ACCURACY ASPECTS OF PROCESSING AND FILTERING OF MULTIBEAM DATA: GRID modeling VERSUS TIN BASED modeling
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1. Introduction

Requirements for hydrographical Digital Terrain modeling processing:

1. “Fast” modeling (real-time and/or post-processing)
2. Allow “Editing” (manual and/or automatic)
3. Give the option of “Intelligent” filtering (reduction) of data
4. “Accurate” volume computation => “accountability”
   => GRID and TIN (triangular irregular network) modeling
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2. Grid modeling: Principle

- Multibeam =>
  - Equidistant coordinates in international grid system
    - UTM
  - Conventional reference plane (or GRS80 ellipsoid)
- Output = equidistant grid data =>
  - Store only Depth values (typically 2 byte/point: 65536 depth values)
  - Grid interval distance is decisive parameter
2. Grid modeling: Filtering: Why?

- Huge amount of points (e.g. Kongsberg EM3002)
  - 40 Hz
  - 500 pts./swap
  - 20,000 pts./sec. or 72 million pts./hour

+ Interpolation by multibeam system software to equidistant grid
  => More or Less points depending on grid interval distance

2. Grid modeling: Filtering: How?

- **Modify grid interval** distance
  
  e.g. 1 by 1 m => 5 by 5 m

  Reduction by 96%

  Loss of resolution can cause loss of seabottom details

- **Use of “smarter” algorithms**
  
  - Depth is weighted average of all depths of initial cells
    - Weighting factor = inverse distance to power n (2?)
  
  - Minimum depth (control survey, not for volume computat.)
  
  - Use model with variable grid intervals => complex
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3. TIN based modeling: principle

- What is a triangulation?
- Why Delaunay is the best triangulation?
- Property of a Delaunay triangulation
- Different Algorithms
What is a triangulation (TIN)?

- network of irregular triangles, created by connecting the points (vertices) of a dataset so that
  - no triangle sides are intersecting
  - no triangles are superposed
  - the union of all triangles fill up the hull of the triangulation

Why Delaunay triangulation?

Advantages:
- Mathematically well defined
- Unique for a given dataset
- Data-sequence independent
- Independent control possible
- Variable density

Drawbacks:
- Complexity by storing points (E,N,H) and triangles (<> grid)
Property of Delaunay triangulation

For each triangle, the circumscribing circle does not contain any other vertex.
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Circumscribing rule is equivalent to the Min-max rule of Lawson.
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Local optimisation leads to global optimisation

Delaunay-triangulation-algorithms
- Incremental
- Divide-and-conquer
- Sweepline
- Giftwrapping
3. TIN based modeling: Filtering

**Greedy insertion**
- Start situation = convex hull of triangulation
- Selective adding by using a rule (Min. Diff. in Depth or Vol.)

**Vertex decimation**
- Start situation = complete Delaunay triangulation
- Then selective elimination of points
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4. Grid modeling: Advantages

Grid model is more easy to implement (than TIN)

- Higher processing speed
- Higher visualisation speed
  => Algorithms can be raster based instead of vector based
- Real-time modeling
- Real-time editing
- Higher software developing speed => lower cost
4. Grid modeling: Drawbacks

Accuracy?

- **Loss** of the initial measured points

- Choice of **grid interval** distance is of capital importance
  - Too small $\Rightarrow$ huge amounts of redundant data
  - Too big $\Rightarrow$ loss of details

- **Variable grid interval model** could solve this, but at the cost of complexity, computer memory and processing time!

4. TIN based modeling: Advantages and drawbacks

- **Original measured points** are kept
- **No interpolated points**
- **Adaptive** model
  - Locally higher point density $\Rightarrow$ smaller triangles $\Rightarrow$ more details
  - Locally lower point density $\Rightarrow$ big triangles $\Rightarrow$ saving computer memory
- **More complex** model
  - Higher computer memory requirements
  - Slower in processing
  - Algorithms difficult to implement
4. Grid versus TIN

5. Accuracy Aspects of TIN models

- How to compute a volume in a TIN?
- Standard deviation ($\sigma$) of the computed volume
- Mathematical «best» and «worst» $\sigma$ case
- Border Effects
- Example
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How to compute a volume in a TIN?

With $A_j$ as the planimetric surface of a triangle $j$, $f_{ref}$ as the height of the horizontal reference plane and $f_i$ as the elevation of the 3 vertices $i$ of the triangle, the volume $V_j$ generated by one triangle $j$ is equal to

$$\frac{1}{3}(f_1 + f_2 + f_3 - f_{ref})A_j = V_j$$
How to compute a volume in a TIN?

The total volume $V$ is the sum of the volumes of all individual prisms, thus

$$
\frac{1}{3} \sum_i f_i \left( \sum_{j \in A_i} A_j \right) - f_{ref} A_{tot} = V
$$

If we call $B_i$ the sum of the surfaces of all triangles with point $i$ as vertex or

$$
B_i = \left( \sum_{j \in A_i} A_j \right)
$$

Then we can write

$$
\frac{1}{3} \sum_{i=1}^{n} f_i B_i - f_{ref} A_{tot} = V
$$

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Assuming that all $f_i$ are independent, the variance of the volume can be found

\[
\frac{1}{3} \sum_{i=1}^{n} f_i B_i - f_{ref} A_{tot} = V \quad \Rightarrow \quad Var(V) = \frac{1}{9} \sum_i Var(f_i) B_i^2
\]

The standard deviation and variance $Var(f_i)$ of the elevation of a point is usually assumed to be constant so that, with $n$ the total number of points

\[
Var(V) = \frac{Var(f)}{9} \sum_i B_i^2
\]

\[
\sigma(V) = \frac{\sigma(f)}{9} \sum_{i=1}^{n} B_i^2
\]

This form is useful in the case of a TIN model based on non-equidistant points.

\[
\sigma(V) = \frac{1}{\sqrt{n}} \sigma(f) A_{tot} \sqrt{1+\left(\frac{\sigma(B)}{B}\right)^2}
\]

The latter form is applicable to TIN’s of irregular spaced points but is also particularly suited in the case of a TIN model based on equidistant points.

**TIN with regular spaced points**

\[
\sigma(V) = \frac{1}{\sqrt{n}} \sigma(f) A_{tot} \sqrt{1+\left(\frac{\sigma(B)}{B}\right)^2}
\]

Assuming a TIN of regular spaced points, and without the consideration of border issues, a minimum of the standard deviation can be found for a layout where all rectangular cells of the TIN have an identical direction of the diagonal. In this case, every non-border point has 6 neighboring triangles and as all triangles have the same surface, $\sigma(B) = 0$, and it can be found that

\[
\sigma(V) = \frac{1}{\sqrt{n}} \sigma(f) A_{tot}
\]
5. Accuracy Aspects of TIN models

• How to compute a volume in a TIN?

• Standard deviation (σ) of the computed volume

• Mathematical « best » and « worst » σ case

• Border Effects

• Example
5. Accuracy Aspects of TIN models

- How to compute a volume in a TIN?
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## 5. Accuracy Aspects of TIN models

- How to compute a volume in a TIN?
- Standard deviation (σ) of the computed volume
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- Example
The following small data set with 5 irregular spaced points given in (E, N, H) is considered:

<table>
<thead>
<tr>
<th>E</th>
<th>N</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1: coordinates of the example

A standard deviation of 0.5 for each of the height values is given.

The volume between a zero-level reference and this surface is easily computed as: $1/3 \times \text{planimetric surface} \times \text{prism height}$. In this example, where the prism height is equal to 6, the prism volume = $1/3 \times 5 \times 5 \times 6 = 50$. 

Example: volume
Example: $\sigma(Vol)$

The variance of the volume is $\text{Var}(V) = \frac{1}{5} \sum \text{Var}(f_i) B_i^2$. $\text{Var}(f_i)$ is computed as the square of the standard deviation of the heights or $0.5^2=0.25$. The five different $B_i$ for point 1, 2, ..., 5 are

- $B_1 = 0.5 \times 5 \times 1 + 0.5 \times 5 \times 3 = 10$
- $B_2 = 0.5 \times 5 \times 3 + 0.5 \times 5 \times 4 = 17.5$
- $B_3 = 0.5 \times 5 \times 4 + 0.5 \times 5 \times 2 = 15$
- $B_4 = 0.5 \times 5 \times 2 + 0.5 \times 5 \times 1 = 7.5$
- $B_5 = 0.5 \times 5 \times 1 + 0.5 \times 5 \times 3 + 0.5 \times 5 \times 4 + 0.5 \times 5 \times 2 = 25$

As a check, the sum of the $B_i$ is always 3 times the total surface ($75 = 25 \times 3$).
The sum of the $B_i^2$ is $10^2 + 17.5^2 + 15^2 + 7.5^2 + 25 = 1312.5$.
Hence the $\text{Var}(V)$ is $0.25 \times 1312.5 / 9 = 36.458$.
The standard deviation for the volume of 50 is the root of 36.458 or approx. 6.038.
Thus, the volume between the zero-level and the prism surface is $50 \pm 6.038$.

Example: alternatives

If point 5 had been the central point with coordinates (2.5, 2.5, 6), the volume would have been the same. $B_1$, $B_2$, $B_3$, and $B_4$ would be all equal to $0.5 \times 5 \times 2.5 + 0.5 \times 5 \times 2.5 = 12.5$, $B_5$ being equal to 25. $\text{Var}(V)$ is then $0.25 \times (4 \times 12.5^2 + 25^2) / 9 = 34.722$ and the standard deviation is now slightly reduced to 5.893 (instead of 6.038).

The theoretical "lower limit case" would yield $\sigma_{\text{min}}(V) = \text{sqrt}(0.25 \times 25^2 / 5) = 5.590$.
The theoretical "upper limit case" would yield $\sigma_{\text{max}}(V) = \text{sqrt}(0.25 \times 25^2 / 3) = 7.217$.

However, these two latter cases are purely hypothetical cases, both layouts being geometrically impossible.

**Conclusion:** $\sigma$ spread (< 10%) is small: "Best case" is good approximation and easy to compute!
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- Hydrographic impose specific requirements to the processing
- Multibeam or homogeneous data coverage => Grid modeling
  - Straightforward (easy implementation) => faster
  - Less flexible (fixed grid interval distance)
- Singlebeam or non-homogeneous data coverage => TIN
  - More complex (more difficult implementation) => slower
  - Flexible (variable triangle size)
- Accuracy of TIN Volume
  - Fast $\sigma$ approximation
  - Accurate $\sigma$ computation
  - Border effects are neglectable
Next meeting point:

HYDRO 2012, Rotterdam, 12-15 November 2012
Organised by the Hydrographic Society Benelux

www.hydro12.com
www.hydrographicsocietybenelux.eu

Acknowledgments

The Institute for the Promotion of Innovation by Science and Technology in Flanders (IWT) funded project n° IWT990159 ‘Survey System for Dredging’ (1999-2002) with
- Ghent University, Geography Department, as scientific partner.
- DEME, Survey Department as private partner.
- The present fundamental research fits in the larger, international Eureka project «Dredging Survey 2000 (EU203511)».

Thanks to Gert Brouns who carried out during 18 months research work concerning the editing of triangulation models.

Financial support from BOF/GOA 01GA0405, funded by Ghent University, for the research work of Denis Constales.
References (1/2)


References (2/2)


Thank you for your attention!

Questions?