Investigate of Transition Curves with Lateral Change of Acceleration for Highways Horizontal Geometry

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SUMMARY

Circular horizontal curves and simple superelevation application are used adequately for safety and comfort in design and application of roads without the need of high speed when combining alignments. Nevertheless they are just not appropriate for speed roads.

In the recent years with developments in sciences on account of high speeds on motorways and railways have got better the horizontal geometry of roads with respect to road dynamic. This condition are provided only suitable transition curves. Generally transition curves are used two straight lines with a circle but this conceive not adequate for high speed project and road dynamic. So it was necessary to investigations new solutions.

New transition curves are defined with lateral change of acceleration. This equation can be used all stipulation related to horizontal geometry and vehicle motion. New curve1 and curve 2 (Tari 1, Tari2) and classical transition curve (Clothoid and sinüzoidal) are compared with lateral change of acceleration. In this article known mathematics functions (curvature and superelevation functions) are examined for classical and new transition curve and then developed new programme.

Besides all transition curves which mentioned in this article are examined with diagrams of lateral change of acceleration. New and classical transition curves according to these diagrams are compared and understood that suitable by known criterias. To give an example of new curve 1 (Tari 1) and sinüzoidal curve Programme codes and illustrated with diagrams.
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1. INTRODUCTION

Nowadays design and application of the highways and railways horizontal geometry is very considerable for high speed road projects. In design of the combining alignments using an arc of circular curves not adequate for safety and comfort wherefore effects of instantaneous centrifugal force. However this solution failed as the speed increased and it causes problem related to vehicle dynamic. Transition curves like clothoid and sinüzoid which is the mentioned in this article classical curves are used for better solutions for traveling security and comfort on highways and railways route design.

Transition curves are used for joining a straight lines with a circle (figure1.1) The function of these curves is provide to flexibility the effects of centrifugal force due to the reasons of sudden changes in curvature. Classical curves solve this problem partly. The ideal case regarding vehicle road dynamics is that there is no breaking points on the diagrams of curvature belongs to curved sections and diagrams of curvature and lateral change of acceleration should have contionus diagram. New transition curves; (curve 1 and curve 2) solve this problem exactly for remove to instantaneous effects of centrifugal force and no breaking points on the diagrams of lateral change of acceleration. (Tari 1997) Besides this curves provides that security and travelling comfort for high speed route design in this projects.

In this article known mathematics functions; curvature, superelevation and lateral change of acceleration functions are examined for classical and new transition curve and then developed new programme. Besides all transition curves which mentioned in this article are examined with diagrams of lateral change of acceleration. Value of velocity, curve lengt, curve radius and superelevation are entered by all of the programme users in this new software. New and classical transition curves according to these diagrams are compared and understood that suitable by known criterias. To give an example of new curve 1 (Tari 1) and sinüzoidal curve programme codes and illustrated with diagrams.

2. LATERAL CHANGE OF ACCELERATION

Lateral change of acceleration is very considerable criterion when designed to horizontal geometry for road vehicle dynamic. It is the change of resultant acceleration occurring along the curve normal respect to time. This changes are formed by unbalanced forces, vehicle mass and velocity moving on a curved orbit (Tari 1997, Baykal 1996). Generally lateral change of acceleration is shown as;
\[ z = \frac{da}{dT} n \]  \hspace{1cm} (2.1)

where;

\[ z = \text{lateral change of acceleration} \]
\[ a = \text{resultant acceleration} \]
\[ T = \text{time} \]
\[ n = \text{unit vector along the curve normal} \]

vehicles moving on a superelevated road are the gravitational force \( (P=mg) \), the centrifugal force \( (F=mkv^2) \) and the motor force \( (F_t=ma) \). according to figure 2.1 resultant force \( (R) \) of the gravitational and centrifugal forces can be divided \( D \) and \( F_n \) components. \( D \) component is balanced with the reaction of the road platform.

![Figure 2.1 Forces acting on vehicle moving on superelevated road](image)

\[ F_t = mkv^2 \]  \hspace{1cm} \text{and} \hspace{1cm} \[ F_n = \alpha \cos \beta \tan \alpha \]  \hspace{1cm} (2.2)

\[ \alpha = \frac{u}{b} \]  \hspace{1cm} and  \hspace{1cm} \[ \cos \beta = \frac{b}{\sqrt{u^2 + b^2}} \]  \hspace{1cm} (2.3)

into (2.2)

\[ a_n = \left( kv^2 - g\frac{u}{b} \right) \frac{b}{\sqrt{u^2 + b^2}} \]  \hspace{1cm} (2.4)

can be derived.

Where,
$u$: superelevation (m)
$b$: horizontal width of the road platform (m)
$g$: gravitational acceleration ($9.81 \text{ m/s}^2$)
$k$: curvature of orbiting curve ($1/\text{m}$)
$a_n$: acceleration along the direction of the curve normal on an inclined road platform ($\text{m/s}^2$)

$$
\vec{a} = \frac{dv}{dT} \hat{t} + \frac{b}{\sqrt{u^2 + b^2}} (k\nu^2 - g\frac{u}{b}) n \tag{2.5}
$$

in this equalization $\hat{t}$ is the unit vector along the direction of the tangent of curve. The equation of lateral change of acceleration is obtained as follows;

$$
z = \frac{da}{dT} n = \frac{bv}{\sqrt{u^2 + b^2}} \left( 3ka_t + \nu^2 \frac{dk}{dl} - \frac{k\nu^2 u + gb}{u^2 + b^2} \frac{du}{dl} \right) \tag{2.6}
$$

this equation of lateral change of acceleration can be used for all conditions related to road horizontal geometry and vehicle motion.

### 3. LATERAL CHANGE OF ACCELERATION FUNCTIONS OF CURVES FOR MOTION WITH CONSTANT VELOCITY MODEL

In this study classical curves (clothoid and sinusoidal) and new transition curves (curve1 and curve2) are examined with constant velocity model for vehicle motion. End of the study new and classical transition curves according to these researchs are compared and illustrated with analytical diagrams.

#### 3.1 Investigate to Lateral Change of Acceleration Function of clothoid curve With Constant Velocity Model

Superelevation and curvature function of first clothoid curve can be expressed as,

$$
k_{k,1}(l) = \frac{l}{L_1 R} \tag{3.1.1}
$$

$$
u_{k,1}(l) = \frac{ul}{L_1} \tag{3.1.2}
$$

the function of lateral change of acceleration of the recommended first clothoid curve is derived from (2.6) by taking consideration (3.1.1) ,(3.1.2) and using the transformation of variable of $t=l/L$ and differentiated with respect $l$ as follows; (Tari 1997)

$$
z_{k,1}(t) = \frac{L_1^2 \nu(v^2 - gR\tan\alpha_m)}{L^3 R(t^2 \tan^2 \alpha_m + \frac{L^2}{L^2})^{3/2}} \quad 0 \leq t < \frac{L_1}{L}
$$
where
\[
\tan \alpha_m = \frac{u}{b} \quad (3.1.4)
\]

\(l\): Horizontal length of arc of orbital curve,
\(L\): Total length of arc of orbital curve.

Superelevation and curvature function of the arc of circle can be expressed as,
\[
k_{k,2}(t) = \frac{1}{R} u_{k,2}(t) = u \quad (3.1.5)
\]

the function of lateral change of acceleration of the recommended arc of circle is derived from (2.6) by taking consideration (3.1.5) as follows;
\[
z_{k,2}(t) = 0 \quad \frac{L_1}{L} \leq t < \frac{L_1 + L_2}{L} \quad (3.1.6)
\]

Superelevation and curvature function of second clothoid curve can be expressed as,
\[
k_{k,3}(l) = \frac{1}{RL_3} (L - l) \quad (3.1.7)
\]
\[
u_{k,3} = \frac{u}{L_3} (L - l) \quad (3.1.8)
\]

the function of lateral change of acceleration of the recommended second clothoid curve is derived from (2.6) by taking consideration (3.1.7) , (3.1.8) as follows;
\[
z_{k,3}(t) = -\frac{L_1^2 v^2 - gR \tan \alpha_m}{L^3 R ((1-t)^2 \tan^2 \alpha_m + \frac{L_1^2}{L^2})^{3/2}} \quad \frac{L_1 + L_2}{L} \leq t \leq 1 \quad (3.1.9)
\]
can be derived. (Tar\(ı\) 1997)

Equalization of (3.1.3),(3.1.6) and (3.1.9) are considered then examined with analytical diagram by new programme. (figure 3.1.1) In this new programme;

Horizontal length of transition curve and arc of circle \(L_1=L_2=L_3=600m\)

Superelevation \(u=0.15m\)

Total length of arc of orbital curve. \(L=1800\)

Radius \(R=1850\)

Velocity \(V=230 \text{ km/sa}\)
3.2 Investigate to Lateral Change of Acceleration Function of sinüzoidal curve With Constant Velocity Model

Superelevation and curvature function of first sinüzoidal curve can be expressed as,

\[
k_{s,1}(l) = \frac{l}{R} \left( 1 - \frac{1}{2\pi} \sin \left( \frac{l}{L_1} \right) \right) \]  
\[
u_{s,1}(l) = u \left( 1 - \frac{1}{2\pi} \sin \left( \frac{l}{L_1} \right) \right) \]  

0 ≤ l ≤ L

\[
N_{s,1} = \frac{L_1}{L} \left( v^2 - gR \tan \alpha_m \right) \left( 1 + Q_{s,1}^2 \tan^2 \alpha_m \right)^{3/2} \]  
\[
Q_{s,1} = \frac{L_1}{L} \frac{L_2}{L_1} \sin \left( \frac{L_2}{L_1} \right) \]  
\[
N_{s,1} = \frac{L_1}{L_1} - \frac{1}{2\pi} \cos \left( \frac{L_2}{L_1} \right) \]  

\[
0 ≤ t < \frac{L_1}{L} \]  

\[
N_{s,1} = \frac{L_2}{L_1} \cos \left( \frac{L_2}{L_1} \right) \]

\[
\frac{L_1}{L_1} \cos \left( \frac{L_2}{L_1} \right) \]  

The function of lateral change of acceleration of the recommended first sinüzoidal curve is derived from (2.6) by taking consideration (3.2.1), (3.2.2) and using the transformation of variable of t = l/L and differentiated with respect l as follows; (Tarı 1997)

\[
z_{s,1}(t) = \frac{N_{s,1}v^2 + gR}{R(1 + Q_{s,1}^2 \tan^2 \alpha_m)} \]  

\[
Q_{s,1} = \frac{L_1}{L_1} \frac{L_2}{L_1} \sin \left( \frac{L_2}{L_1} \right) \]  

\[
N_{s,1} = \frac{L_1}{L_1} \cos \left( \frac{L_2}{L_1} \right) \]  

(3.2.3)

(3.2.4)

(3.2.5)

can be derived.

Superelevation and curvature function of the arc of circle can be expressed as,

\[
k_{s,2}(t) = \frac{1}{R} \]  
\[
u_{s,2}(t) = u \]  

(3.2.6)
the function of lateral change of acceleration of the recommended arc of circle is derived from (2.6) by taking consideration (3.2.6) as follows;

\[ z_{s,2}(t) = 0 \quad \frac{L_1}{L} \leq t < \frac{L_1 + L_2}{L} \]  

Superelevation and curvature function of second sinusoidal curve can be expressed as,

\[ k_{s,3}(l) = \frac{1}{R} \left( \frac{L - l}{L_3} - \frac{1}{2\pi} \sin \left( \frac{L - l}{L_3} \right) \right) \]  

\[ u_{s,3}(l) = u \left( \frac{L - l}{L_3} - \frac{1}{2\pi} \sin \left( \frac{L - l}{L_3} \right) \right) \]

the function of lateral change of acceleration of the recommended second clothoid curve is derived from (2.6) by taking consideration (3.2.8) , (3.2.9) as follows;

\[ z_{s,3}(t) = \frac{N_{s,2} v^2 - g R \tan \alpha_m}{R (1 + Q_{s,2} \tan^2 \alpha_m)^{3/2}} \quad \frac{L_1 + L_2}{L} \leq t \leq 1 \]  

\[ Q_{s,2} = \frac{L - L_1}{L_3} - \frac{1}{2\pi} \sin \left( \frac{L - L_1}{L_3} \right) \]  

\[ N_{s,2} = -\frac{1}{L_3} + \frac{1}{L_3} \cos \left( \frac{L - L_1}{L_3} \right) \]  

Figure 3.2.1 Analytic diagram for sinusoidal curve functions with constant velocity model
3.3 Investigate to Lateral Change of Acceleration Function of curve 1 With Constant Velocity Model

Superelevation and curvature function of first curve 1 can be expressed as,

\[ k_{1,1}(l) = \frac{1}{R} \left\{ a \left( \frac{l}{L_1} \right)^5 + b \left( \frac{l}{L_1} \right)^4 + c \left( \frac{l}{L_1} \right) \right\}, \quad u_{1,1}(l) = \frac{ul^3}{L_1^3} \left( \frac{6l^2}{L_1^3} - \frac{15l}{L_1} + 10 \right) \quad 0 \leq l \leq L_1 \] (3.3.1)

the function of lateral change of acceleration of the recommended first sinuzoidal curve is derived from (2.6) by taking consideration (3.3.1) and using the transformation of variable of \( t = \frac{l}{L} \) and differentiated with respect \( l \) as follows; (Tar 1997)

\[ z_{1,1}(t) = \frac{N_{1,1} \alpha_m}{R(1 + Q^2_{1,1} \tan^2 \alpha_m)^{3/2}} \quad 0 \leq t \leq \frac{L_1}{L} \] (3.3.2)

\[ Q_{1,1} = \frac{6L^5 t^5}{L_1^5} - \frac{15L^4 t^4}{L_1^4} + \frac{10L^3 t^3}{L_1^3}, \quad N_{1,1} = \frac{30L^4 t^4}{L_1^5} - \frac{60L^3 t^3}{L_1^4} + \frac{30L^2 t^2}{L_1^3} \] (3.3.3)

can be derived.

Superelevation and curvature function of the arc of circle can be expressed as,

\[ k_{1,2}(t) = \frac{1}{R}, \quad u_{1,2}(t) = u \] (3.3.4)

the function of lateral change of acceleration of the recommended arc of circle is derived from (2.6) by taking consideration (3.3.4) as follows;

\[ z_{1,2}(t) = 0 \quad \frac{L_1}{L} \leq t \leq \frac{L_1 + L_2}{L} \] (3.3.5)

Superelevation and curvature function of second curve1 can be expressed as,

\[ k_{1,3}(l) = \frac{(L - l)^3}{RL_3^3} \left( \frac{6(L - l)^2}{L_3^2} - \frac{15(L - l)}{L_3} + 10 \right) \quad L_1 + L_2 \leq l \leq L_1 + L_2 + L_3 \] (3.3.6)

\[ u_{1,3}(l) = \frac{u(L - l)^3}{L_3^3} \left( \frac{6(L - l)^2}{L_3^2} - \frac{15(L - l)}{L_3} + 10 \right) \] (3.3.7)

the function of lateral change of acceleration of the recommended second clothoid curve is derived from (2.6) by taking consideration (3.3.6), (3.3.7) as follows;
\[ z_{1,3}(t) = \frac{N_{1,2}v(v^2 - gR \tan \alpha_m)}{R(1 + Q_{1,2}^2 \tan^2 \alpha_m)^{3/2}} \quad \frac{L_1 + L_2}{L} \leq t \leq 1 \] (3.3.8)

\[
\begin{align*}
Q_{1,2} &= \frac{6(L - L_t)^5}{L_3^5} - \frac{15(L - L_t)^4}{L_3^4} + \frac{10(L - L_t)}{L_3^3} \\
N_{1,2} &= \frac{30(L - L_t)^4}{L_3^5} - \frac{60(L - L_t)^3}{L_3^4} + \frac{30(L - L_t)}{L_3^3}
\end{align*}
\] (3.3.9, 3.3.10)

can be written (Tarı 1997).

Figure 3.3.1 Analytic diagram for curve1 functions with constant velocity model

3.4 Investigate to Lateral Change of Acceleration Function of combined curve 2 With Constant Velocity Model

Superelevation and curvature function of combined curve 2 can be expressed as,

\[
k_2(t) = \frac{823543}{6912R} (t^7 - 4t^6 + 6t^5 - 4t^4 + t^3) \quad u_2(t) = \frac{823543u}{6912} (t^7 - 4t^6 + 6t^5 - 4t^4 + t^3)
\] (3.4.1)

the function of lateral change of acceleration of the recommended first sinüzoidal curve is derived from (2.6) by taking consideration (3.4.1) and using the transformation of variable of \( t=\frac{l}{L} \) and differentiated with respect \( l \) as follows; (Tarı 1997)

\[
z_2(t) = \frac{N_2v(v^2 - gR \tan \alpha_m)}{LR(1 + Q_2^2 \tan^2 \alpha_m)^{3/2}} \quad 0 \leq t \leq 1
\] (3.4.2)

\[
\begin{align*}
Q_2 &= \frac{823543}{6912} (t^7 - 4t^6 + 6t^5 - 4t^4 + t^3) \\
N_2 &= \frac{823543}{6912} (7t^6 - 24t^5 + 30t^4 - 16t^3 + 3t^2)
\end{align*}
\] (3.4.3, 3.4.4)

can be written.

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4. CONCLUSION

This study and new programme examined that new and classical transition curves according to these diagrams are understood that suitable by known criterias for horizontal geometry of highways and railways. According to criterion 1, diagram of lateral change of acceleration functions of contunity and discontinuties in the form of jump very considerable for travelling comfort. This curves for suitable this criteria excep to clothoid curves (figure 4.1.a) (Tari 1997). According to criterian 2 amplitudes of lateral change of acceleration is very...
considerable. In literature 0.3-0.6m/sec³ values are the maximum values of lateral change of acceleration for travel comfort (Tari 1997). Curve 2 more suitable than Sinüzoidal and curve 1 for criterian 2 (figure 4.1.b). According to criterian 3, break affect and irregular change of the lateral change of acceleration of diagrams at the point where transition curves and arc of circle are combined. Break affect is determined that differences between slope values of two tangents on this diagram at this point (Tari 1997). Because of this, curve 2 is the best curve for travelling comfort and security for high speed highway and railway projects (figure 4.1.c). New programme support this criterians which are mentioned above.

<table>
<thead>
<tr>
<th>Table 4.1</th>
<th>An example of new curve 1 (Tari 1) and sinüzoidal curve programme codes</th>
</tr>
</thead>
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Private Sub Command2_Click()  
    sinüzoidal curve  
    Dim xstart, xfinish, ystart, yfinish, R, V, Lt, L1, L2, L3, g  
    Scale (xstart, ystart)-(xfinish, yfinish)  
    Line (0, ystart)-(0, yfinish), QBColor(12)  
    Line (xstart, 0)-(xfinish, 0), QBColor(12)  
    Dim x2, y2, Q1, N1, Q2, N2  
    For x2 = 0 To 1 Step 0.001  
        Q1 = (Lt * x2 / L1) - (1 / 2 * 3.14 ^ 2 * Sin(Lt * x2 / L1 * 2 * 3.14))  
        N1 = (1 / L1) - (1 / L1 * Cos(Lt * x2 / L1 * 2 * 3.14))  
        Q2 = ((Lt - Lt * x2) / L3) - (1 / 2 - 3.14 * Sin((Lt - Lt * x2) / L3 * 2 * 3.14))  
        N2 = (-1 / L3) + (1 / L3 * Cos((Lt - Lt * x2) / L3 * 2 * 3.14))  
    Select Case x2  
        Case 0 To (L1 / Lt): y2 = (N1 * V * (V ^ 2 - g * R * 0.1)) / (R * (1 + Q1 ^ 2 * 0.1 ^ 2) ^ 1.5)  
        Case (L1 / Lt) To ((L1 + L2) / Lt): y2 = 0  
        Case ((L1 + L2) / Lt) To (Lt / Lt): y2 = (N2 * V * (V ^ 2 - g * R * 0.1)) / (R * (1 + Q2 ^ 2 * 0.1 ^ 2) ^ 1.5)  
        Case Else  
        End Select  
    PSet (x2, y2), QBColor(1)  
    Next  

Private Sub Command3_Click()  
    tari 1  
    Dim xstart, xfinish, ystart, yfinish, R, V, Lt, L1, L2, L3, g  
    Scale (xstart, ystart)-(xfinish, yfinish)  
    Line (0, ystart)-(0, yfinish), QBColor(12)  
    Line (xstart, 0)-(xfinish, 0), QBColor(12)  
    Dim x3, y3, Q3, N3, Q4, N4  
    For x3 = 0 To 1 Step 0.004  
        Q3 = ((6 * Lt ^ 5 * x3 ^ 5) / L1 ^ 5) - ((15 * Lt ^ 4 * x3 ^ 4) / L1 ^ 4) + ((10 * Lt ^ 3 * x3 ^ 3) / L1 ^ 3)  
        N3 = ((30 * Lt ^ 4 * x3 ^ 4) / L1 ^ 5) - ((60 * Lt ^ 3 * x3 ^ 3) / L1 ^ 4) + ((30 * Lt ^ 2 * x3 ^ 2) / L1 ^ 3)  
        Q4 = ((6 * (Lt - Lt * x3) ^ 5) / (L3 ^ 5)) - ((15 * (Lt - Lt * x3) ^ 4) / (L3 ^ 4)) + ((10 * (Lt - Lt * x3) ^ 3) / (L3 ^ 3))  
        N4 = ((30 * (Lt - Lt * x3) ^ 4) / (L3 ^ 5)) + ((60 * (Lt - Lt * x3) ^ 3) / (L3 ^ 4)) - ((30 * (Lt - Lt * x3) ^ 2) / (L3 ^ 3))  
    Select Case x3  
        Case 0 To (L1 / Lt): y3 = (N3 * V * (V ^ 2 - g * R * 0.1)) / (R * (1 + Q3 ^ 2 * 0.1 ^ 2) ^ 1.5)  
        Case (L1 / Lt) To ((L1 + L2) / Lt): y3 = 0  
        Case ((L1 + L2) / Lt) To (Lt / Lt): y3 = (N4 * V * (V ^ 2 - g * R * 0.1)) / (R * (1 + Q4 ^ 2 * 0.1 ^ 2) ^ 1.5)  
        Case Else  
        End Select  
    PSet (x3, y3), QBColor(4)
REFERENCES


BIOGRAPHICAL NOTES

Eray Can graduated from the Department of Geodesy and Photogrammetry Engineering at Zonguldak Karaelmas University in 1999. He received his Msc degree with the thesis about Highway horizontal geometry in 2005. He is PhD student and currently study on his PhD thesis in this university. His research interests are engineering surveying, highways and railways horizontal geometry. He is research assistant Department of Geodesy and Photogrammetry Engineering at Zonguldak Karaelmas University, Turkey.

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