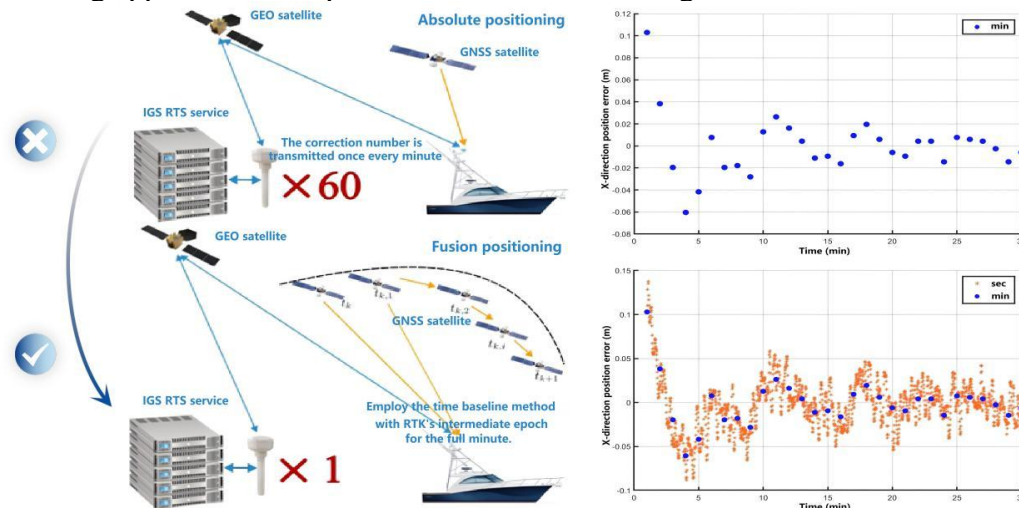


Figure 5. Fusion positioning method and high-frequency precise positioning results

This method reduces the number of short message devices required for continuous single-vessel positioning from 60 units to just 1 unit per operation, lowering the positioning cost per vessel from over one hundred thousand yuan to 3,000 yuan. In 2022, the Shandong Provincial Committee of the Communist Party of China and the Shandong Provincial People's Government issued a notice emphasizing the promotion of installing BDS positioning system terminals on all vessels registered in Shandong. The application of the aforementioned achievements is expected to reduce the cost of vessel retrofitting in Shandong Province by more than one billion yuan.

4. Robust Adaptive GNSS/INS Integrated Navigation Algorithm for Unmanned Surface Vessels

A robust adaptive GNSS/INS integrated navigation and positioning algorithm suitable for unmanned surface vessels is proposed. This algorithm includes a vessel GNSS/INS integrated navigation method based on motion constraints and a robust adaptive filtering algorithm to mitigate the impact of abnormal acceleration. It addresses the challenge of significant errors in the estimation of maritime vehicle motion parameters caused by initial GNSS positioning errors and abnormal acceleration errors in GNSS-denied marine environments, such as under large bridges, near offshore platforms, or inside sea caves. The proposed method improves the GNSS/INS integrated navigation positioning accuracy from the 100-meter level to the 2-meter level.

4.1 Vessel GNSS/INS Integrated Navigation Algorithm Based on Motion Constraints

To address the issue that adding extra sensors in GNSS-denied marine environments increases system redundancy but also raises system cost, a motion constraint algorithm originally applied to land-based wheeled vehicles is introduced for vessels, as illustrated in Figure 6.

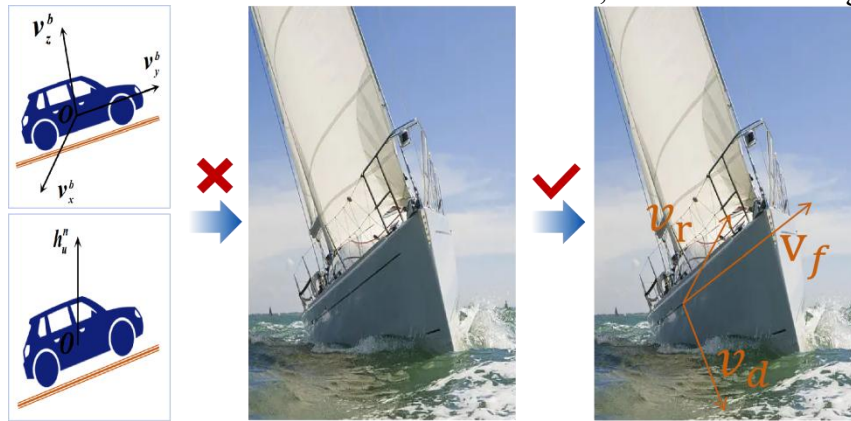


Figure 6. Differences between land-based wheeled vehicle motion and vessel motion

$$\mathbf{v}_{mc}^v = [\mathbf{v}_f^v, \mathbf{v}_r^v, \mathbf{v}_d^v] \quad (2)$$

$$\mathbf{k} = \delta \mathbf{z}_v = \mathbf{v}_{INS}^v - \mathbf{v}_{mc}^v = \mathbf{H}_v \delta \mathbf{x} + \mathbf{n}_v \quad (3)$$

$$\mathbf{k} = \mathbf{H} = [0, \mathbf{C}_b^v \mathbf{C}_n^b, -\mathbf{C}_b^v \mathbf{C}_n^b (\mathbf{v}_{INS}^n \times), -\mathbf{C}_b^v (\mathbf{l}^b), 0, -\mathbf{C}_b^v (\mathbf{l}^b \times) \text{diag}(\mathbf{w}_{ib}^b)] \quad (4)$$

The aforementioned Equation (2) represents the composition of the vessel motion constraint vector. Equation (3) represents the velocity observation vector participating in the

filter update, and Equation (4) represents the corresponding observation matrix composition. In the variables of these equations, the superscripts v , n , and b denote the vessel coordinate system, the navigation coordinate system, and the IMU body coordinate system, respectively. v_{mc}^v represents the velocity under motion constraints in the vessel coordinate system, where v_f^v, v_r^v and v_d^v in v_{mc}^v are the forward, right, and down velocities in the v -frame, respectively. v_{INS}^v is the velocity calculated by the inertial navigation system through mechanization in the v -frame. The velocity observation vector δz_v can be expressed as the difference between the velocity estimated by INS (v_{INS}^v) and the velocity under motion constraints (v_{mc}^v). H_v is the corresponding observation matrix. δx is the state vector of the Kalman filter. C_b^v is the direction cosine matrix for rotating from the vessel coordinate system to the body coordinate system. C_n^b is the direction cosine matrix for rotating from the body coordinate system to the navigation coordinate system. l^b is the projection in the b -frame of the lever arm vector from the IMU measurement center to the origin of the vessel coordinate system. w_{ib}^b is the angular velocity sensed by the gyroscope.

The aforementioned algorithm overcomes the challenge that motion constraint algorithms, due to phenomena such as sideslip, cannot meet the positioning requirements of moving vessels in complex marine environments. For the first time, the optimal design motion constraint conditions and motion constraint parameters for vessels were determined, and a vessel GNSS/INS integrated navigation algorithm based on motion constraints was proposed. Without adding extra sensors, this algorithm significantly improves the positioning accuracy and robustness of vessel GNSS/INS integrated navigation in GNSS-denied environments, enhancing positioning accuracy from the 100-meter level (achieved with unconstrained algorithms) to the 10-meter level.

4.2 Vessel GNSS/INS Integrated Navigation Algorithm Using Robust Adaptive

Filtering to Mitigate the Impact of Abnormal Acceleration

To address the issues that, in GNSS-denied marine environments, motion constraint algorithms cannot adaptively adjust constraint parameters based on the actual motion state of the maritime vehicle, and that outliers inevitably occur in neural network algorithms, a vessel GNSS/INS integrated navigation algorithm using robust adaptive filtering to suppress the influence of abnormal acceleration is proposed. This algorithm utilizes a fitting model trained by a neural network to predict GNSS pseudo-measurements. Simultaneously, it employs the motion constraint algorithm to detect and correct outliers present in the neural network predictions, ensuring the normal update and iteration of the filter. This achieves adaptive filtering regulation for the vessel GNSS/INS integrated navigation algorithm, improving positioning accuracy from the 10-meter level (achieved with the GNSS/INS integrated navigation algorithm augmented by motion constraints) to the 2-meter level, as illustrated in Figure 7.

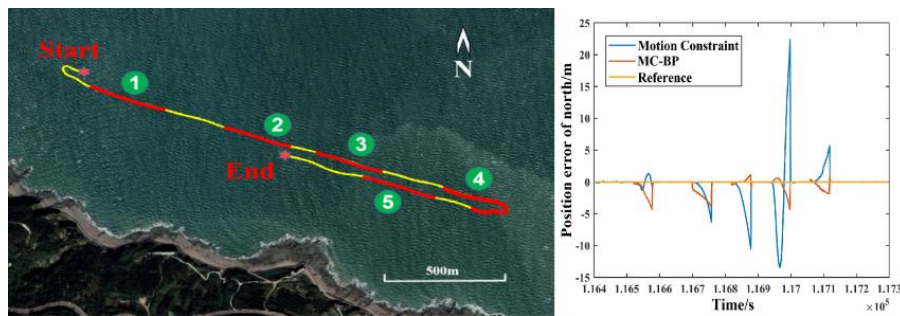


Figure 7. Positioning accuracy

5. Conclusion

Based on the aforementioned innovative achievements—the theoretical model for the non-isotropic characteristics of tropospheric delay with high spatiotemporal resolution over marine areas, the high-frequency marine fusion positioning method based on single-station BDS short message communication, and the robust adaptive GNSS/INS integrated navigation model suitable for unmanned surface vessels—a BDS-based marine precise positioning hardware and software platform has been established. This platform resolves issues related to the high cost and time delays of lifesaving equipment, providing precise positioning technical support for operational tasks such as maritime search and rescue and the intelligent operation and maintenance of marine ranches.

(1) BDS Multifunctional Marine Dynamic Element Monitoring System

To address the current limitations in continuous online marine monitoring capabilities, the singularity of marine environmental monitoring elements, and the long-standing technological lag in domestic marine monitoring equipment, the development and demonstration application of a BDS multifunctional buoy-based marine environment dynamic monitoring system were undertaken. This system achieves high-precision BDS/GNSS buoy-based measurements of tides, currents, and waves, enhancing the security of positioning and communication. To tackle the issues of low accuracy in traditional GNSS position-derived velocity and significant contamination of raw Doppler observations by instrument noise, a Doppler velocity measurement algorithm for BDS buoys based on adaptive robust Kalman filtering was proposed. This algorithm improves velocity measurement accuracy from the cm/s level to the mm/s level under high-dynamic conditions for BDS buoys.

(2) Offshore Platform Safety Protection System and Life Rescue Equipment

Addressing the challenges of long response times, inaccuracy, and high costs in emergency response to current maritime accidents, we have developed an offshore platform safety protection system, emergency search and rescue terminals, and portable strap-down user terminals. This development relies on operationally operational marine dynamic environment forecasts, high-precision atmospheric delay correction product prediction technology, BDS short message-based precise real-time positioning technology for open seas, and vessel GNSS/INS navigation and positioning technology. This enables distress information to be transmitted to a control center within one minute, reducing the cost of distress terminals to under 3,000 yuan. The BDS-based intelligent maritime location services have been successfully applied to a marine search and rescue environmental support service platform, contributing to

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the efficient handling of 21 maritime emergencies, including vessel oil spills and search and rescue operations, significantly enhancing comprehensive maritime emergency management capabilities.

The innovative BDS high-precision marine location service system we have developed integrates high-precision tropospheric delay estimation models, low-cost BDS marine fusion positioning methods, robust adaptive vessel GNSS/INS integrated navigation algorithms, marine environment monitoring, and lifesaving equipment. This forms a precise, rapid, economical, robust, and safe BDS marine positioning system with independent intellectual property rights. It provides precise positioning technical support for operational tasks such as maritime search and rescue and the intelligent operation and maintenance of marine ranches, achieving significant social and economic benefits.

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BIOGRAPHICAL NOTES

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